Physics 7314/7360
Homework 8 25 Oct 2007
Due Thurs, Nov 1

1. Read in A&H: the rest of Ch 8, Appendices J and L, and 9.1 through 9.3.

   True/False: I read this material.

2. AH 7.7

3. AH 7.8 (for u only, not v)

4. Show that the combinations $\hat{x} \pm i\hat{y}$ rotate around the $\hat{z}$ axis in the same way as states $|j, m\rangle$ with $j = 1$ and $m = \pm 1$. (This is simple; use what you know about the rotation matrix. This exercise is useful for understanding helicity in the following problem.)

5. The Lagrangian for a free, massive, vector boson is

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} M^2 A^\mu A_\mu \,.$$

   Using the Euler-Lagrange equations discussed in lecture,

   (a) Find the three independent solutions in momentum space for a free particle, $\epsilon^\mu(\mathbf{k}, \lambda) \exp(-ik \cdot x)$, in a frame in which $\mathbf{k}$ is along the z axis. Choose the ϵ^μ vectors to be orthonormal,

$$\epsilon(\mathbf{k}, \lambda)^\mu \, \epsilon(\mathbf{k}, \lambda')^*_\mu = -\delta_{\lambda,\lambda'}$$

   and with definite helicity.

   (b) Give the form of these solutions in the particle's rest frame, and in the limit where the magnitude of $\mathbf{k}$ is large.

   (c) For a massless field ($M = 0$), this becomes a gauge field. The equation

$$\partial_\mu A^\mu = 0$$

   is no longer an equation of motion, but we may impose it as a gauge condition ("Lorentz gauge"). We may also impose $A^0 = 0$ (cf AH section 7.3). There are now only two independent solutions. Give these in a frame in which $\mathbf{k}$ is along the z axis (again, with definite helicity).

6. Optional: Add one non-renormalizable interaction to the $\lambda\phi^4$ theory with a coupling constant of order $1/\Lambda^2$. Show, using a few one-loop diagrams and simple power-counting (and with Λ as the cutoff) that it contributes corrections to diagrams that are only of order $1/\Lambda^2$, in spite of large loop contributions which go as positive powers of Λ.