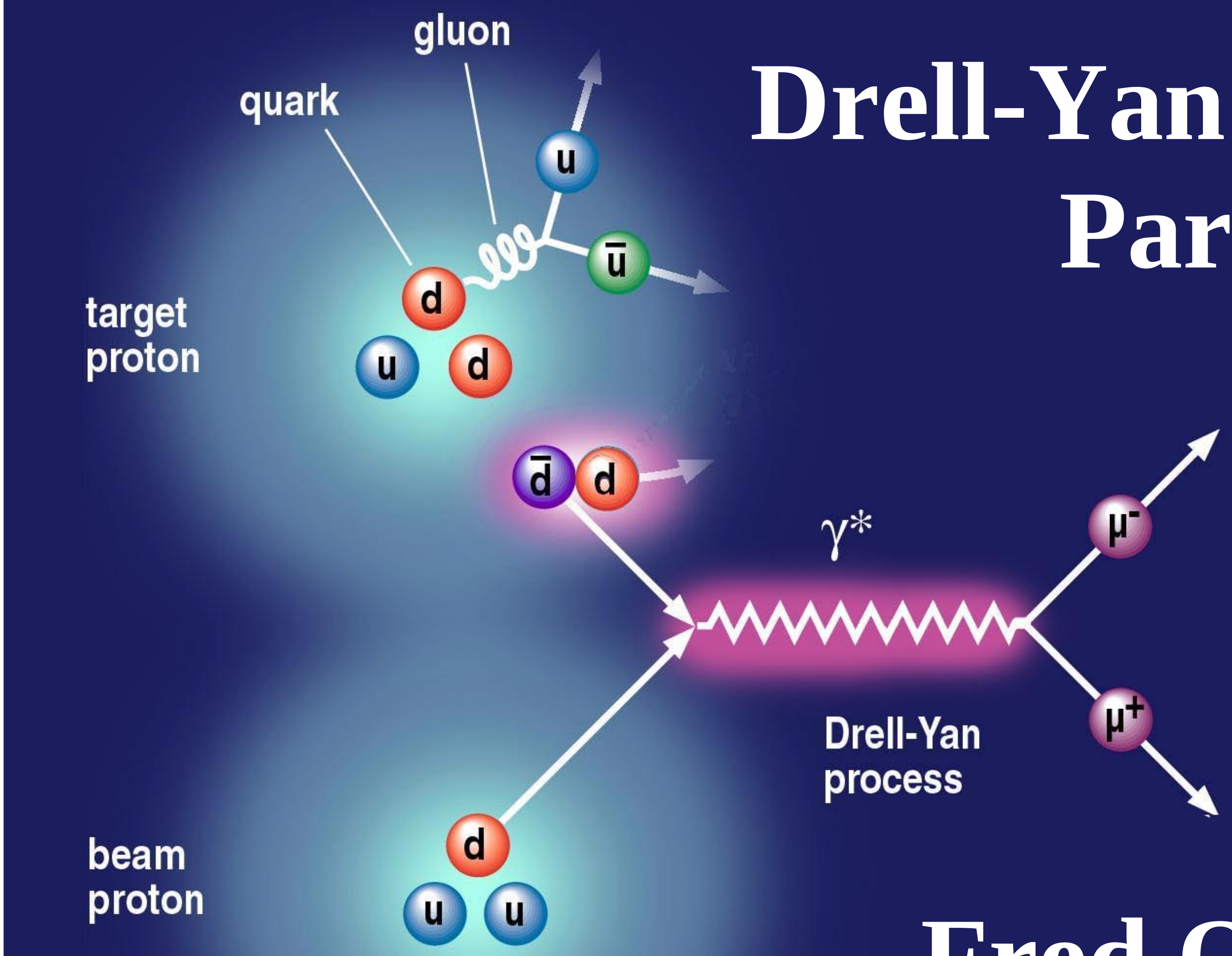
Drell-Yan Process: Part I



Fred Olness
SMU

OpenOffice.org Source Project

Power Point

Part I: Drell-Yan Process

History:

Discovery of J/ψ, Upsilon, W/Z, and "New Physics" ???

Calculation of $q\, q \rightarrow \mu^+\mu^-$ **in the Parton Model**

Scaling form of the cross section

Rapidity, longitudinal momentum, and x_F

Comparison with data:

NLO QCD corrections essential (the K-factor)

σ(pd)/σ(pp) important for d-bar/ubar

W Rapidity Asymmetry important for slope of d/u at large x

Where are we going?

P_T Distribution

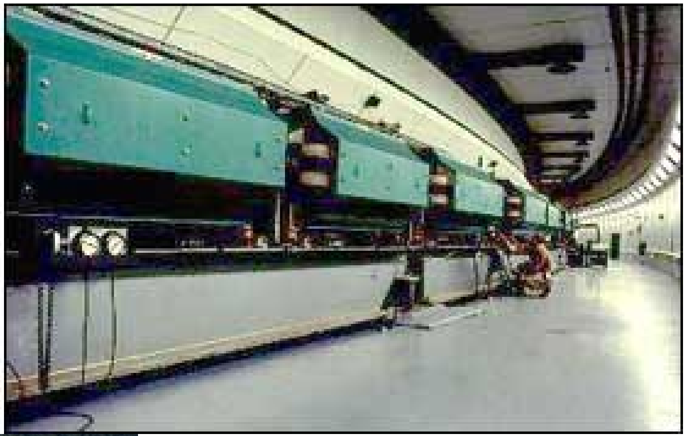
W-mass measurement

Resummation of soft gluons

Historical Background

Our story begins in the late 1960's at CERN

Brookhaven National Lab
Alternating Gradient Synchrotron

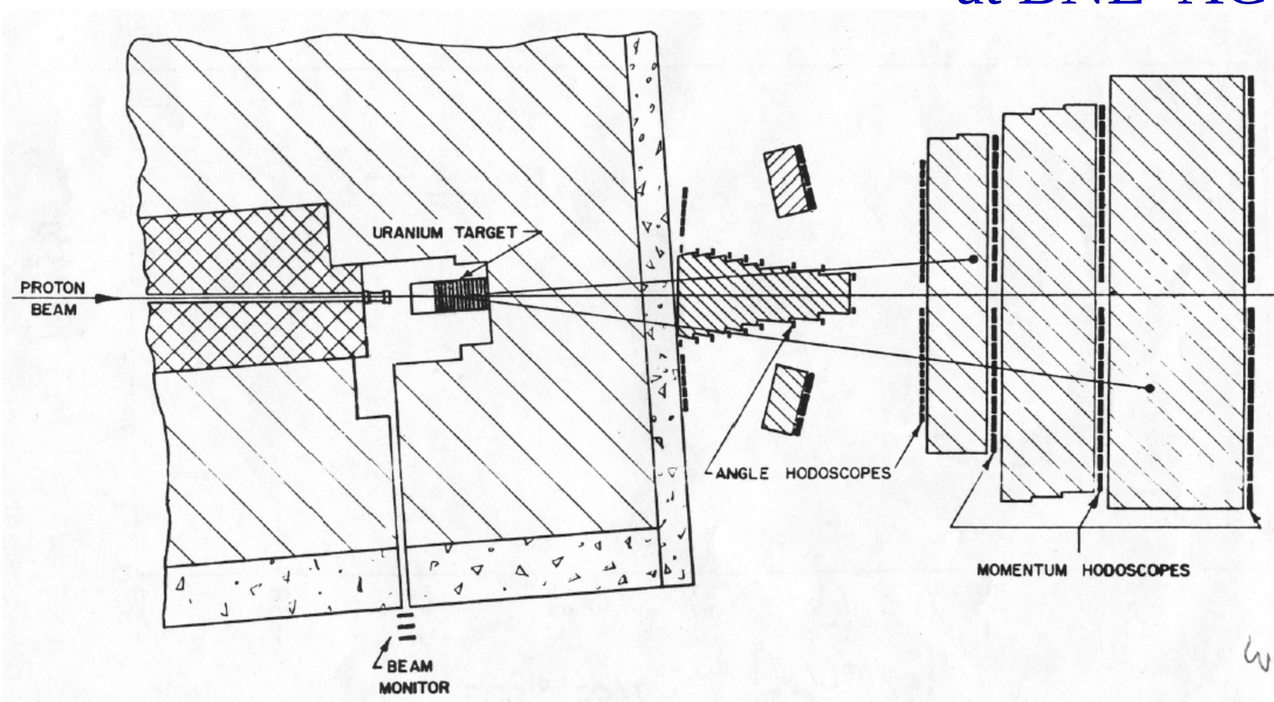


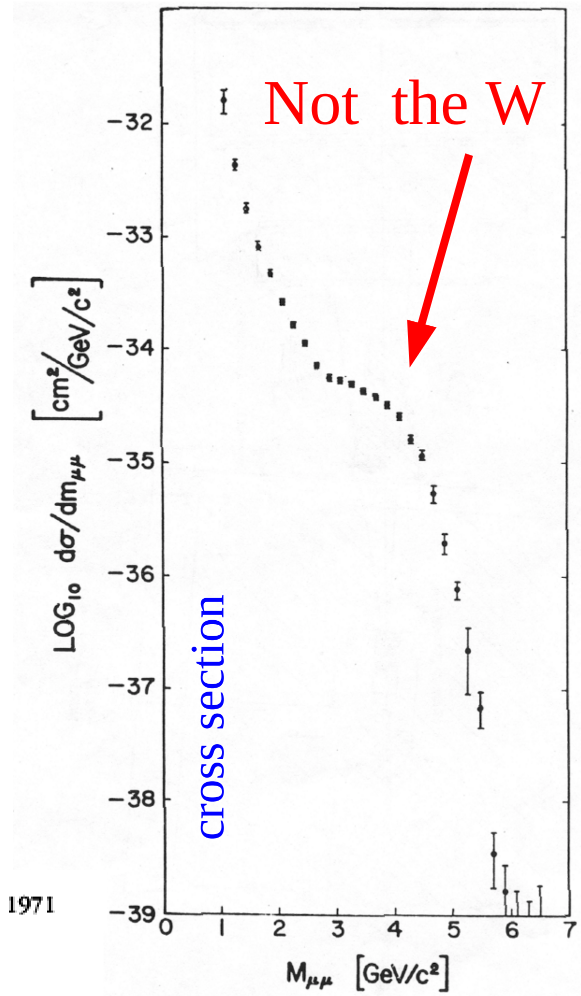
An Early Experiment:

The Goal: $p + N \rightarrow W + X$

They found: $p + N \rightarrow \mu^+ \mu^- + X$

at BNL AGS





$M_{\mu\mu}$ GeV

VOLUME 27, NUMBER 11 PHYSICAL REVIEW LETTERS 13 SEPTEMBER 1971

Production of Intermediate Bosons in Strong Interactions*

L. M. Lederman and B. G. Pope†
Columbia University, New York, New York 10533
(Received 14 June 1971)

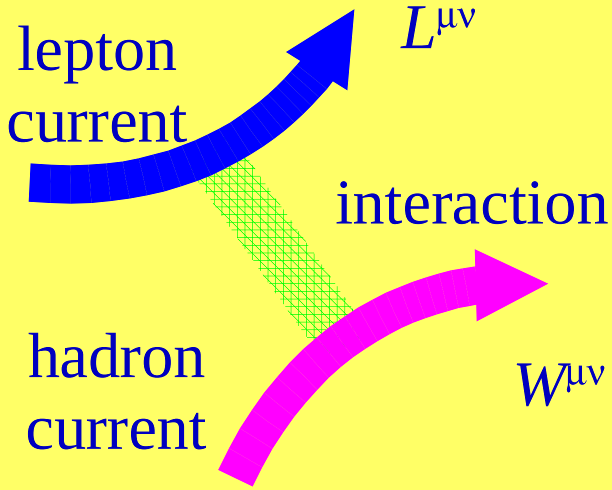
Several searches for the weak intermediate boson (W) have been carried out using the reaction

$$p + Z \rightarrow W + \text{anything}, \quad (1)$$

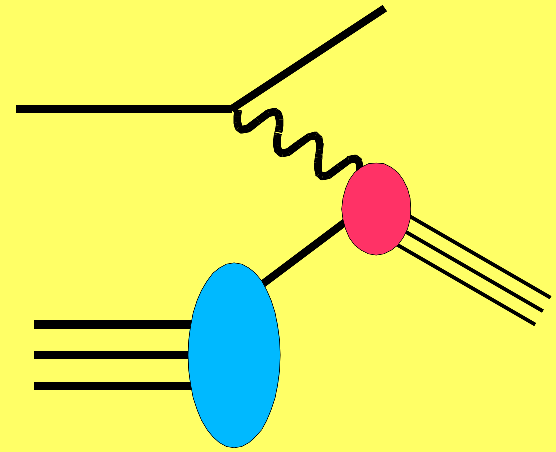
with the decay of the W into muons as the signature.[1,2] Failure to observe a muon signal from any

What is the explanation???

In DIS, we have two choices for an interpretation:



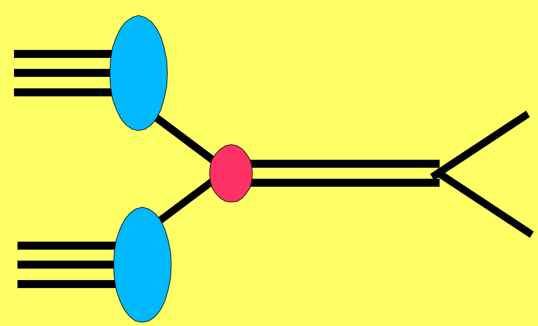
Conserved Current Interactions



The Parton Model

What about Drell-Yan???

? ? ?

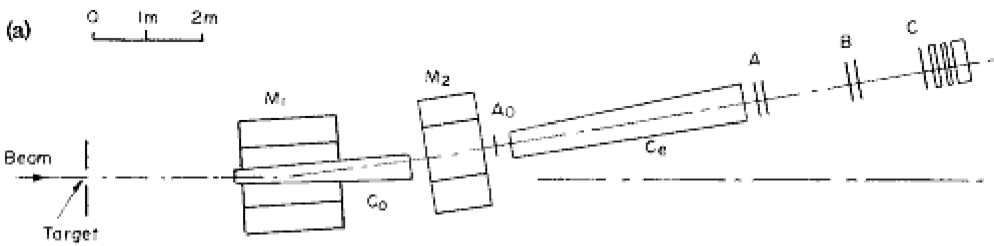


The Parton Model

Discovery of the J/Psi Particle

The Process: $p + Be \rightarrow e^+ e^- X$

very narrow width
$\Rightarrow$ long lifetime



at BNL AGS

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECE

Experimental Observation of a Heavy Particle J†

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen
J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technolog
Cambridge, Massachusetts 02139

and

Y. Y. Lee
Brookhaven National Laboratory, Upton, New York 11973
(Received 12 November 1974)

We report the observation of a heavy particle J, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + \mathrm{Be} \rightarrow e^+ + e^- + x$ by measuring the e^+e^- mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

This experiment is part of a large program to ... daily with a thin Al foil. The beam spot

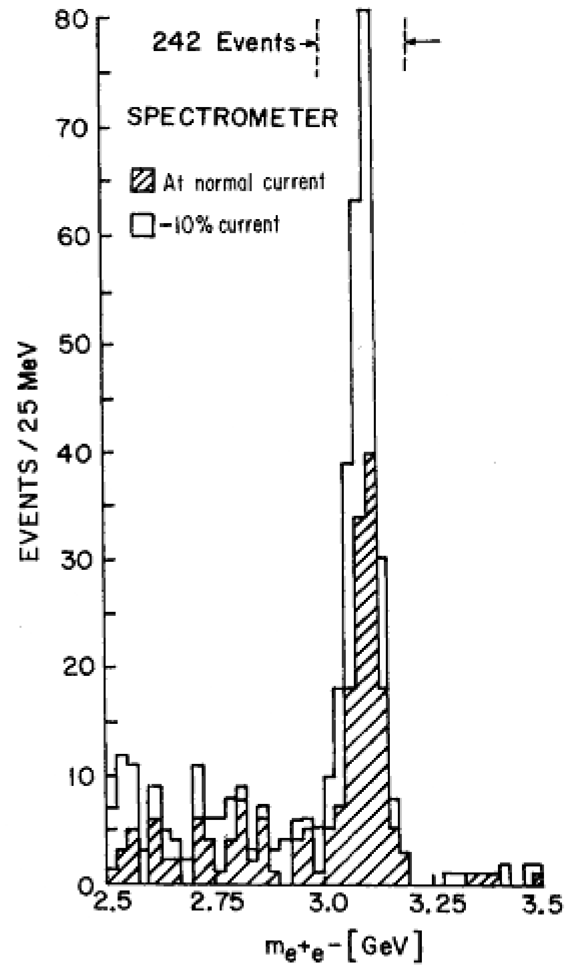
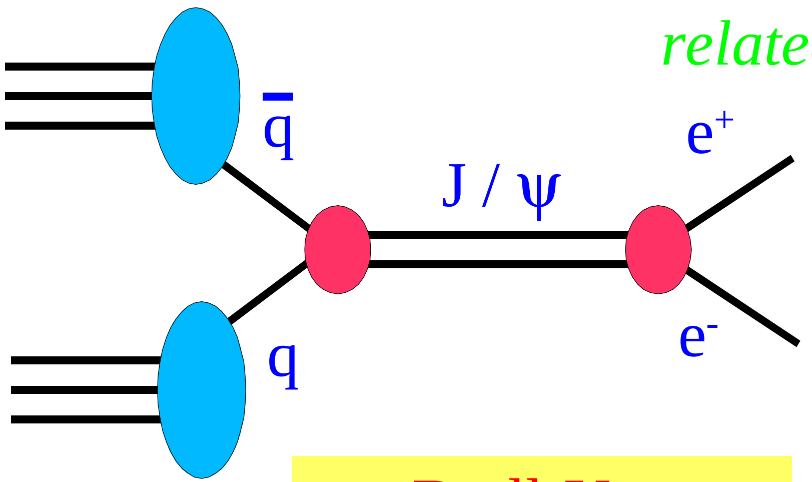
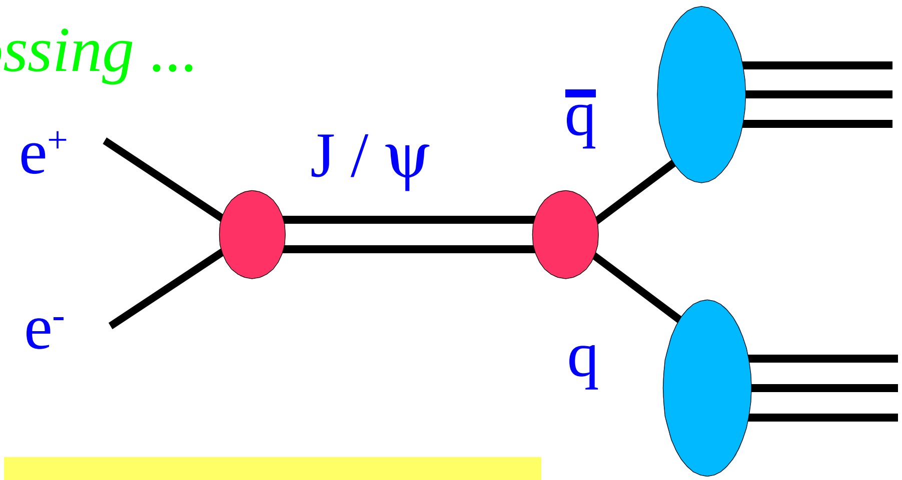


FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

The November Revolution

related by crossing ...





Drell-Yan
Brookhaven AGS

e^+e^- Production
SLAC SPEAR
Frascati ADONE

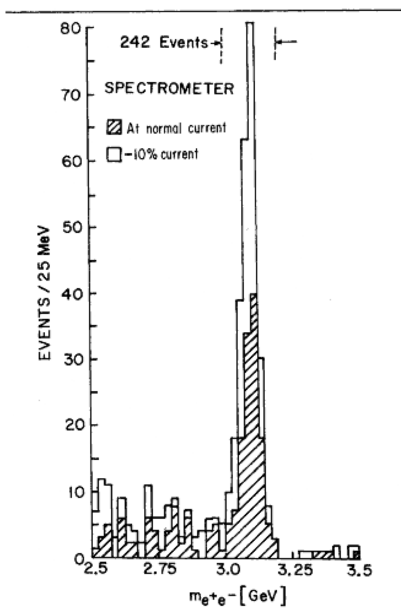
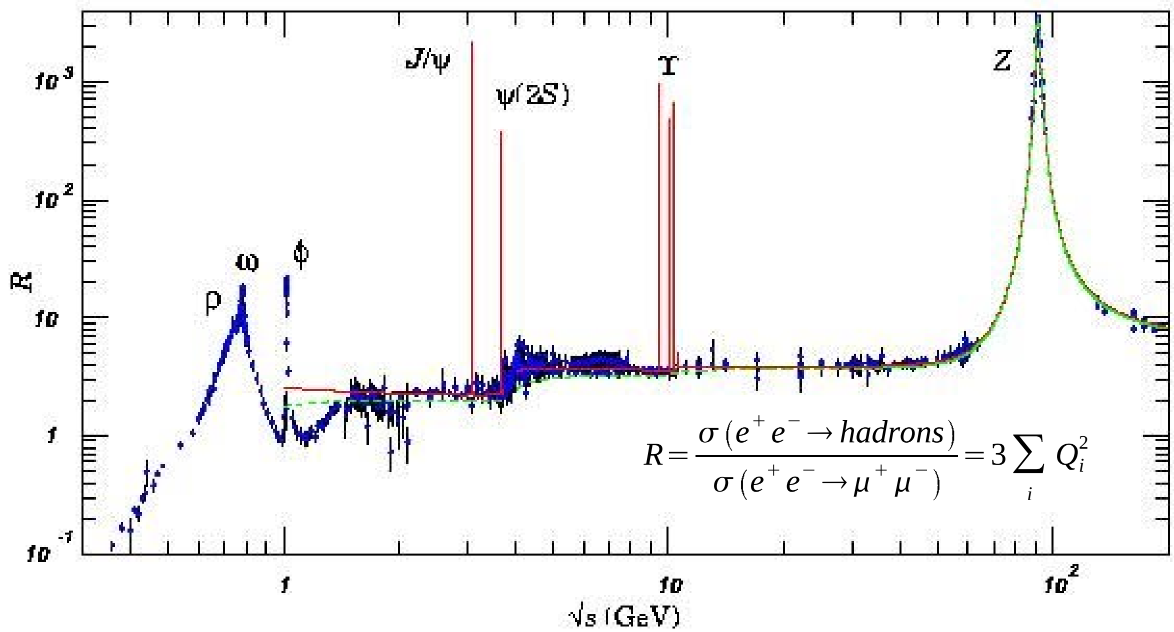


FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.



More Discoveries with Drell-Yan

1974: The J/Psi (charm) discovery

$p+N \rightarrow J/\psi$

... *1976 Nobel Prize*

1977: The Upsilon (bottom) discovery

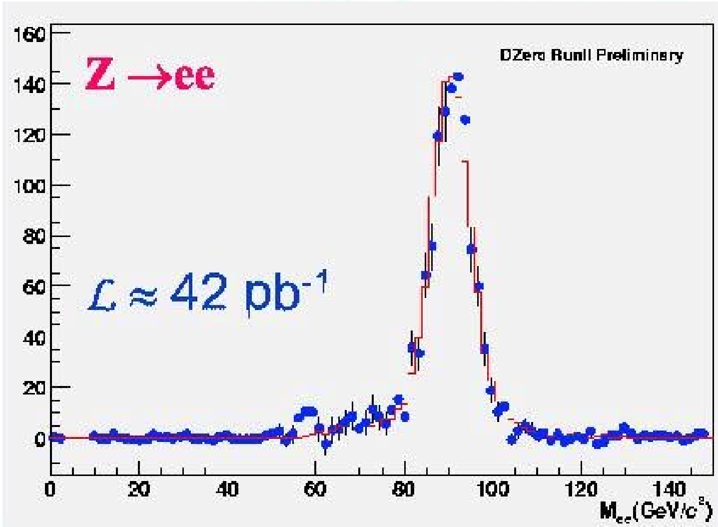
$p+N \rightarrow \Upsilon$

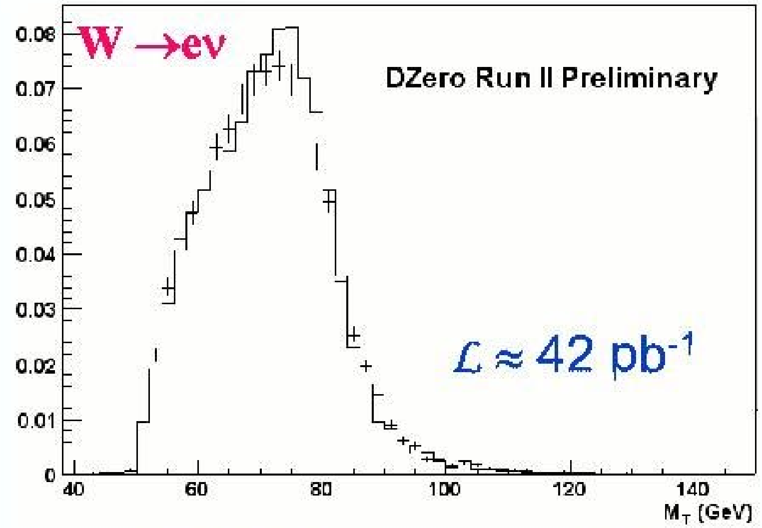
1983: The W and Z discovery

$p + \bar{p} \rightarrow W/Z$

... *1984 Nobel Prize*

W/Z in the electron channel





- 1139 $Z \rightarrow ee$ candidates
  - $|\eta^e|<1.1$, $E_T>25$ GeV, no track match required
- $\varepsilon(Z)\approx 8\%$, bkgd ~ 18%

- 27370 $W \rightarrow e\nu$ candidates
  - $|\eta^e|<1.1$, E_T & $\not{E}_T$ >25 GeV
- $\varepsilon(W)\approx 16\%$
- bkgd ~ 3% QCD, ~1.5% τ

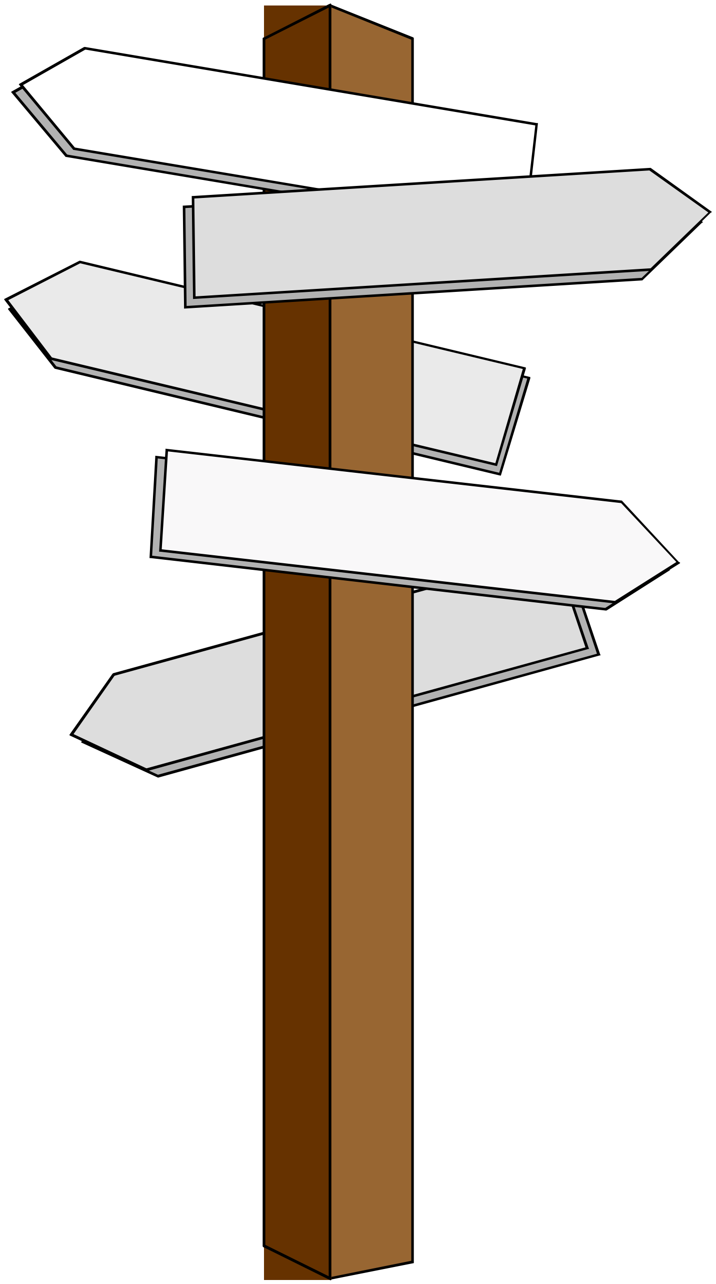
$$\boxed{\sigma(W)Br(W \rightarrow e\nu) = 3054 \pm 100(N_W) \pm 86(sys) \pm 305(lumi)\ \text{pb}}$$

$$\boxed{\sigma(Z)Br(Z \rightarrow ee) = 294 \pm 11(N_Z) \pm 8(sys) \pm 29(lumi)\ \text{pb}}$$

Where do we find

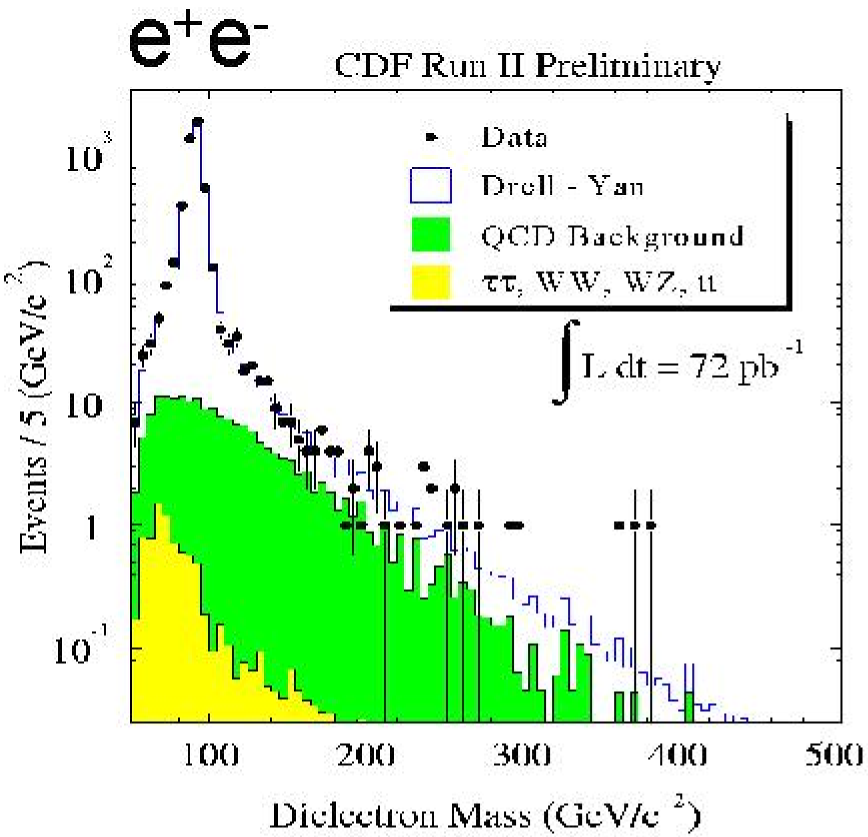
New Physics??

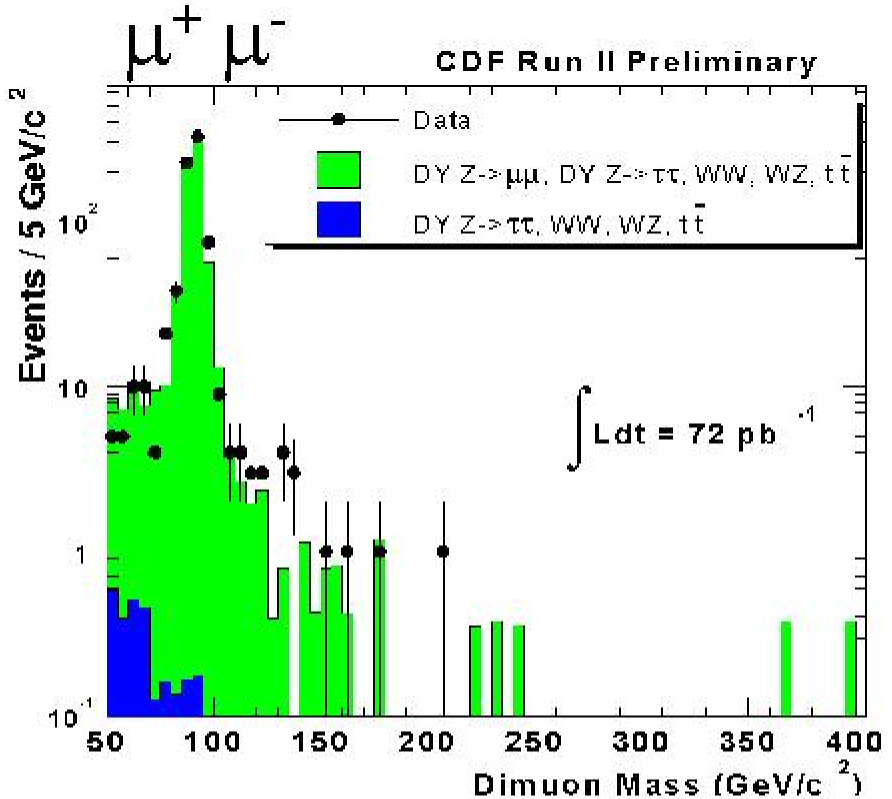
- New Higgs Bosons
- New W' or Z'
- SUSY
- ... unknown...



Search in Drell-Yan Spectrum

- High Mass Dileptons
  - electrons & muons used
- Sensitive to Z' and Randall-Sundrum Graviton
- No excess observed



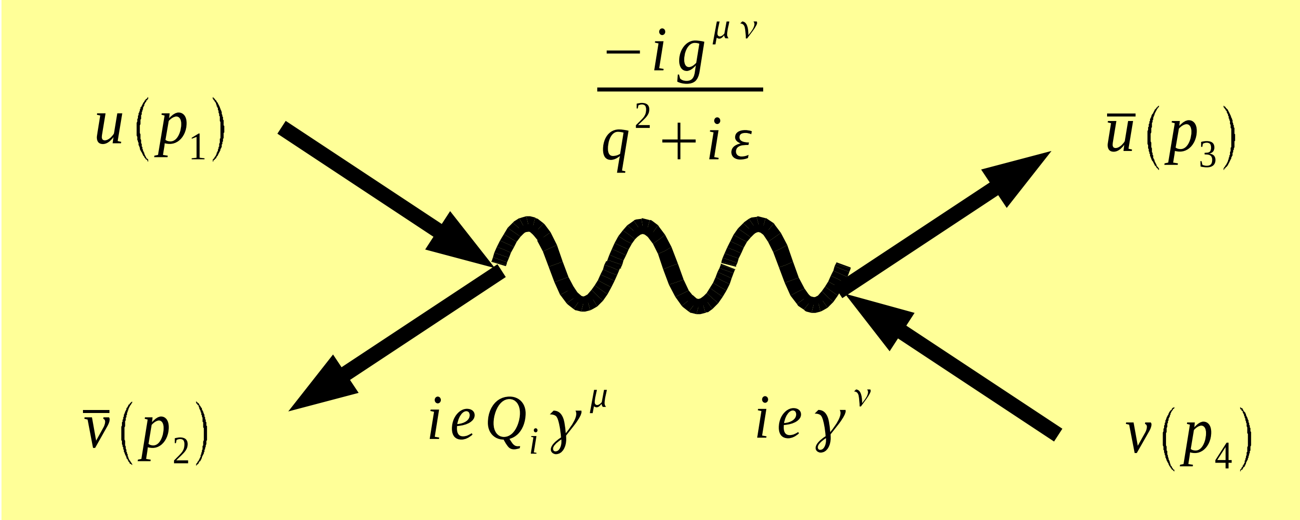


Let's

Calculate

First, we'll compute the partonic $\hat{\sigma}$ in the partonic CMS

Let's compute the Born process: $q+\bar{q}\to e^{+}+e^{-}$



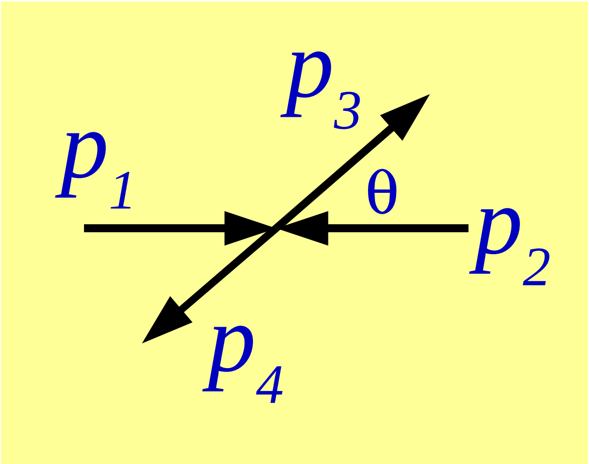
Gathering factors and contracting $g^{\mu\nu}$, we obtain:

$$-iM = iQ_i\,\frac{e^2}{q^2}\,\{\bar{v}(p_2)\gamma^{\mu}u(p_1)\}\;\{\bar{u}(p_3)\gamma_{\mu}v(p_4)\}$$

Squaring, and averaging over spin and color, ...

$$\overline{|M|^2} = \left(\frac{1}{2}\right)^2 3\left(\frac{1}{3}\right)^2 Q_i^2\,\frac{e^4}{q^4}\,Tr\left[\not{p}_2\gamma^{\mu}\not{p}_1\gamma^{\nu}\right]\;Tr\left[\not{p}_3\gamma_{\mu}\not{p}_4\gamma_{\nu}\right]$$

Let's work out some parton level kinematics



$$p_1 = \frac{\sqrt{\hat{s}}}{2}(1,0,0,+1)$$

$$p_2 = \frac{\sqrt{\hat{s}}}{2}(1,0,0,-1)$$

$$p_3 = \frac{\sqrt{\hat{s}}}{2}\left(1,+\sin(\theta),0,+\cos(\theta)\right)$$

$$p_4 = \frac{\sqrt{\hat{s}}}{2}\left(1,-\sin(\theta),0,-\cos(\theta)\right)$$

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$$

Defining the Mandelstam variables ...

$$\hat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

$$\hat{t} = (p_1 - p_3)^2 = (p_2 - p_4)^2$$

$$\hat{u} = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

$$\hat{t} = -\frac{\hat{s}}{2}\left(1 - \cos(\theta)\right)$$

$$\hat{u} = -\frac{\hat{s}}{2}\left(1 + \cos(\theta)\right)$$

We'll now compute the matrix element M

Manipulating the traces, we find ...

$$
\begin{aligned}
&Tr\left[\not{p}_2 \gamma^{\mu} \not{p}_1 \gamma^{\nu}\right] \; Tr\left[\not{p}_3 \gamma_{\mu} \not{p}_4 \gamma_{\nu}\right] \\
&= 4\left[p_1^{\mu} p_2^{\nu} + p_2^{\mu} p_1^{\nu} - g^{\mu\nu}(p_1 \cdot p_2)\right] \times 4\left[p_3^{\mu} p_4^{\nu} + p_4^{\mu} p_3^{\nu} - g^{\mu\nu}(p_3 \cdot p_4)\right] \\
&= 2^5 \left[(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)\right] \\
&= 2^3 \left[\hat{t}^2 + \hat{u}^2\right]
\end{aligned}
$$

Where we have used:

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$$

$$
\begin{aligned}
\hat{s} &= 2(p_1 \cdot p_2) = 2(p_3 \cdot p_4) \\
\hat{t} &= 2(p_1 \cdot p_3) = 2(p_2 \cdot p_4) \\
\hat{u} &= 2(p_1 \cdot p_4) = 2(p_2 \cdot p_3)
\end{aligned}
$$

Putting all the pieces together, we have:

$$\overline{|M|^2} = Q_i^2 \, \alpha^2 \, \frac{2^5 \pi^2}{3} \left(\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right)$$

with

$$q^2 = (p_1 + p_2)^2 = \hat{s}$$

$$\alpha = \frac{e^2}{4\pi}$$

... and put it together to find the cross section

$$\boxed{d\hat{\sigma} \simeq \frac{1}{2\hat{s}} \overline{|M|^2} \, d\Gamma}$$

In the partonic CMS system

$$d\Gamma = \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \, \delta(p_1 + p_2 - p_3 - p_4) = \frac{d\cos(\theta)}{16\pi}$$

Recall,

$$\hat{t} = \frac{-\hat{s}}{2}\left(1 - \cos(\theta)\right) \quad and \quad \hat{u} = \frac{-\hat{s}}{2}\left(1 + \cos(\theta)\right)$$

so, the differential cross section is ...

$$\boxed{\frac{d\hat{\sigma}}{d\cos(\theta)} = Q_i^2 \alpha^2 \frac{\pi}{6} \frac{1}{\hat{s}} \left(1 + \cos^2(\theta)\right)}$$

and the total cross section is ...

$$\hat{\sigma} = Q_i^2 \alpha^2 \frac{\pi}{6} \frac{1}{\hat{s}} \int_{-1}^{1} d\cos(\theta) \left(1 + \cos^2(\theta)\right) = \boxed{\frac{4\pi\alpha^2}{9\hat{s}} Q_i^2 \equiv \hat{\sigma}_0}$$

Some Homework:

#1) Show:

$$\frac{d^3p}{(2\pi)^3 2E} = \frac{d^4p}{(2\pi)^4} (2\pi)\, \delta^+(p^2 - m^2)$$

This relation is often useful as the RHS is manifestly Lorentz invariant

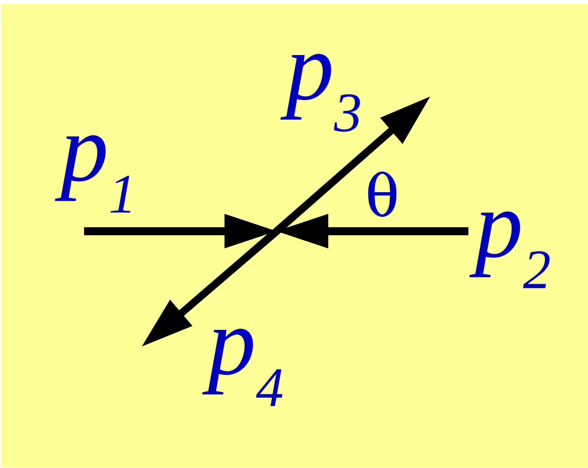
#2) Show that the 2-body phase space can be expressed as:

$$d\Gamma = \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} (2\pi)^4\, \delta(p_1 + p_2 - p_3 - p_4) = \frac{d\cos(\theta)}{16\pi}$$

Note, we are working with massless partons, and θ *is in the partonic CMS frame*

Some More Homework:

#3) Let's work out the general $2 \to 2$ kinematics for general masses.



a) Start with the incoming particles.
Show that these can be written in the general form:

$$p_1 = (E_1, 0, 0, +p) \qquad p_1^2 = m_1^2$$
$$p_2 = (E_2, 0, 0, -p) \qquad p_2^2 = m_2^2$$

... with the following definitions:

$$E_{1,2} = \frac{\hat{s} \pm m_1^2 \mp m_2^2}{2\sqrt{\hat{s}}} \qquad p = \frac{\Delta(\hat{s}, m_1^2, m_2^2)}{2\sqrt{\hat{s}}}$$

$$\Delta(a,b,c) = \sqrt{a^2 + b^2 + c^2 - 2(ab + bc + ca)}$$

Note that $\Delta(a,b,c)$ is symmetric with respect to its arguments, and involves the only invariants of the initial state: s, m_1^2, m_2^2.

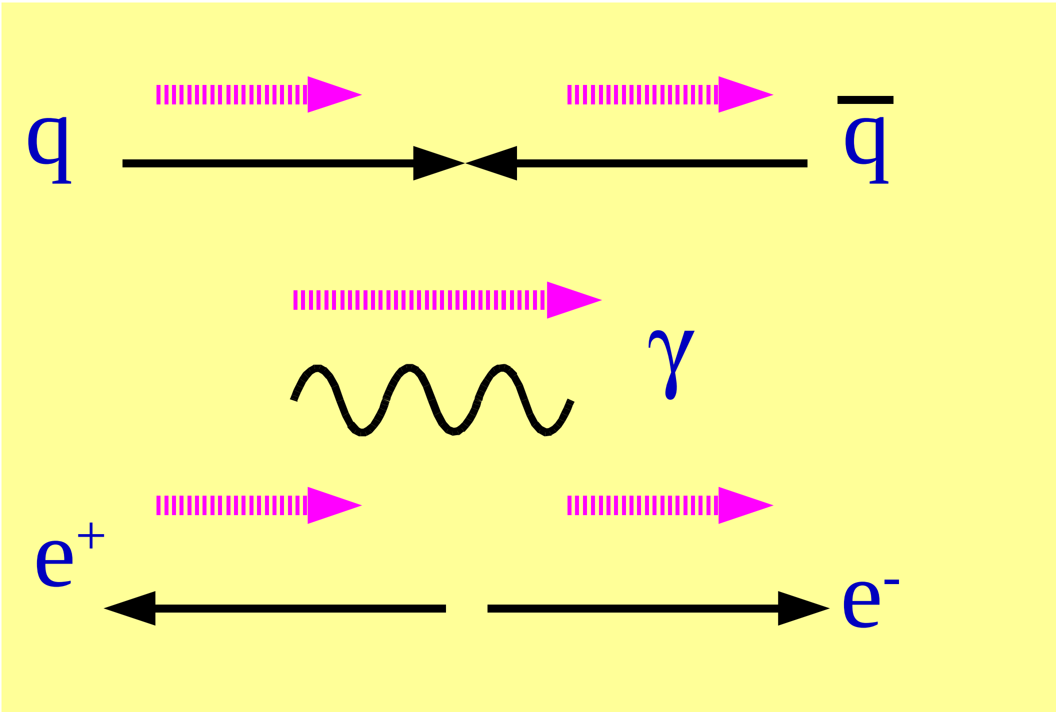
b) Next, compute the general form for the final state particles, p_3 and p_4. Do this by first aligning p_3 and p_4 along the z-axis (as p_1 and p_2 are), and then rotate about the y-axis by angle θ.

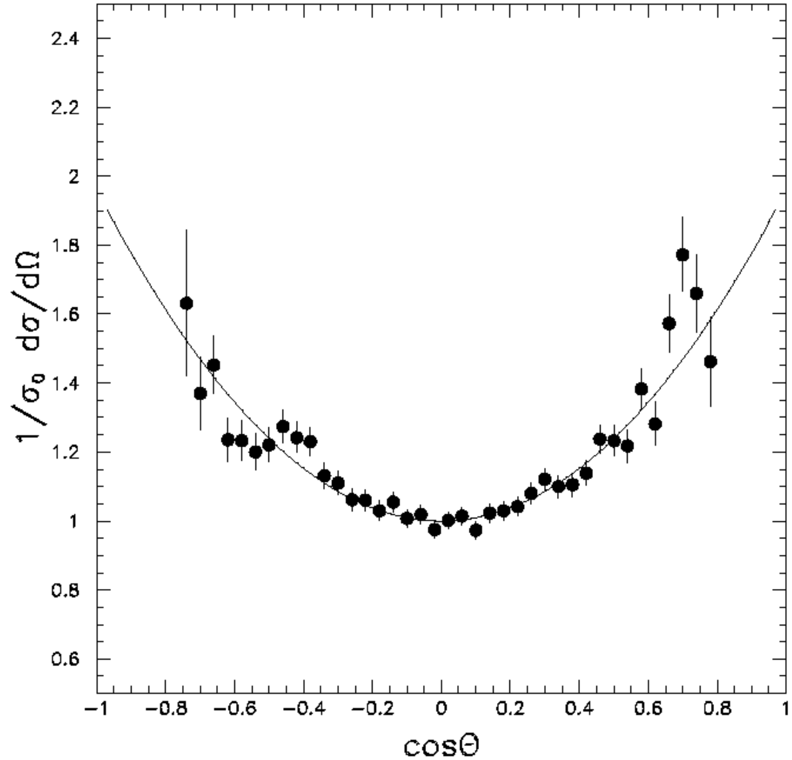
What does the angular dependence tell us?

Observe, the angular dependence: $q + \bar{q} \rightarrow e^{+} + e^{-}$

$$\frac{d\hat{\sigma}}{d\cos(\theta)} = Q_i^2 \alpha^2 \frac{\pi}{6} \frac{1}{\hat{s}} \left(1 + \cos^2(\theta)\right)$$

Characteristic of scattering of spin ½ constitutients by a spin 1 vector

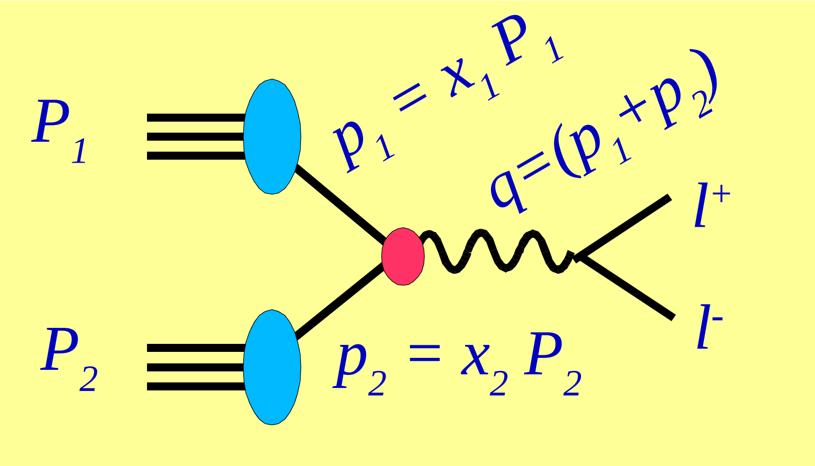




Note, for the photon, the mirror image of the above is also valid; hence the symmetric distribution.
The W has V-A couplings, so we'll find: (1+cosθ)²

Next, we'll compute the hadronic CMS

Kinematics in the Hadronic Frame



$$P_1 = \frac{\sqrt{s}}{2}(1,0,0,+1) \qquad P_1^2 = 0$$

$$P_2 = \frac{\sqrt{s}}{2}(1,0,0,-1) \qquad P_2^2 = 0$$

$$s = (P_1 + P_2)^2 = \frac{\hat{s}}{x_1 x_2} = \frac{\hat{s}}{\tau}$$

Therefore

$$\tau = x_1 x_2 = \frac{\hat{s}}{s} \equiv \frac{Q^2}{s}$$

Fractional energy^2 between partonic and hadronic system

$$\frac{d\sigma}{dQ^2} = \sum_{q,\bar{q}} \int dx_1 \int dx_2 \left\{ q(x_1)\bar{q}(x_2) + \bar{q}(x_1) q(x_2) \right\} \hat{\sigma}_0 \, \delta(Q^2 - \hat{s})$$

Hadronic cross section

Parton distribution functions

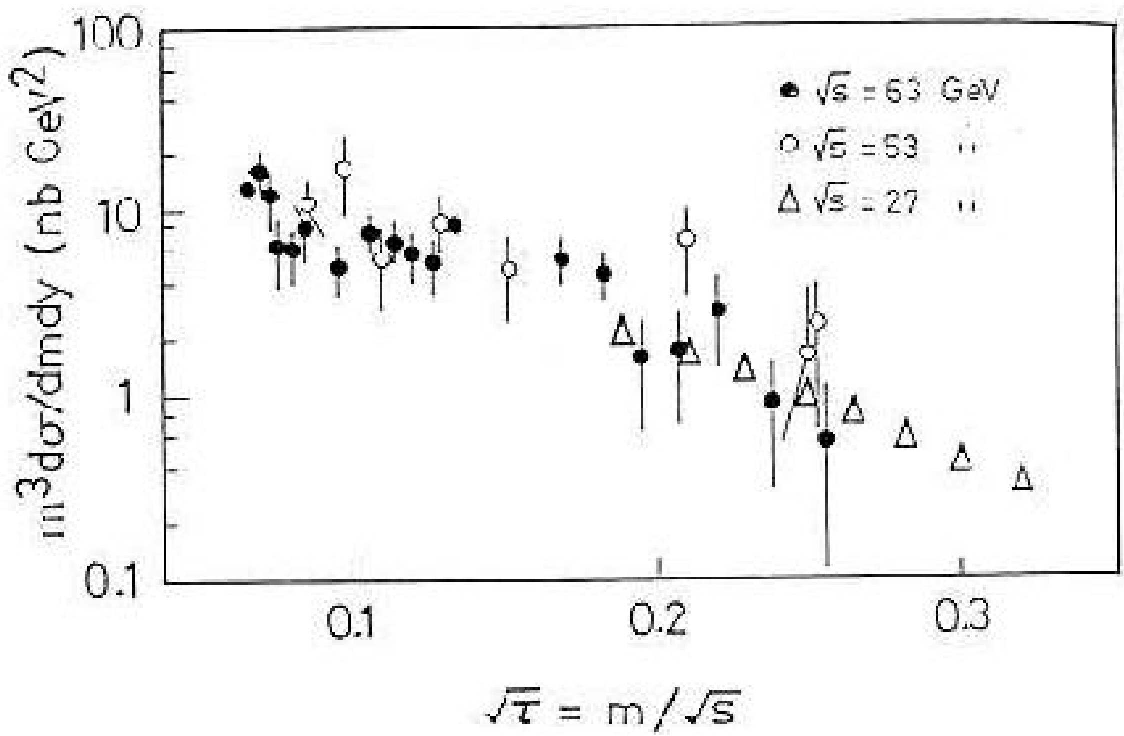
Partonic cross section

Scaling form of the Drell-Yan Cross Section

Using: $\hat{\sigma}_0 = \dfrac{4\pi\alpha^2}{9\hat{s}} Q_i^2$ and $\delta(Q^2 - \hat{s}) = \dfrac{1}{s\,x_1}\,\delta\left(x_2 - \dfrac{\tau}{x_1}\right)$

we can write the cross section in the scaling form:

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9} \sum_{q,\bar{q}} Q_i^2 \int_\tau^1 \frac{dx_1}{x_1}\, \tau \left\{ q(x_1)\bar{q}(\tau/x_1) + \bar{q}(x_1) q(\tau/x_1) \right\}$$



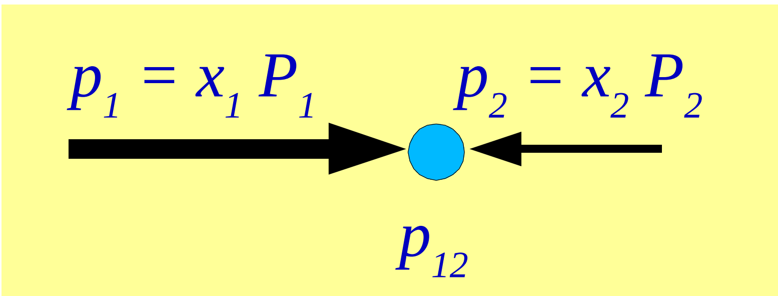
Notice the RHS is a function of only τ, not Q.

This quantity should lie on a universal scaling curve.

Cf., DIS case, & scattering of point-like constituents

Longitudinal Momentum Distributions

Partonic CMS has longitudinal momentum w.r.t. the hadron frame



$$p_{12} = (p_1 + p_2) = (E_{12}, 0, 0, p_L)$$

$$E_{12} = \frac{\sqrt{s}}{2}(x_1 + x_2)$$

$$p_L = \frac{\sqrt{s}}{2}(x_1 - x_2) \equiv \frac{\sqrt{s}}{2} x_F$$

x_F is a measure of the longitudinal momentum

The rapidity is defined as:

$$y = \frac{1}{2} \ln \left\{ \frac{E_{12} + p_L}{E_{12} - p_L} \right\} = \frac{1}{2} \ln \left\{ \frac{x_1}{x_2} \right\}$$

$$x_{1,2} = \sqrt{\tau}\, e^{\pm y}$$

$$d x_1 \, d x_2 = d\tau \, dy$$

$$d Q^2 \, dx_F = dy \, d\tau \; s \; \sqrt{x_F^2 + 4\tau}$$

$$\frac{d\sigma}{dQ^2 \; dx_F} = \frac{4\pi\alpha^2}{9 Q^4} \frac{1}{\sqrt{x_F^2 + 4\tau}} \, \tau \sum_{q,\bar{q}} Q_i^2 \left\{ q(x_1) \bar{q}(\tau/x_1) + \bar{q}(x_1) q(\tau/x_1) \right\}$$

So, we're ready to compare with data

(or so we think...)

Let's compare data and theory

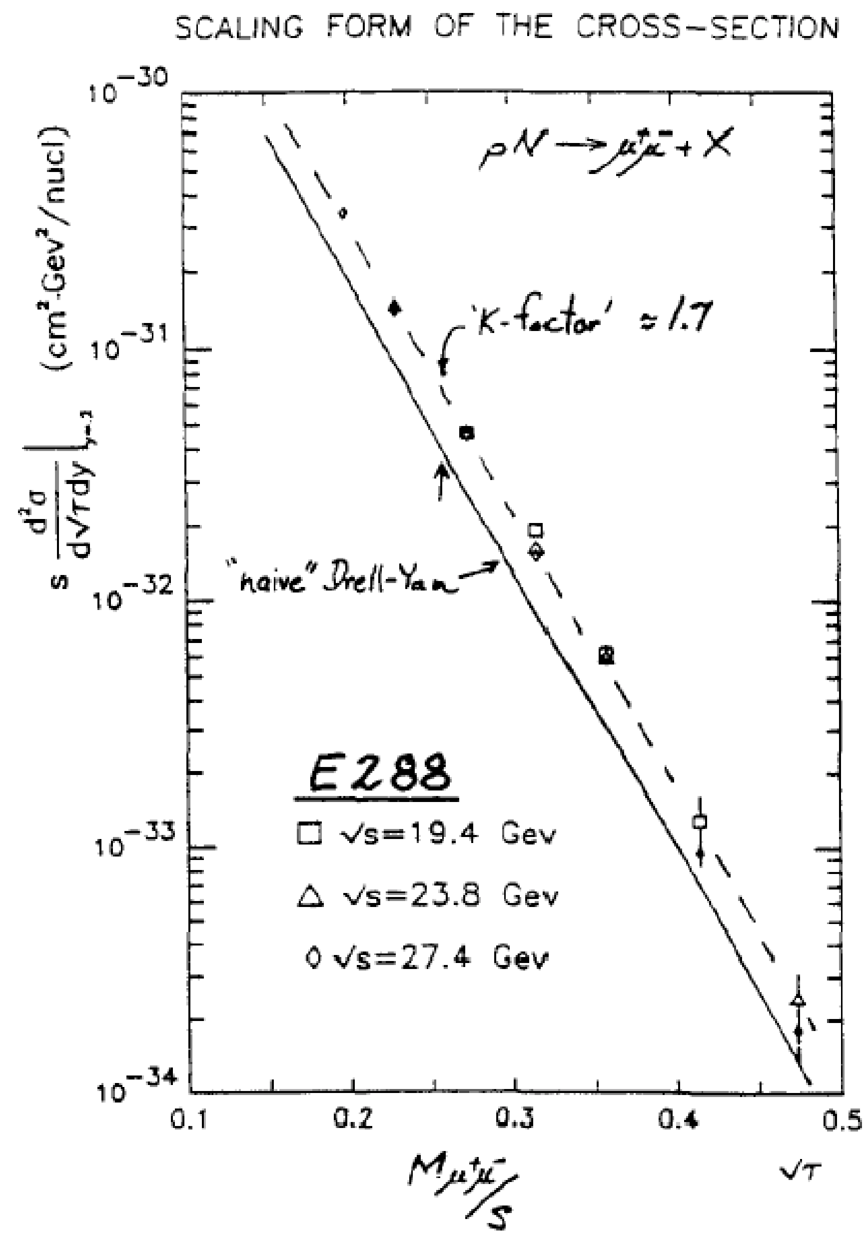


Table 1.2: Experimental K-factors.

Experiment	Interaction	Beam Momentum	$K = \sigma_{\text{meas.}}/\sigma_{\text{DY}}$
E288 [Kap 78]	$p\ Pt$	300/400 GeV	~ 1.7
WA39 [Cor 80]	$\pi^{\pm}\ W$	39.5 GeV	~ 2.5
E439 [Smi 81]	$p\ W$	400 GeV	1.6 ± 0.3
NA3 [Bad 83]	$(\bar{p}$ - $p)Pt$	150 GeV	2.3 ± 0.4
	$p\ Pt$	400 GeV	$3.1 \pm 0.5 \pm 0.3$
	$\pi^{\pm}\ Pt$	200 GeV	2.3 ± 0.5
	$\pi^{-}\ Pt$	150 GeV	2.49 ± 0.37
	$\pi^{-}\ Pt$	280 GeV	2.22 ± 0.33
NA10 [Bet 85]	$\pi^{-}\ W$	194 GeV	$\sim 2.77 \pm 0.12$
E326 [Gre 85]	$\pi^{-}\ W$	225 GeV	$2.70 \pm 0.08 \pm 0.40$
E537 [Ana 88]	$\bar{p}\ W$	125 GeV	$2.45 \pm 0.12 \pm 0.20$
E615 [Con 89]	$\pi^{-}\ W$	252 GeV	1.78 ± 0.06

J. C. Webb, Measurement of continuum dimuon production in 800-GeV/c proton nucleon collisions, arXiv:hep-ex/0301031.

Oooops,

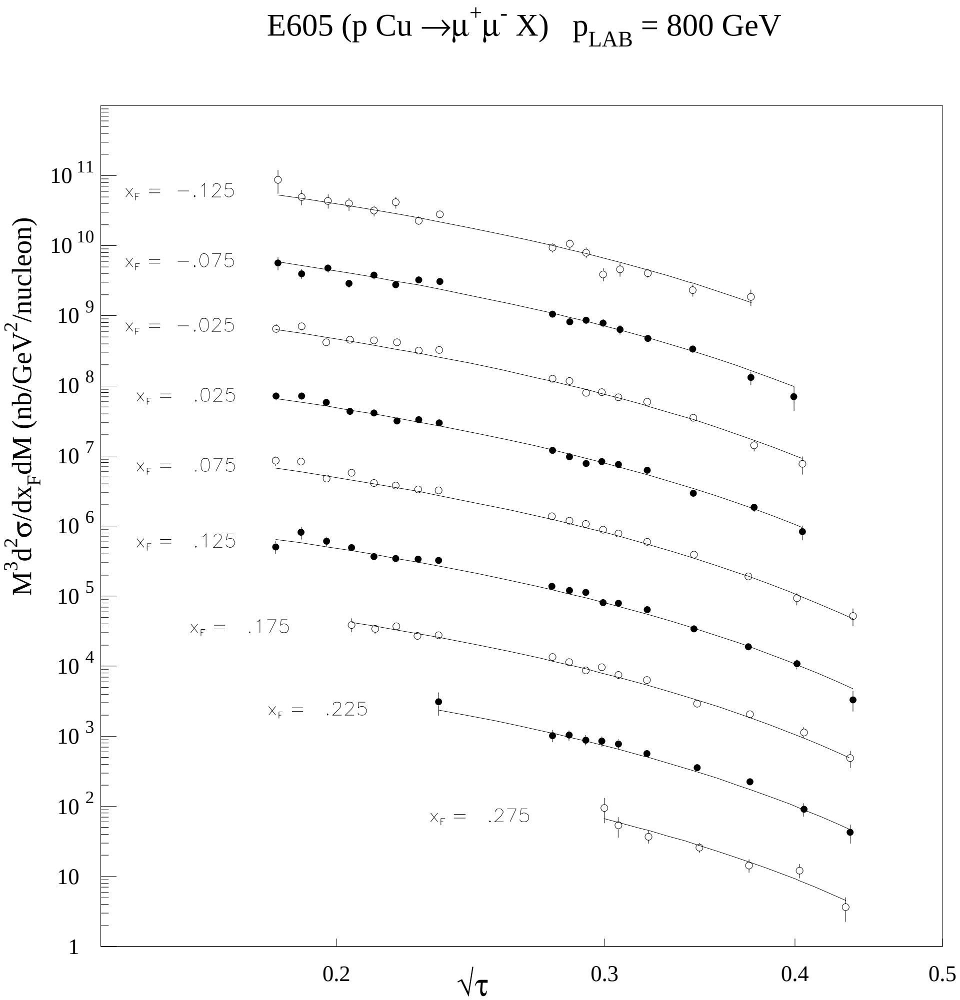
we need the

QCD corrections

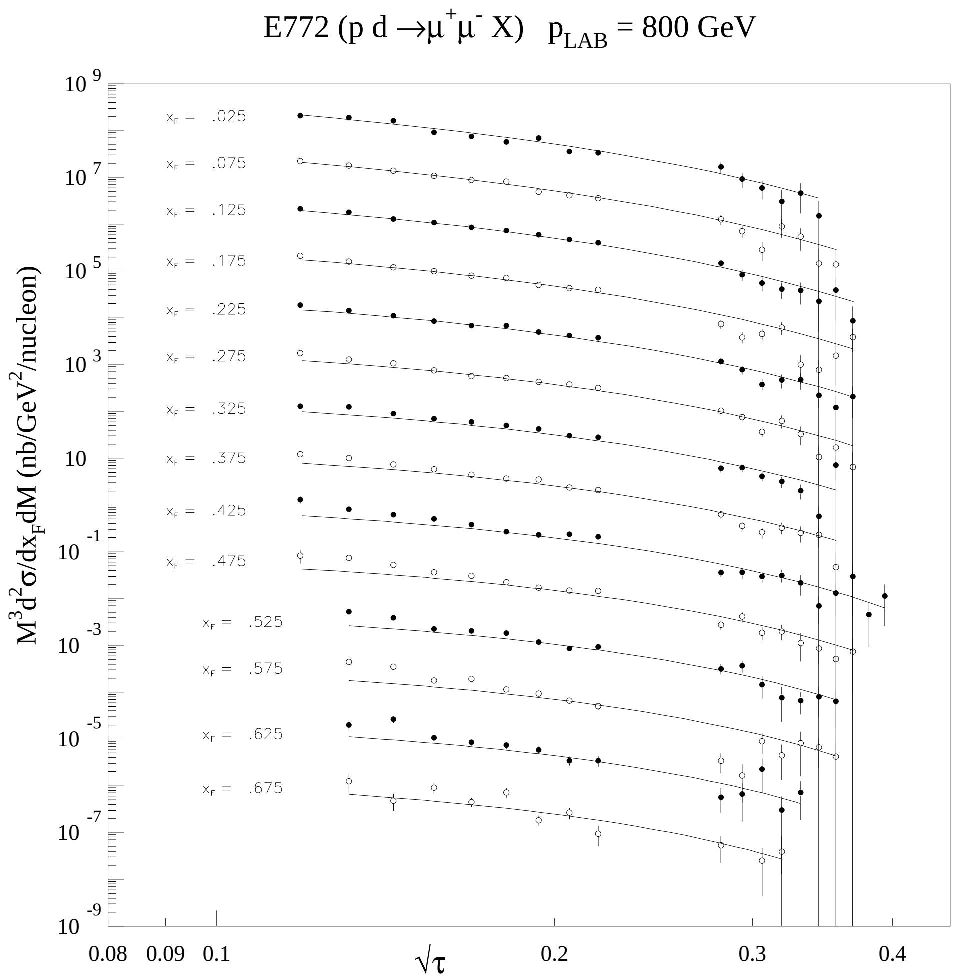
$$K = 1 + \frac{2\pi\alpha_s}{3}(\ldots) + \ldots = ? = e^{2\pi\alpha_s/3}$$

Excellent agreement between data and theory

p + Cu at 800 GeV



p + d at 800 GeV



pp & pN processes sensitive to anti-quark distributions

A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne,
Eur. Phys. J. C23, 73 (2002);
Eur. Phys. J. C14, 133 (2000);
Eur. Phys. J. C4, 463 (1998)

Drell-Yan can give us unique and detailed information about PDF's.

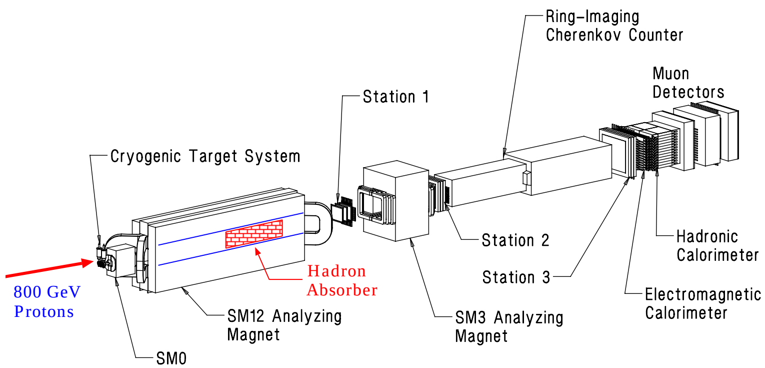
We'll now examine two examples:

1) Ratio of pp/pd cross section

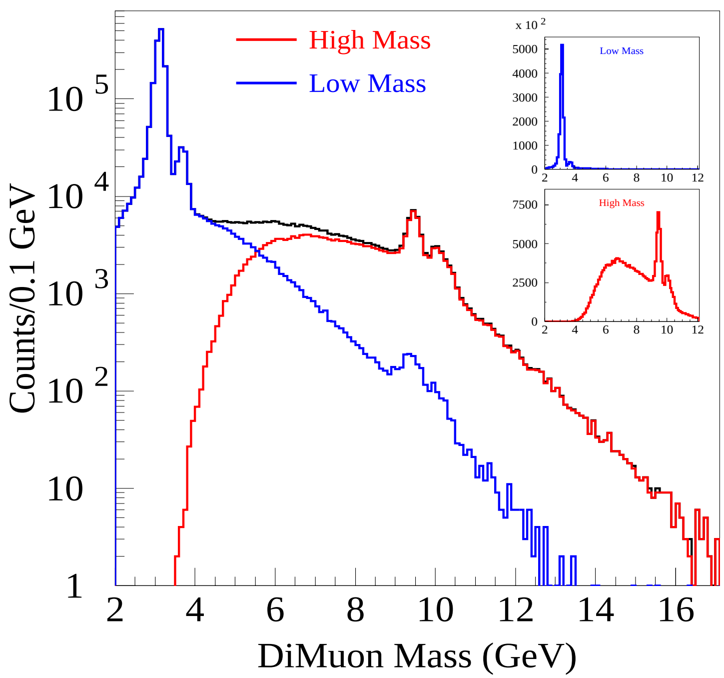
2) W Rapidity Asymmetry

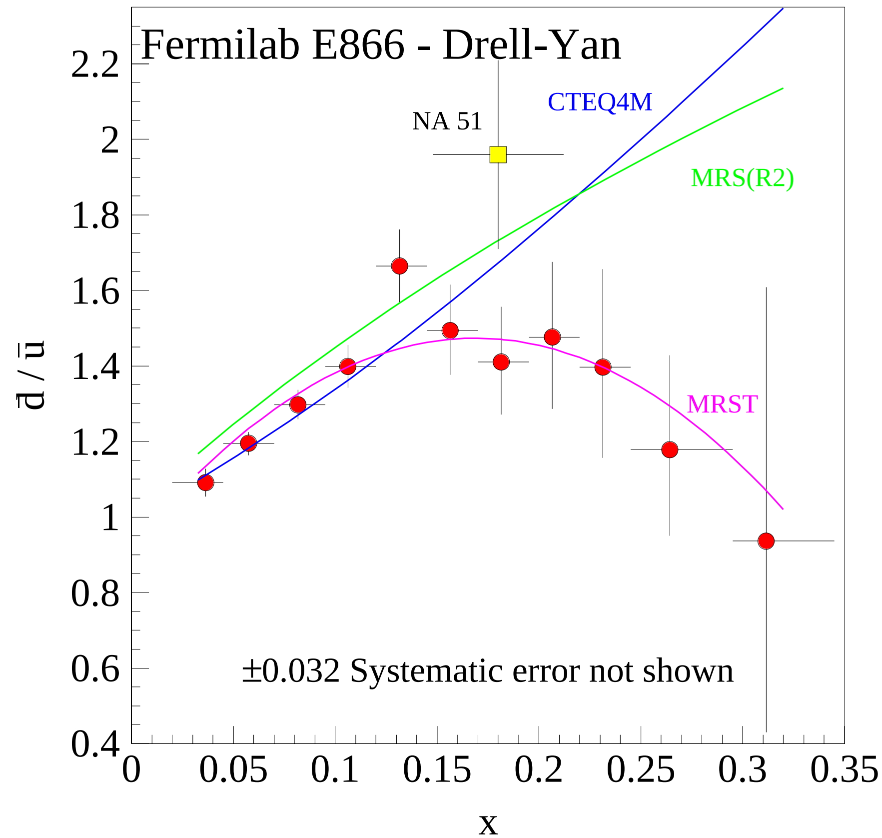
A measurement of $\bar{d}(x)/\bar{u}(x)$
Antiquark asymmetry in the Nucleon Sea
FNAL E866/NuSea

ACU, ANL, FNAL, GSU, IIT, LANL, LSU, NMSU, UNM, ORNL, TAMU, Valpo.



800 GeV $p + p$ and $p + d \to \mu^+\mu^- X$





Cross section ratio of pp *vs.* pd

Obtain the neutron PDF via isospin symmetry:

$$u \Leftrightarrow d$$
$$\bar{u} \Leftrightarrow \bar{d}$$

In the limit $x_1 >> x_2$:

$$\sigma^{pp} \propto \frac{4}{9} u(x_1)\bar{u}(x_2) + \frac{1}{9} d(x_1)\bar{d}(x_2)$$

$$\sigma^{pn} \propto \frac{4}{9} u(x_1)\bar{d}(x_2) + \frac{1}{9} d(x_1)\bar{u}(x_2)$$

For the ratio we have:

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2} \frac{\left(1 + \frac{1}{4}\frac{d_1}{u_1}\right)}{\left(1 + \frac{1}{4}\frac{d_1}{u_1}\frac{\bar{d}_2}{\bar{u}_2}\right)} \left(1 + \frac{\bar{d}_2}{\bar{u}_2}\right) \approx \frac{1}{2}\left(1 + \frac{\bar{d}_2}{\bar{u}_2}\right)$$

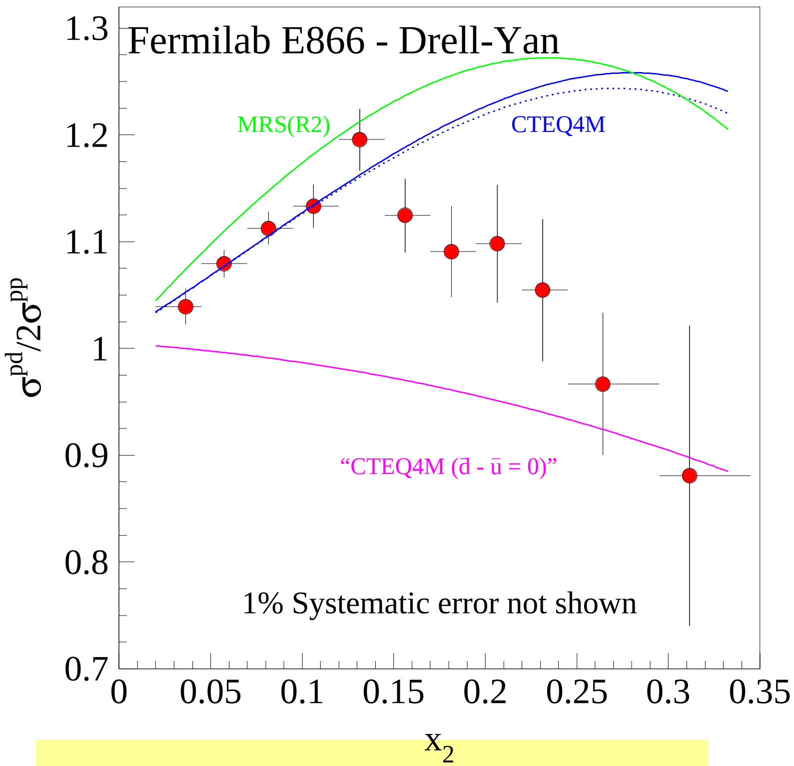
As promised, this provides information about the sea-quark distributions

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2}\left(1 + \frac{\bar{d}_2}{\bar{u}_2}\right)$$

EXERCISE: Verify the above.

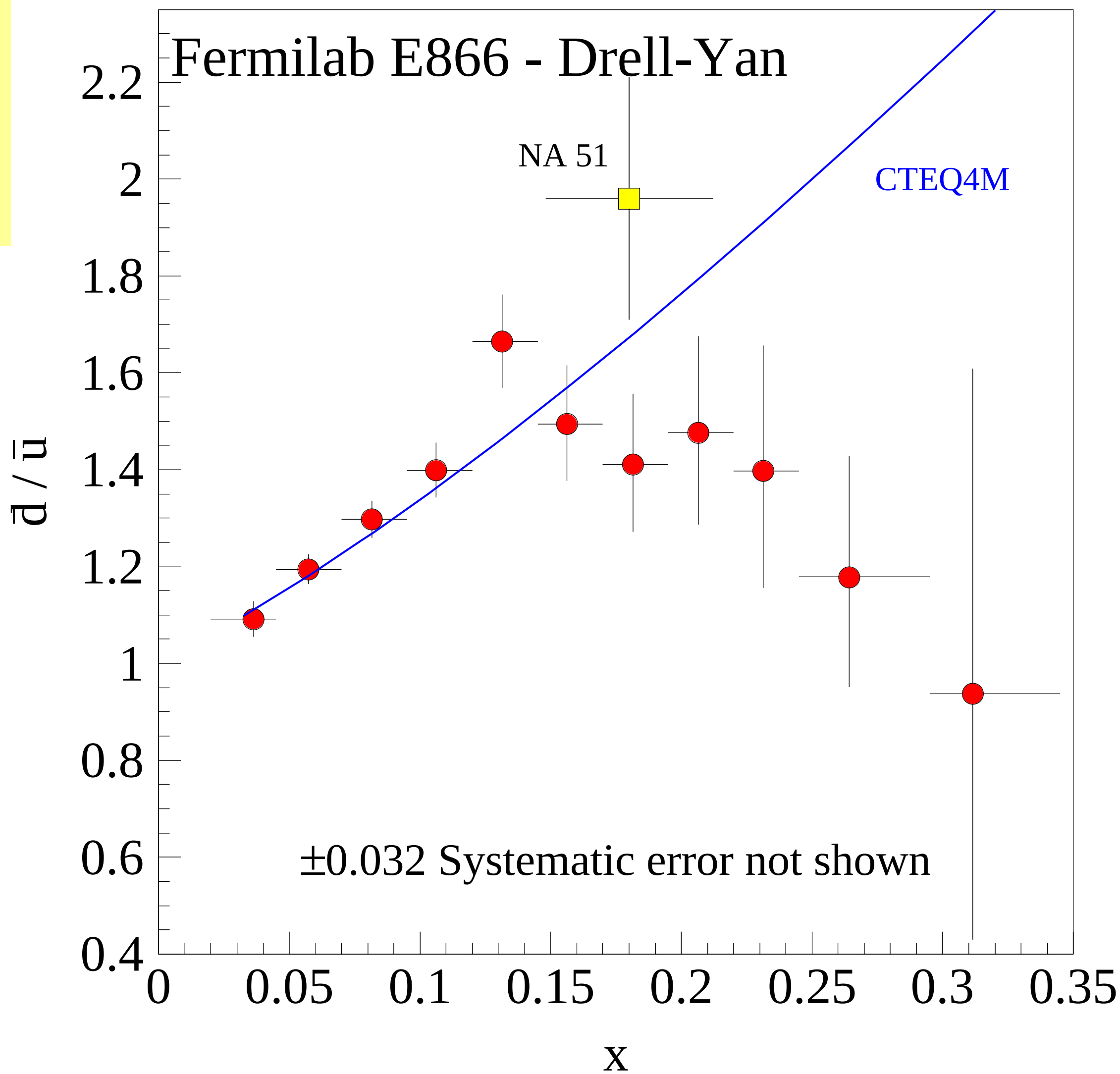
Does the theory match the data???

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2}\left(1+\frac{\bar{d}_2}{\bar{u}_2}\right)$$



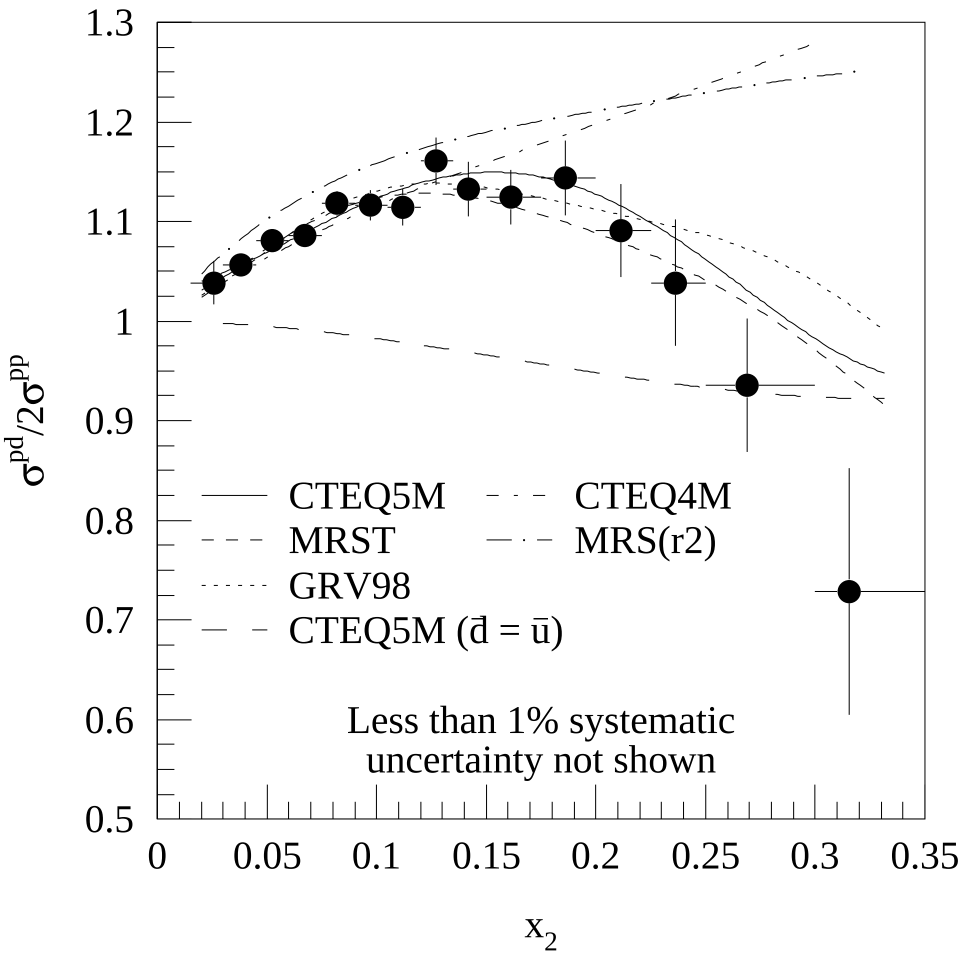
Implies R<1 for large x:

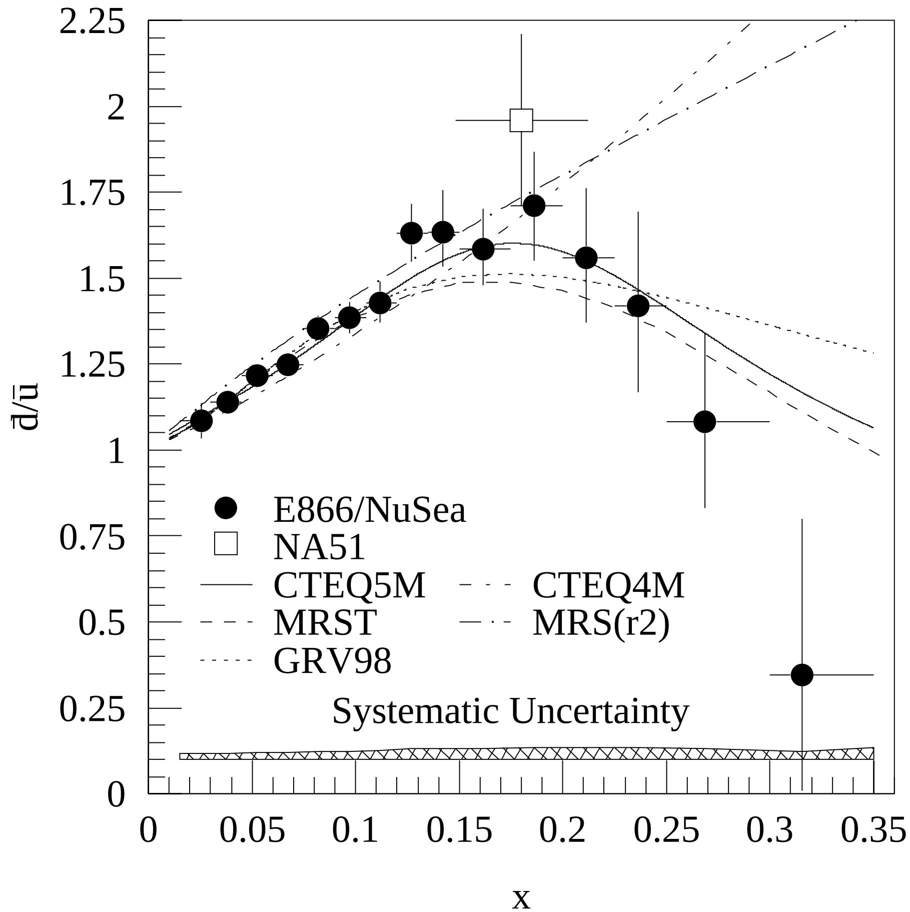
$$\bar{d} \ll \bar{u}$$



E.A. Hawker, et al. [FNAL E866/NuSea Collaboration], Measurement of the light antiquark flavor asymmetry in the nucleon sea, PRL 80, 3715 (1998)

E866 required significant changes in the hi-x sea distributions





With increased flexibility in the parameterization of the sea-quark distributions, good fits are obtained

E.A. Hawker, et al. [FNAL E866/NuSea Collaboration], Measurement of the light antiquark flavor asymmetry in the nucleon sea, PRL 80, 3715 (1998)

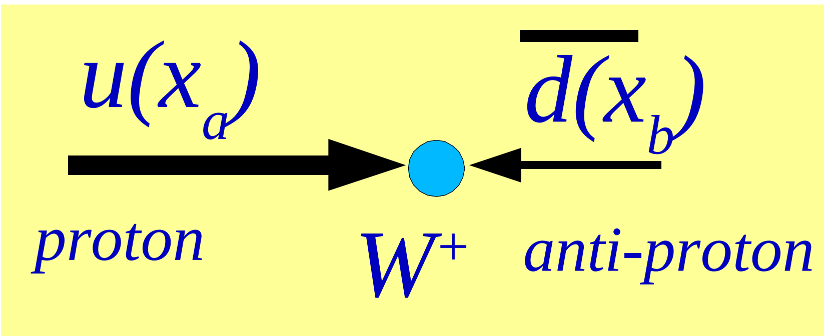
H. L. Lai, et al.} [CTEQ Collaboration], Global {QCD} analysis of parton structure of the nucleon: CTEQ5 parton distributions, EPJ C12, 375 (2000)

Next ...

2) W Rapidity Asymmetry

Where do the W's and Z's come from ???

$$\frac{d\sigma}{dy}(W^{\pm}) = \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\sum_{q\bar{q}}|V_{q\bar{q}}|^2\left[q(x_a)\,\bar{q}(x_b) + q(x_b)\,\bar{q}(x_a)\right]$$



For anti-proton:

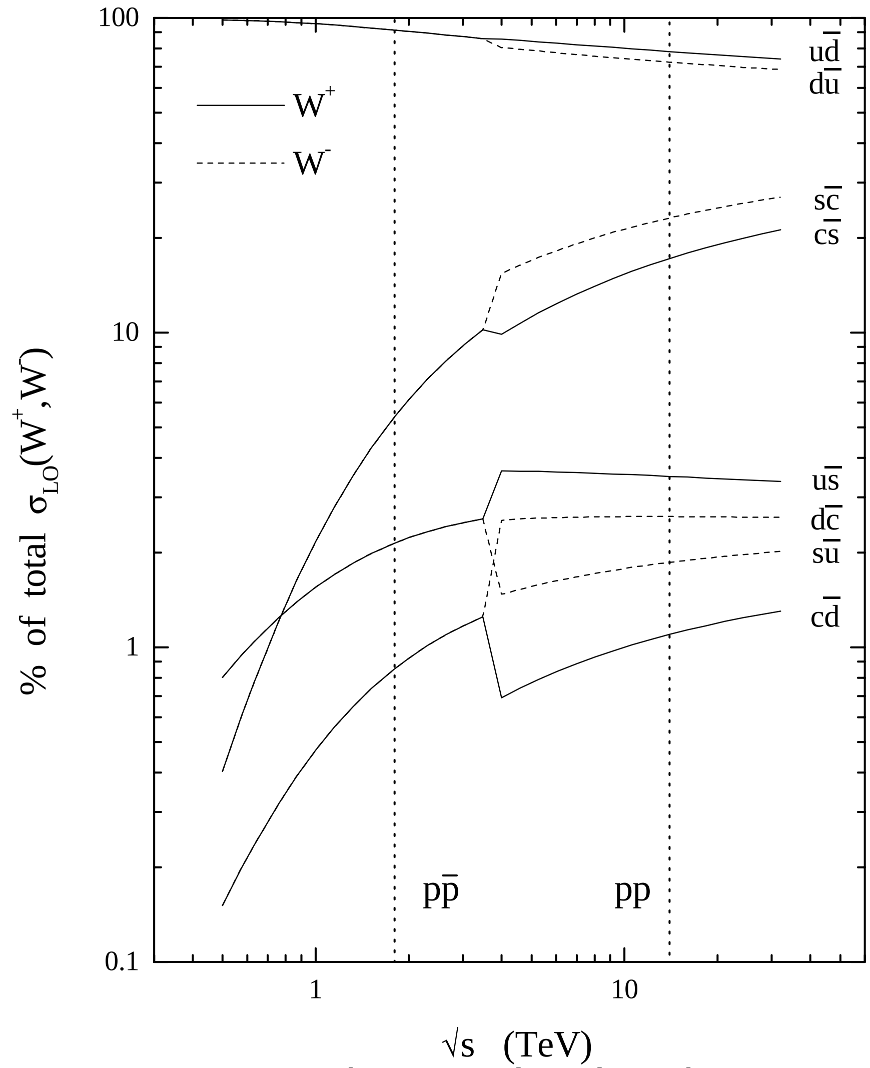
$$u(x) \Leftrightarrow \bar{u}(x) \qquad d(x) \Leftrightarrow \bar{d}(x)$$

Therefore

$$\frac{d\sigma}{dy}(W^{+}) \approx \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\left[u(x_a)\,d(x_b)\right]$$

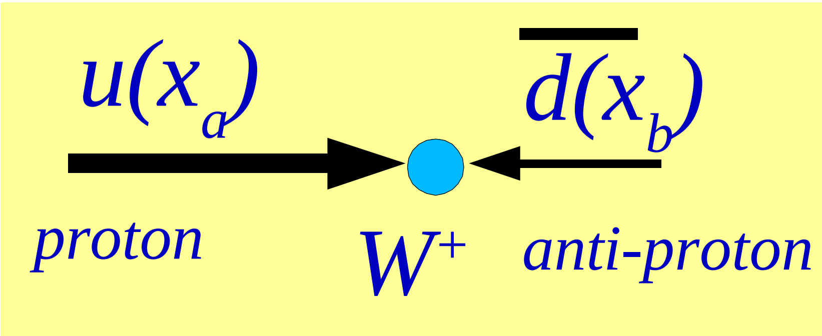
$$\frac{d\sigma}{dy}(W^{-}) \approx \frac{2\pi}{3}\frac{G_F}{\sqrt{2}}\left[d(x_a)\,u(x_b)\right]$$

flavour decomposition of W cross sections



A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C23, 73 (2002); Eur. Phys. J. C4, 463 (1998)

A bit of calculation



$$A(y) = \frac{\frac{d\sigma}{dy}(W^+) - \frac{d\sigma}{dy}(W^-)}{\frac{d\sigma}{dy}(W^+) + \frac{d\sigma}{dy}(W^-)}$$

With the previous approximation,

$$A \approx \frac{u(x_a)d(x_b) - d(x_a)u(x_b)}{u(x_a)d(x_b) + d(x_a)u(x_b)} = \frac{R_{du}(x_b) - R_{du}(x_a)}{R_{du}(x_b) + R_{du}(x_a)}$$

where $\quad R_{du}(x) = \dfrac{d(x)}{u(x)}$

We can make Taylor expansions:

$$x_{1,2} = x_0 e^{\pm y} \simeq x_0(1 \pm y)$$

$$R_{du}(x_{1,2}) \approx R_{du}(x_0) \pm y\, x_0\, R'_{du}(\sqrt{\tau})$$

Thus, the asymmetry is:

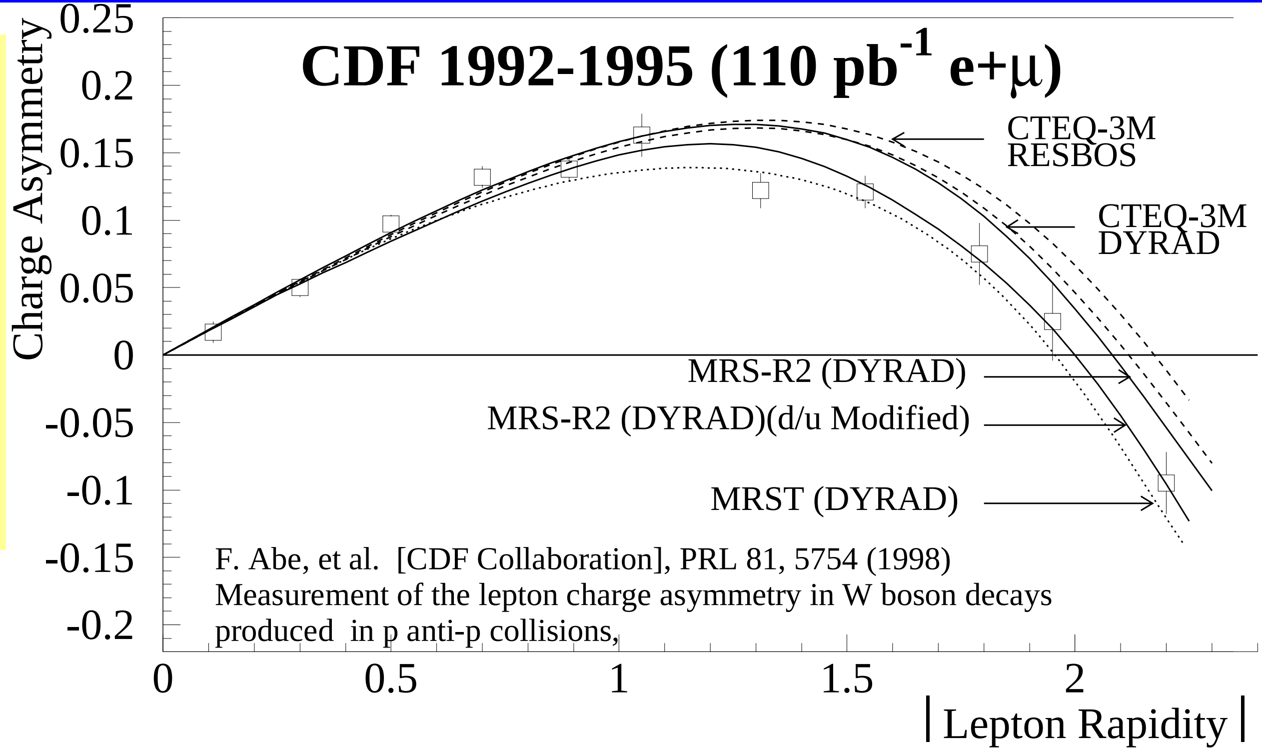
$$\boxed{A(y) = -y\, x_0 \frac{R'_{du}(x_0)}{R_{du}(x_0)}}$$

EXERCISE: Verify the above.

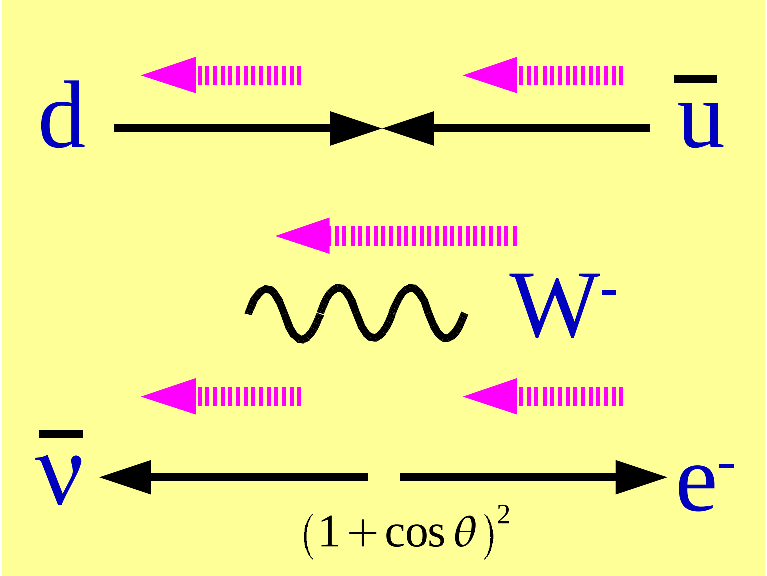
Charged Lepton Asymmetry

Unfortunately,
we don't measure the W
directly since $W \rightarrow e\nu$.

Still the lepton contains
important information



$$A(y) = \frac{\frac{d\sigma}{dy}(l^+) - \frac{d\sigma}{dy}(l^-)}{\frac{d\sigma}{dy}(l^+) + \frac{d\sigma}{dy}(l^-)}$$

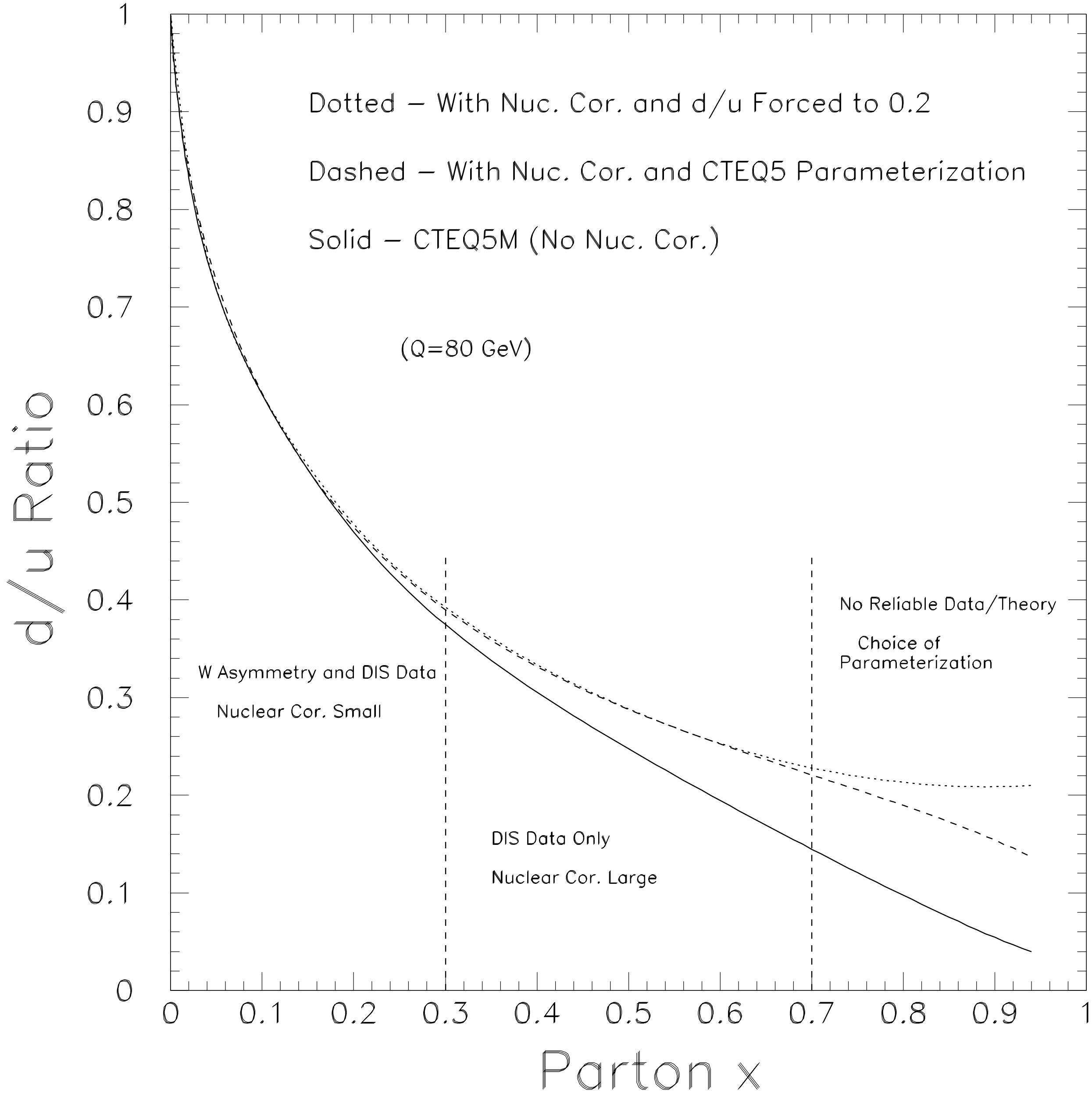


d/u Ratio at High-x

The form of the
d/u ratio at large x
as a function of

1) Parameterization

2) Nuclear Corrections



S. Kuhlmann, et al., Large-x parton distributions, PL B476, 291 (2000)

End of Part I: Where have we been???

History:

Discovery of J/ψ, Upsilon, W/Z, and "New Physics" ???

Calculation of $q\ q \to \mu^+\mu^-$ in the Parton Model

Scaling form of the cross section

Rapidity, longitudinal momentum, and x_F

Comparison with data:

NLO QCD corrections essential (the K-factor)

σ(pd)/σ(pp) important for d-bar/ubar

W Rapidity Asymmetry important for slope of d/u at large x

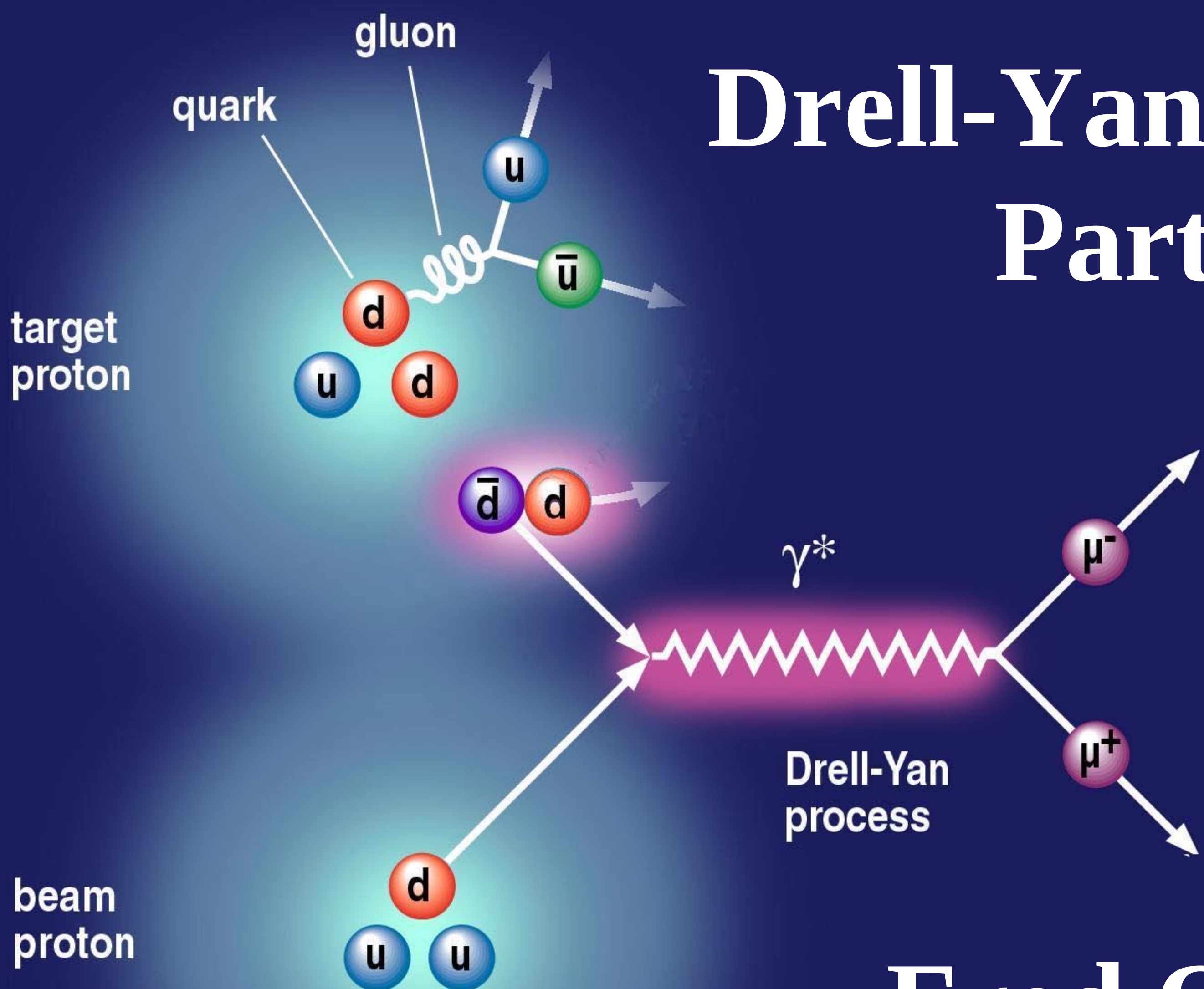
Where are we going?

P_T Distribution

W-mass measurement

Resummation of soft gluons

Drell-Yan Process: Part II



Fred Olness

SMU

Part II: W Boson Production as an example

Finding the W Boson Mass:

The Jacobian Peak, and the W Boson P_T

Multiple Soft Gluon Emissions

Single Hard Gluon Emission

Road map of Resummation

Summing 2 logs per loop: multi-scale problem (Q, q_T)

Correlated Gluon Emission

Non-Perturbative physics at small q_T.

Transverse Mass Distribution:

Improvement over P_T distribution

What can we expect in future?

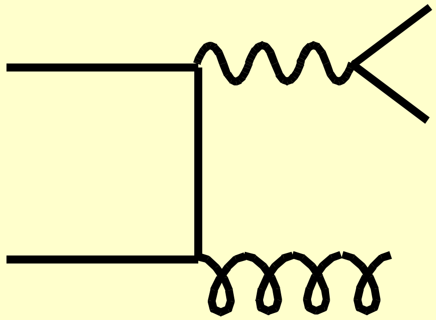
Tevatron Run II

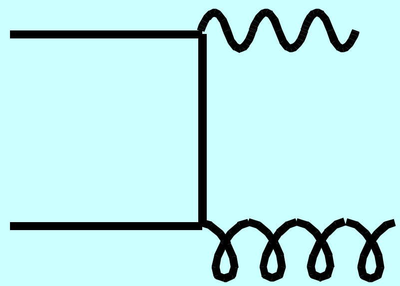
LHC

Side Note: From $pp \to \gamma/Z/W$, we can obtain $pp \to \gamma/Z/W \to l^+ l^-$

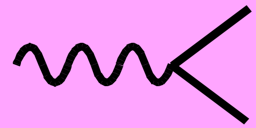
Schematically:

$$d\sigma(q\bar{q} \to l^+ l^- g) = d\sigma(q\bar{q} \to \gamma^* g) \times d\sigma(\gamma^* \to l^+ l^-)$$





$\otimes$



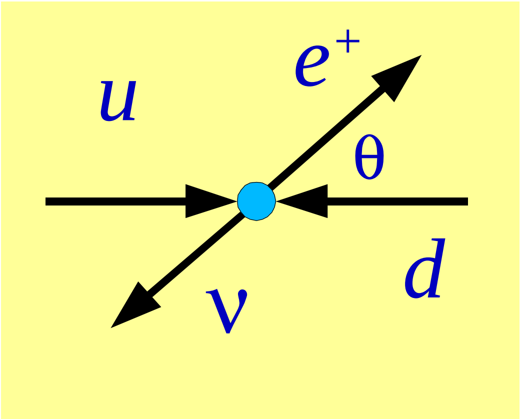
For example:

$$\frac{d\sigma}{dQ^2\, d\hat{t}}(q\bar{q} \to l^+ l^- g) = \frac{d\sigma}{d\hat{t}}(q\bar{q} \to \gamma^* g) \times \frac{\alpha}{3\pi Q^2}$$

Part II: W Boson Production as an example

How do we measure the W-boson mass?

$$u+\overline{d}\rightarrow W^{+}\rightarrow e^{+}\nu$$

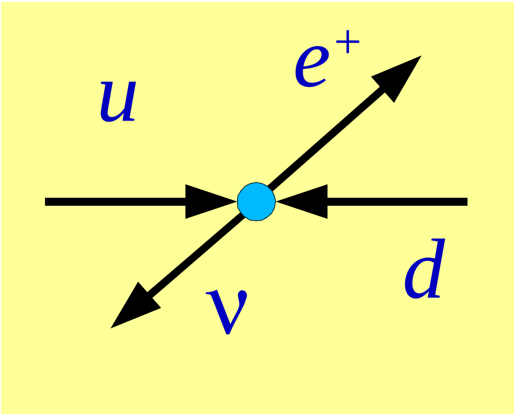


- Can't measure W directly
- Can't measure ν directly
- Can't measure longitudinal momentum

We can measure the P_T of the lepton

How can we use this to extract the W-Mass???

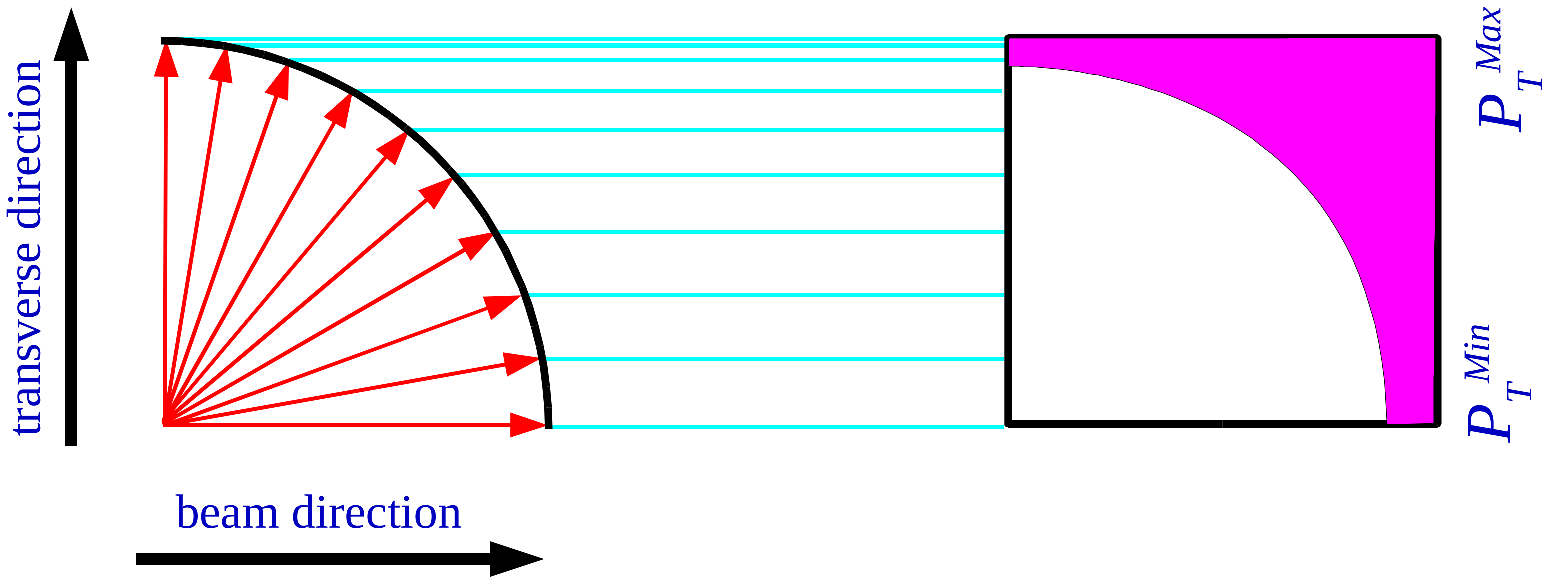
The Jacobian Peak



Suppose lepton distribution is uniform in θ

The dependence is actually $(1+\cos\theta)^2$, but we'll take care of that later

What is the distribution in P_T?



We find a peak at $P_T^{max} \approx M_W/2$

The Jacobian Peak

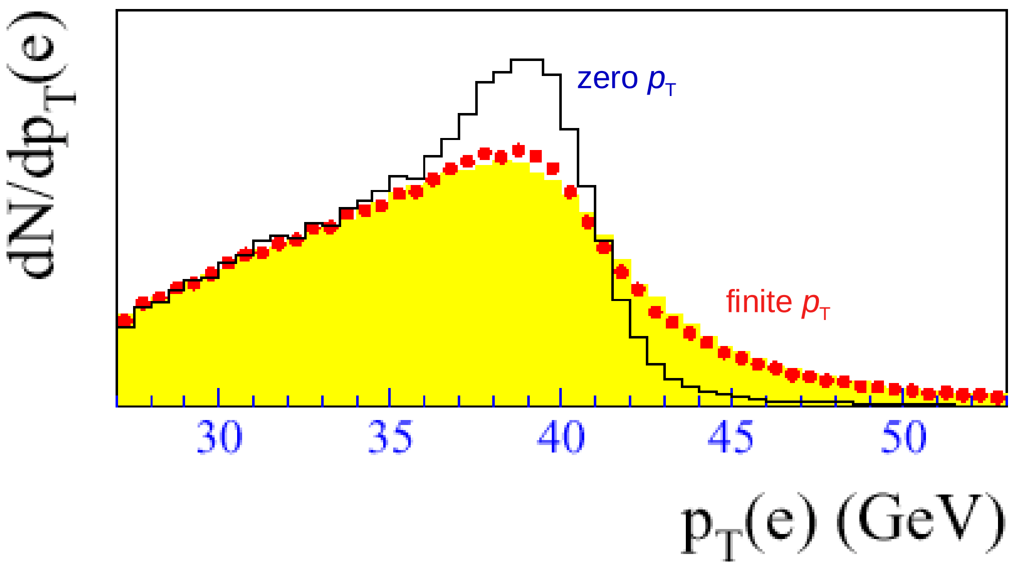
Now that we've got the picture, here's the math ... *(in the W CMS frame)*

$$p_T^2 = \frac{\hat{s}}{4}\sin^2\theta \qquad \cos\theta = \sqrt{1-\frac{4p_T^2}{\hat{s}}} \qquad \frac{d\cos\theta}{dp_T^2} = \frac{2}{\hat{s}}\frac{1}{\cos\theta}$$

So we discover the P_T distribution has a singularity at cosθ=0, or θ=π/2

$$\frac{d\sigma}{dp_T^2} = \frac{d\sigma}{d\cos\theta}\times\frac{d\cos\theta}{dp_T^2} \approx \frac{d\sigma}{d\cos\theta}\times\frac{1}{\cos\theta}$$

singularity!!!



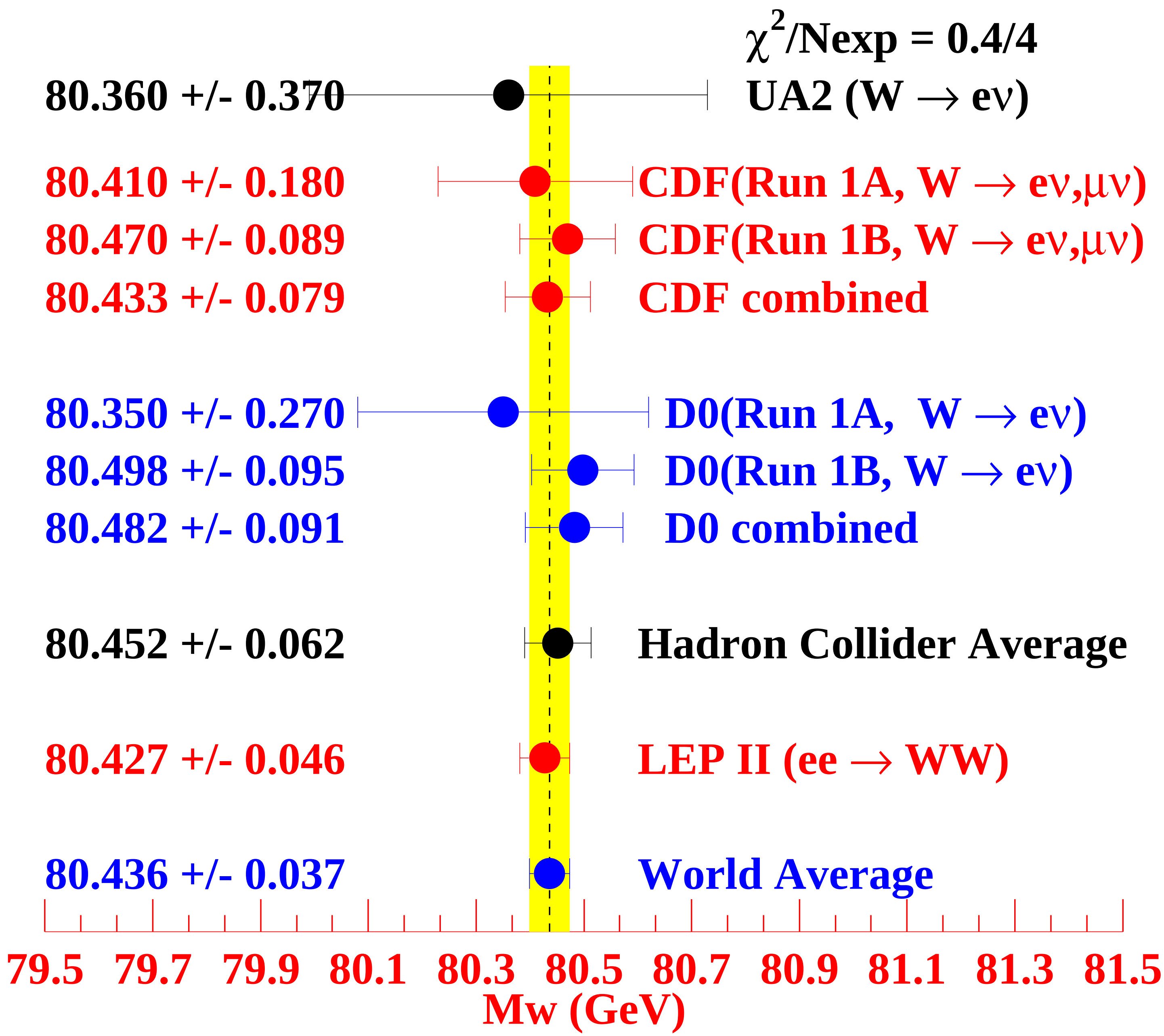
BUT !!!

Measuring the Jacobian peak is complicated if the W boson has finite P_T.

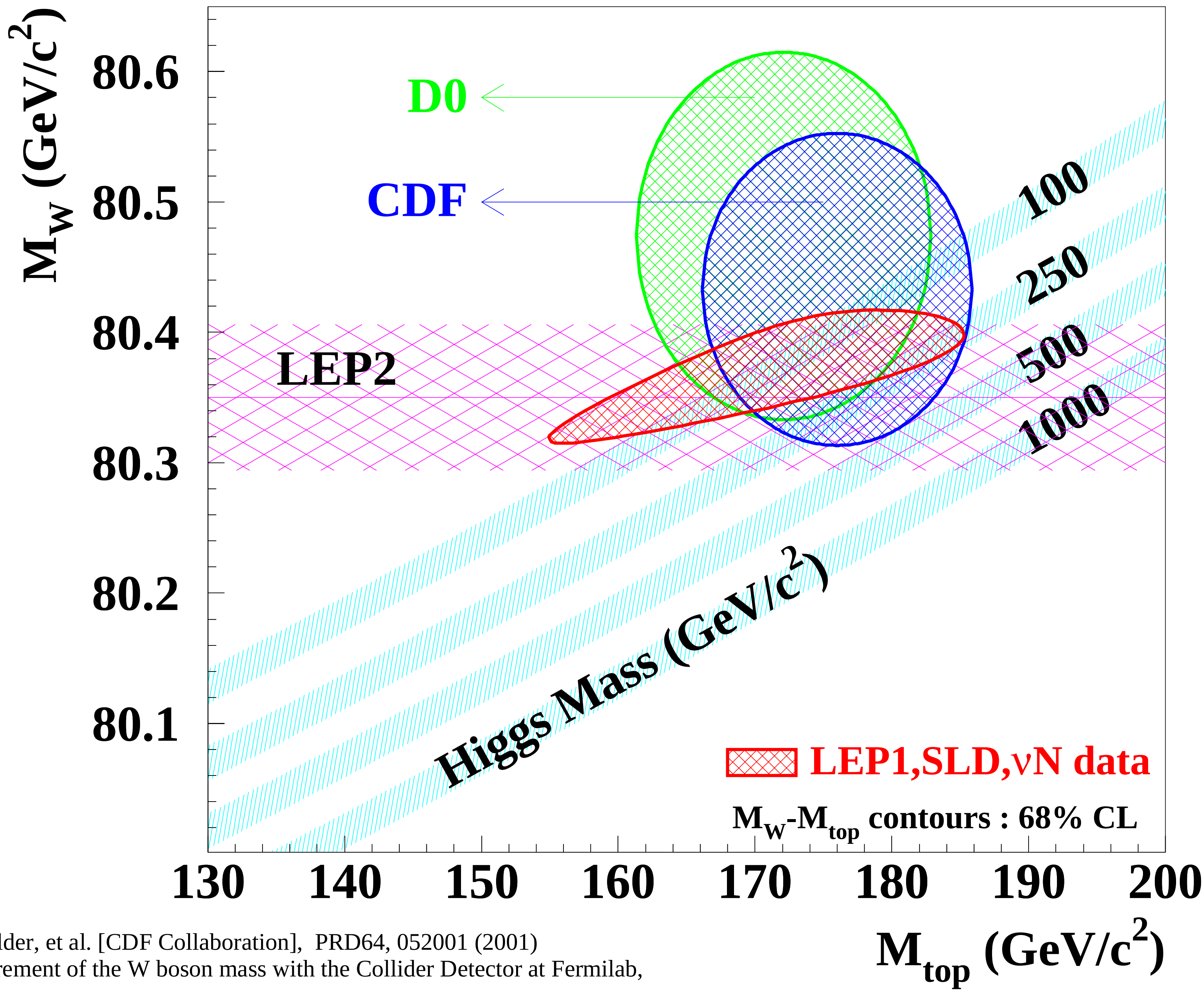
1) The W-mass is important fundamental quantity of the Standard Model

2) P_T Distribution is important for measuring the W-mass

The W-Mass is an important fundamental quantity



The W-Mass is an important fundamental quantity



T.Affolder, et al. [CDF Collaboration], PRD64, 052001 (2001)
Measurement of the W boson mass with the Collider Detector at Fermilab,

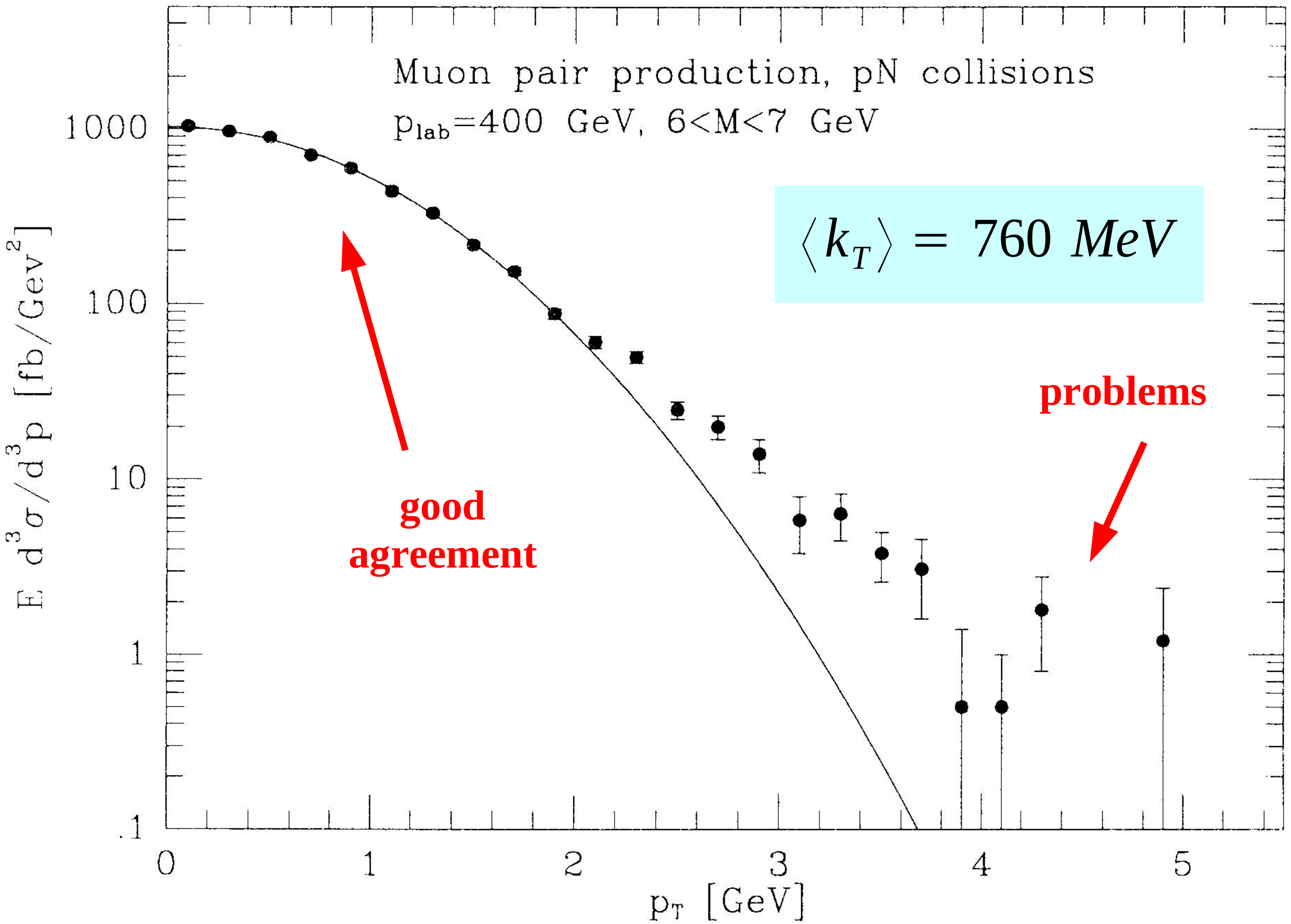
What gives the W

P_T ???

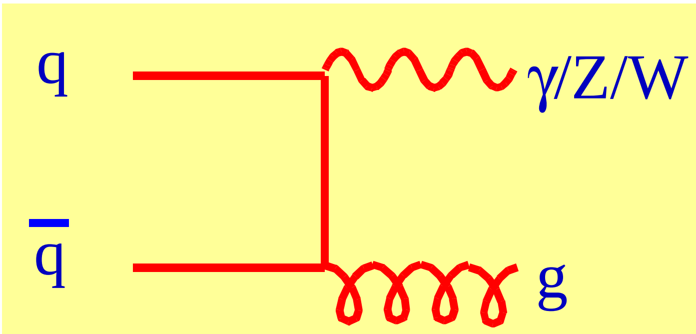
What about the intrinsic k_T of the partons?

Assume a Gaussian form:

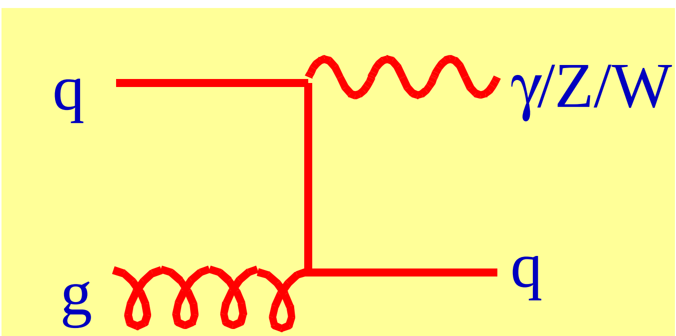
$$\frac{d^2\sigma}{d^2 p_T} \approx \sigma_0 \, e^{-p_T^2}$$



For high P_T, we need a hard parton emission



annihilation



Compton

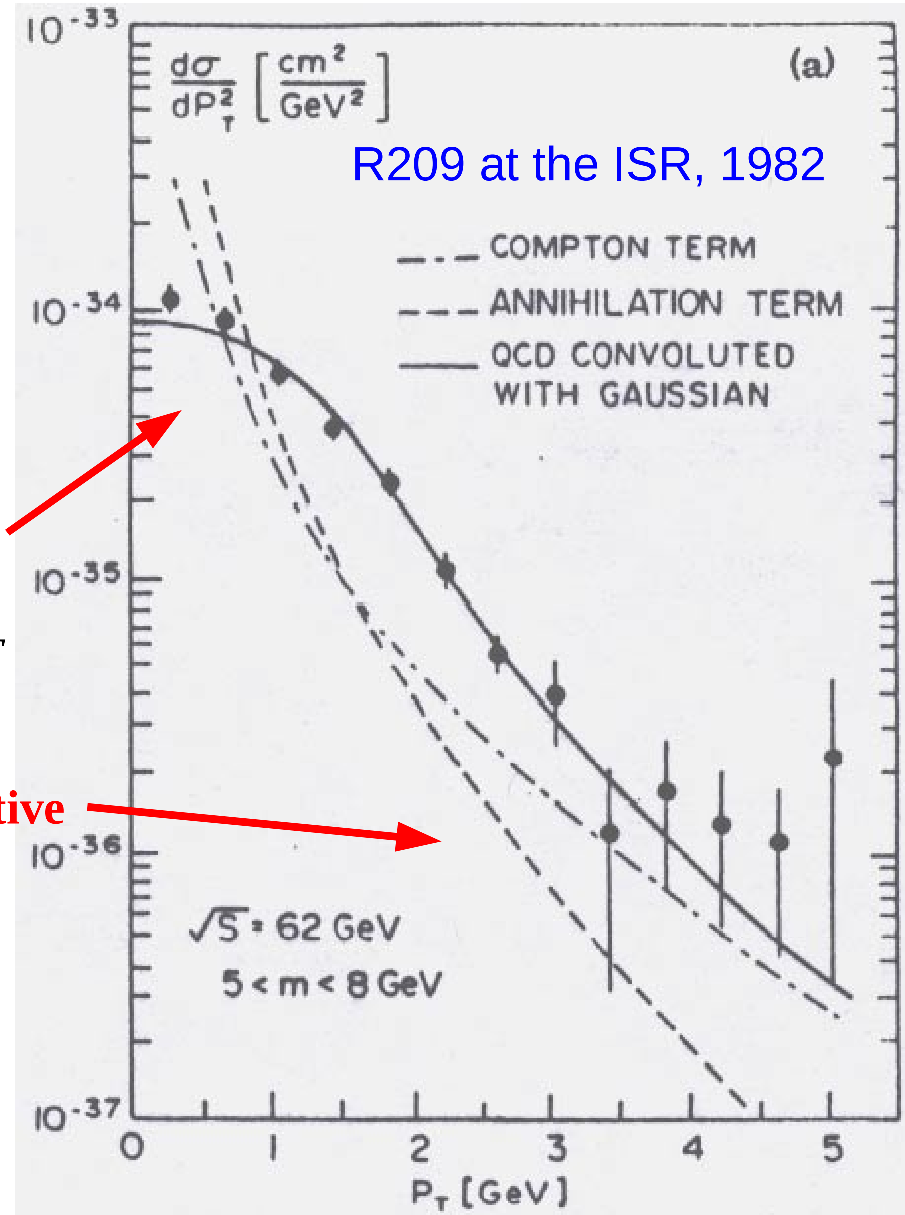
Combination of Gaussian & QCD corrections

Gaussian

$$e^{-p_T^2}$$

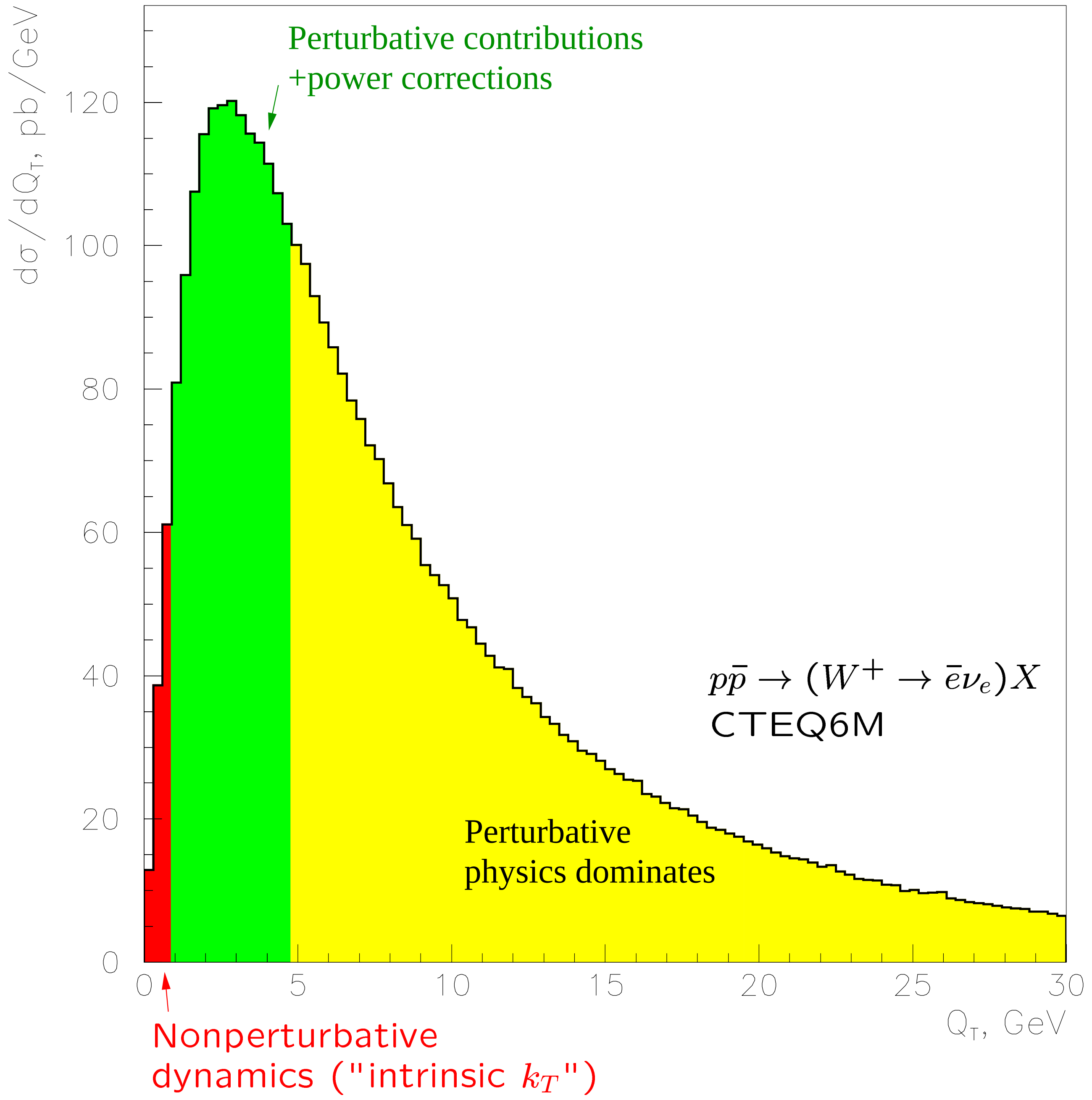
Perturbative

$$\frac{1}{p_T^2}$$



The complete P_T spectrum for the W boson

The full P_T spectrum for the W-boson showing the different theoretical regions



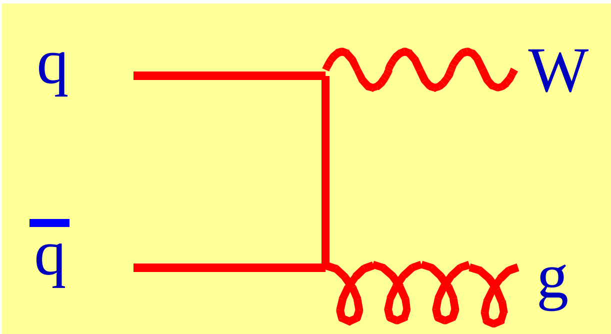
Road map for Resummation

$\Rightarrow$

BEFORE

AFTER

NLO P_T distribution for the W boson



In the limit $P_T \to 0$

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \frac{4\alpha_s}{3\pi}\, \frac{\ln s/p_T^2}{p_T^2}$$

singular

finite

$$\int_0^s \frac{d\sigma}{d\tau\, dy\, dp_T^2}\, dp_T^2 = \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} + O(\alpha_s)$$

$$\int_0^{p_T^2} \frac{d\sigma}{d\tau\, dy\, dp_T^2}\, dp_T^2 = \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \left\{ 1 - \int_{p_T^2}^{s} \frac{4\alpha_s}{3\pi}\, \frac{\ln s/p_T^2}{p_T^2}\, d\,p_T^2 \right\}$$

$$= \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \left\{ 1 - \frac{2\alpha_s}{3\pi} \ln^2 \frac{s}{p_T^2} \right\}$$

effect of gluon emission

$$= \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \exp\left\{ \frac{2\alpha_s}{3\pi} \ln^2 \frac{s}{p_T^2} \right\}$$

assume this exponentiates

Parisi & Petronzio, NP B154, 427 (1979)
Dokshitzer, D'yakanov, Troyan, Phy. Rep. 58, 271 (1980)

Resummation of soft gluons: Step #1

Differentiating the previous expression for $d^2\sigma/d\tau\, dy$

Sudakov Form Factor

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \frac{4\alpha_s}{3\pi}\,\frac{\ln s/p_T^2}{p_T^2} \times \exp\left\{-\frac{2\alpha_s}{3\pi}\ln^2\frac{s}{p_T^2}\right\}$$

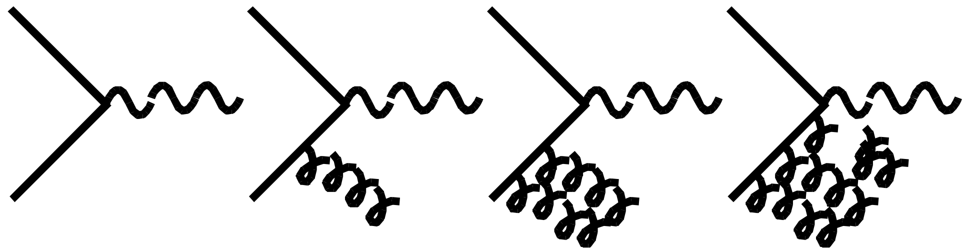
finite at p_T=0

We just resummed (exponentiated) an infinite series of soft gluon emissions

$$e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2!} - \frac{(\alpha_s L^2)^3}{3!} + \ldots$$

$$L = \ln\frac{s}{p_T^2}$$

Soft gluon emissions treated as uncorrelated



I've skipped over some details ..

Parisi & Petronzio, NP B154, 427 (1979)
Dokshitzer, D'yakanov, Troyan, Phy. Rep. 58, 271 (1980)

Curci, Greco, Srivastava, PRL 43, 834 (1979); NP B159, 451 (1979)
Jeff Owens, 2000 CTEQ Summer School Lectures

We skipped over a few details ...

1) We summed only the leading logarithmic singularity, $\alpha_s L^2$.

We'll need to do better to ensure convergence of perturbation series

2) We assumed exponentiation; proof of this is non-trivial.

The existence of two scales $(Q,p_T)\equiv(Q,q_T)$ yields 2 logs per loop

3) Gluon emission was assumed to be uncorrelated.

This leads to too strong a suppression at $P_T=0$.

Will need to impose momentum conservation for P_T.

4) In the limit $P_T \rightarrow 0$, terms of order $\alpha_S(\mu=P_T) \rightarrow \infty$;

Must handle this Non-Perturbative region.

1) We summed only the leading logarithmic singularity

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_s L}{q_T^2}\left\{1 + \alpha_s^1 L^2 + \alpha_s^2 L^4 + \ldots\right\}$$

we resum these terms

$$\frac{\alpha_s L}{q_T^2}\left\{ \quad + \alpha_s^1 L^1 + \alpha_s^2 L^3 + \alpha_s^3 L^5 + \ldots\right\}$$

we miss these terms

$$L = \ln \frac{s}{p_T^2}$$

The terms we are missing are suppressed by $\alpha_s L$, not α_s!

If (somehow) we could sum the sub-leading log ...

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_s L}{q_T^2}\, e^{\alpha_s (L^2 + L)}$$

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2}\left\{\alpha_s^1 (L^1 + 1) + \alpha_s^2 (L^3 + L^2) + \alpha_s^3 (L^5 + L^4) + \ldots\right\}$$

we resum these terms

$$\frac{1}{q_T^2}\left\{ \quad + \alpha_s^2 (L^1 + 1) + \alpha_s^3 (L^3 + L^2) + \alpha_s^4 (L^5 + L^4) + \ldots\right\}$$

we miss these terms

Now, the terms we are missing are suppressed only by α_s!

2) We assumed exponentiation; proof is non-trivial

Review where the logs come from

Review one-scale problem *(Q)*

resummation via RGE

Review two-scale problem (Q,q_T)

resummation via RGE+ Gauge Invariance

Where do the Logs come from?

Total Cross Section: $\sigma(e^+e^-)$ at 3 Loops

$$\sigma(Q^2)=\sigma_0\Bigg\{1+\frac{\alpha_s(Q^2)}{4\pi}(3C_F)+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2\left[-C_F^2\left(\frac{3}{2}\right)+C_FC_A\left(\frac{123}{2}-44\zeta(3)\right)-C_FTn_f(-22+16\zeta(3))\right]$$

$$+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^3\Bigg[C_F^3\left(-\frac{69}{2}\right)+C_F^2C_A(-127-572\zeta(3)+880\zeta(5))$$

$$+C_FC_A^2\left(\frac{90445}{54}-\frac{10948}{9}\zeta(3)+\frac{440}{3}\zeta(5)\right)$$

$$+C_F^2Tn_f(-29+304\zeta(3)-320\zeta(5))+C_FC_ATn_f\left(-\frac{31040}{27}+\frac{7168}{9}\zeta(3)+\frac{160}{3}\zeta(5)\right)$$

$$+C_FT^2n_f^2\left(\frac{4832}{27}-\frac{1216}{9}\zeta(3)\right)-C_F\pi^2\left(\frac{11}{3}C_A-\frac{4}{3}Tn_f\right)^2+\frac{\left(\sum_f Q_f\right)^2}{\left(N\sum_f Q_f^2\right)}\frac{D}{16}\left(\frac{176}{3}-128\zeta(3)\right)\Bigg]\Bigg\}. \qquad (5.1)$$

Rev. Mod. Phys., Vol. 67, No. 1, January 1995

One mass scale: Q^2. No logarithms!!!

Drelly-Yan at 2 Loops:

$$
\begin{aligned}
H_{q\bar{q}}^{(2),S+V}(z) = {} & \left[\frac{\alpha_s}{4\pi}\right]^2 \delta(1-z)\Bigg\{ C_A C_F \left[\left[\tfrac{193}{3} - 24\zeta(3)\right]\ln\left[\frac{Q^2}{M^2}\right] - 11\ln^2\left[\frac{Q^2}{M^2}\right] - \tfrac{12}{5}\zeta(2)^2 + \tfrac{592}{9}\zeta(2) + 28\zeta(3) - \tfrac{1535}{12}\right] \\
& + C_F^2 \left[[18 - 32\zeta(2)]\ln^2\left[\frac{Q^2}{M^2}\right] + [24\zeta(2) - 176\zeta(3) - 93]\ln\left[\frac{Q^2}{M^2}\right] \right. \\
& \left. + \tfrac{8}{5}\zeta(2)^2 - 70\zeta(2) - 60\zeta(3) + \tfrac{511}{4} \right] \\
& + n_f C_F \left[2\ln^2\left[\frac{Q^2}{M^2}\right] - \tfrac{34}{3}\ln\left[\frac{Q^2}{M^2}\right] + 8\zeta(3) - \tfrac{112}{9}\zeta(2) + \tfrac{127}{6} \right] \Bigg\} \\
& + C_A C_F \left[-\tfrac{44}{3}\mathcal{D}_0(z)\ln^2\left[\frac{Q^2}{M^2}\right] + \left\{ \left[\tfrac{536}{9} - 16\zeta(2)\right]\mathcal{D}_0(z) - \tfrac{176}{3}\mathcal{D}_1(z) \right\}\ln\left[\frac{Q^2}{M^2}\right] \right. \\
& \left. - \tfrac{176}{3}\mathcal{D}_2(z) + \left[\tfrac{1072}{9} - 32\zeta(2)\right]\mathcal{D}_1(z) + \left[56\zeta(3) + \tfrac{176}{3}\zeta(2) - \tfrac{1616}{27}\right]\mathcal{D}_0(z) \right] \\
& + C_F^2 \left[[64\mathcal{D}_1(z) + 48\mathcal{D}_0(z)]\ln^2\left[\frac{Q^2}{M^2}\right] + \{192\mathcal{D}_2(z) - 96\mathcal{D}_1(z) - [128 + 64\zeta(2)]\mathcal{D}_0(z)\}\ln\left[\frac{Q^2}{M^2}\right] \right. \\
& \left. + 128\mathcal{D}_3(z) - (128\zeta(2) + 256)\mathcal{D}_1(z) + 256\zeta(3)\mathcal{D}_0(z) \right] \\
& + n_f C_F \left[\tfrac{8}{3}\mathcal{D}_0(z)\ln^2\left[\frac{Q^2}{M^2}\right] + \left[\tfrac{32}{3}\mathcal{D}_1(z) - \tfrac{80}{9}\mathcal{D}_0(z)\right]\ln\left[\frac{Q^2}{M^2}\right] + \tfrac{32}{3}\mathcal{D}_2(z) - \tfrac{160}{9}\mathcal{D}_1(z) - \left[\tfrac{224}{27} - \tfrac{32}{3}\zeta(2)\right]\mathcal{D}_0(z) \right] .
\end{aligned}
\tag{7.14}
$$

Two mass scales: $\{Q^2, M^2\}$. Logarithms!!!

Renormalization Group Equation

More Differential Quantities ⇒ More Mass Scales ⇒ More Logs!!!

$$\frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right)$$

$$\frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right) \quad and \quad \ln\left(\frac{q_T^2}{\mu^2}\right)$$

How do we resum logs? Use the Renormalization Group Equation

For a physical observable R:

$$\mu \frac{dR}{d\mu} = 0$$

Using the chain rule:

$$\left\{ \mu^2 \frac{\partial}{\partial \mu^2} + \underbrace{\left[\mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2} \right]}_{\beta(\alpha_s(\mu))} \frac{\partial}{\partial \alpha_s(\mu^2)} \right\} R(\mu^2, \alpha_s(\mu^2)) = 0$$

Solution⇒ $\ln\left(\frac{Q^2}{\mu^2}\right) = \int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \frac{dx}{\beta(x)}$

Renormalization Group Equation: *OVER SIMPLIFIED!*

$$\left\{\mu^2 \frac{\partial}{\partial \mu^2} + \beta\left(\alpha_s(\mu)\right) \frac{\partial}{\partial \alpha_s(\mu^2)}\right\} R(\mu^{2,} \alpha_s(\mu^2)) \;=\; 0$$

If we expand R in powers of α_s, and we know β, we then know μ dependence of R.

$$R(\mu, Q, \alpha_s(\mu^2)) \;=\; R_0 + \alpha_s(\mu^2)\, R_1\left[\ln\left(Q^2/\mu^2\right) + c_1\right] + \alpha_s^2(\mu^2)\, R_2\left[\ln^2\left(Q^2/\mu^2\right) + \ln\left(Q^2/\mu^2\right) + c_2\right] + O(\alpha_s^3(\mu^2))$$

Since μ is arbitrary, choose μ=Q.

$$R(Q, Q, \alpha_s(Q^2)) \;=\; R_0 + \alpha_s(Q^2)\, R_1\left[0 + c_1\right] + \alpha_s^2(Q^2)\, R_2\left[0 + 0 + c_2\right] + \ldots$$

We just summed the logs

Two-Scale Problems

For $R(\mu, Q, \alpha_s)$, we could resum $\ln(Q^2/\mu^2)$ by taking $Q=\mu$.

What about $R(\mu, Q, q_T, \alpha_s)$; how do we resum $\ln(Q^2/\mu^2)$ and $\ln(q_T^2/\mu^2)$.

Are we stuck? Can't have $\mu^2=Q^2$ and $\mu^2=q_T^2$ at the same time!

Solution: Use Gauge Invariance; cast in similar form to RGE

Use axial-gauge with axial vector ξ.

This enters the cross section in the form: $(\xi \bullet p)$.

$$\sigma\left(x, \frac{Q^2}{\mu^2}, \frac{(p\cdot\xi)^2}{\mu^2}, \ldots\right)$$

$$\frac{d\sigma}{d\mu^2}=0$$

RGE allows us to vary μ to resum logs

$$\frac{d\sigma}{d(p\cdot\xi)^2}=0$$

Gauge invariance allows us to vary $(\xi \bullet p)$ to resum logs

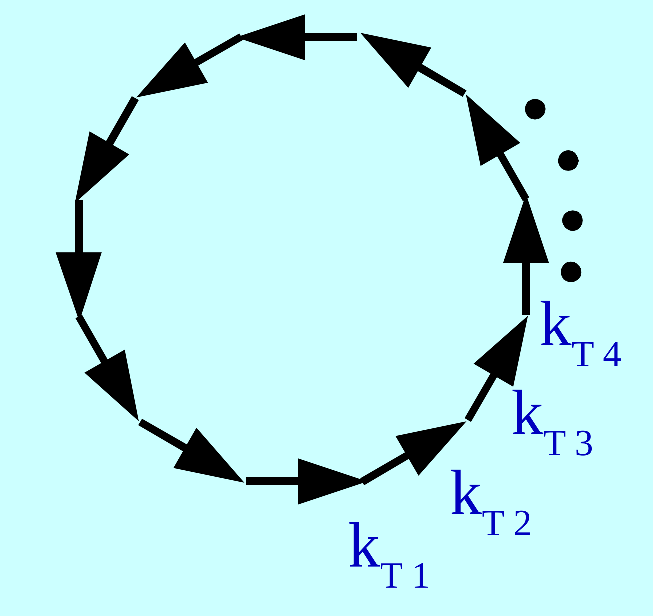
It is covenient to transform to impact parameter space (b-space) to implement this mechanism

The details will fill multiple lectures:
See Sterman TASI 1995; Soper CTEQ 1995

3) We assumed gluon emission was uncorrelated

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \frac{\ln s/p_T^2}{p_T^2} \times \exp\left\{ -\frac{2\alpha_s}{3\pi} \ln^2 \frac{s}{p_T^2} \right\}$$

This leads to too strong a suppression at P_T=0.
Need to impose momentum conservation for P_T.



A particle can receive finite k_T kicks,
yet still have P_T=0

A convenient way to impose transverse momentum conservation is in impact parameter space (b-space) via the following relation:

$$\delta^{(2)}\left(\sum_{i=1}^{n} \vec{k}_{i\,T} - \vec{p}_T \right) = \frac{1}{(2\pi)^2} \int d^2 b\, e^{-i\vec{b}\cdot\vec{p}_T} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{i\,T}}$$

4) We encounter Non-Perturbative Physics

$$S(b,Q) = \int_{\sim 1/b^2}^{\sim Q^2} \frac{d\mu^2}{\mu^2} \left\{ A(\alpha_s(\mu^2)) \ln\left(\frac{Q^2}{\mu^2}\right) + B(\alpha_s(\mu^2)) \right\}$$

as b $\rightarrow\infty$, $\alpha_S(\sim 1/b) \rightarrow \infty$. **PROBLEM!!!**

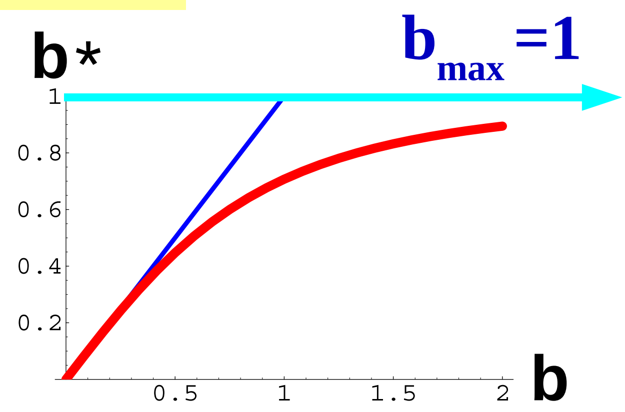
Solution: Use a Non-Perturbative Sudakov form factor (S_{NP}) in the region of large b (small q_T)

$$\tilde{\sigma}(b) \sim e^{S(b)} \rightarrow e^{S(b_*)} * e^{S_{NP}(b)}$$

with

$$b_* = \frac{b}{\sqrt{1 + b^2/b^2_{max}}}$$

Note, as b $\rightarrow \infty$, $b_* \rightarrow b_{max}$.



A Brief *(but incomplete)* History of Non-Perturbative Corrections

Original CSS:

$$S_{NP}^{CSS}(b) = h_1(b,\xi_a)+h_2(b,\xi_b)+h_3(b)\ln Q^2$$

J. Collins and D. Soper, *Nucl.Phys.* **B193** 381 (1981);
erratum: **B213** 545 (1983); J. Collins, D. Soper, and G. Sterman, *Nucl. Phys.* **B250** 199 (1985).

Davies, Webber, and Stirling (DWS):

$$S_{NP}^{DWS}(b) = b^2\left[g_1+g_2\ln\left(b_{max}Q^2\right)\right]$$

C. Davies and W.J. Stirling, Nucl. Phys. B244 337 (1984);
C. Davies, B. Webber, and W.J. Stirling, Nucl. Phys. B256 413 (1985).

Ladinsky and Yuan (LY):

$$S_{NP}^{LY}(b) = g_1 b\left[b+g_3\ln\left(100\,\xi_a\xi_b\right)\right]+g_2 b^2\ln\left(b_{max}Q\right)$$

G.A. Ladinsky and C.P. Yuan, Phys. Rev. D50 4239 (1994);
F. Landry, R. Brock, G.A. Ladinsky, and C.P.Yuan, Phys. Rev. D63 013004 (2001).

"BLNY":

$$S_{NP}^{BLNY}(b) = b^2\left[g_1+g_1g_3\ln\left(100\,\xi_a\xi_b\right)+g_2\ln\left(b_{max}Q\right)\right]$$

F. Landry, "Inclusion of Tevatron Z Data into Global Non-Perturbative QCD Fitting", Ph.D. Thesis, Michigan State University, 2001.
F. Landry, R. Brock, P. Nadolsky, and C.P.Yuan, PRD67, 073016 (2003)

"q_T resummation":

$$\widetilde{F}^{NP}(q_T) = 1-e^{-\tilde{a}\,q_T^2}$$

(not in b-space)

R.K. Ellis, Sinisa Veseli, Nucl.Phys. B511 (1998) 649-669
R.K. Ellis, D.A. Ross, S. Veseli, Nucl.Phys. B503 (1997) 309-338

Functional Extrapolation:

J. Qui, X. Zhang, PRD63, 114011 (2001); E. Berger, J. Qiu, PRD67, 034023 (2003)

Analytical Continuation:

A. Kulesza, G. Sterman, W. vogelsang, PRD66, 014011 (2002)

Recap: Where have we been???

1) We now summed the two leading logarithmic singularities, $\alpha_s(L^2+L)$.

2) We still assumed exponentiation; but sketched ingredients of proof.

The existence of two scales $(Q,p_T)\equiv(Q,q_T)$ yields 2 logs per loop

Use Renormalization Group + Gauge Invariance

Transformation to b-space

3) Gluon emission was assumed to be uncorrelated.

Impose momentum conservation for P_T. *(In b-space)*

4) Introduced Non-Perturbative function for small q_T (large b) region.

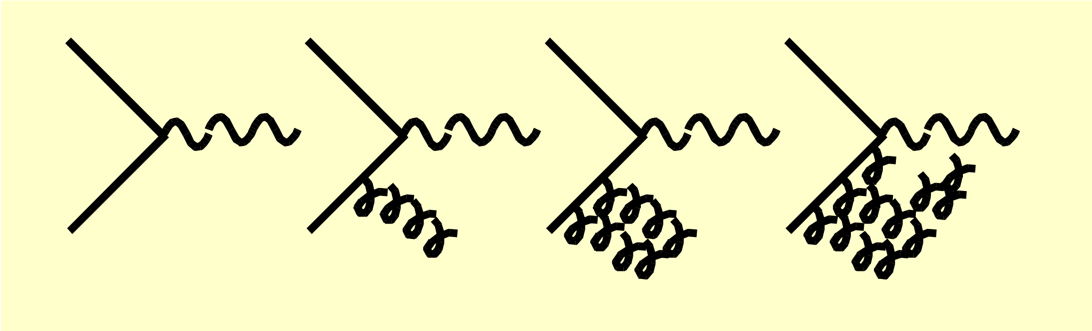
What do we get for the cross section

$$\frac{d\sigma}{dy\, dQ^2\, dq_T^2} = \frac{1}{(2\pi)^2} \int_0^\infty d^2b \; e^{i b\cdot q_T} \, \widetilde{W}(b,Q) \; e^{-S(b_*,Q)+S_{NP}(b,Q)}$$

with

$$-S(b,Q) = -\int_{\sim 1/b^2}^{\sim Q^2} \frac{d\mu^2}{\mu^2} \left\{ A \ln\left(\frac{Q^2}{\mu^2}\right) + B \right\}$$

where we have resummed the soft gluon contributions



I've left out A LOT of material

Putting it all together: $\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$

Let's expand out the resummed expression:

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_s L}{q_T^2}\, e^{\alpha_s (L^2 + L)} \sim \frac{1}{q_T^2}\left\{\alpha_s L \;+\; \alpha_s^2 (L^3 + L^2) + \ldots\right\}$$

Compare the above with the perturbative and asymptotic results:

$$d\sigma_{resum} \sim \left\{\alpha_s L \;+\; \alpha_s^2 (L^3 + L^2 + \;0 + 0) \;+\; \alpha_s^3 (L^5 + L^4) + \ldots\right\}$$

$$d\sigma_{pert} \sim \left\{\alpha_s L \;+\; \alpha_s^2 (L^3 + L^2 + L^1 + 1) \;+\; \alpha_s^3 (0 + 0)\right\}$$

$$d\sigma_{asym} \sim \left\{\alpha_s L \;+\; \alpha_s^2 (L^3 + L^2 + \;0 + 0) \;+\; \alpha_s^3 (0 + 0)\right\}$$

Note that σ_{ASYM} removes overlap between σ_{RESUM} and σ_{PERT}.

We expect:

σ_{RESUM} is a good representation for $q_T \sim 0$

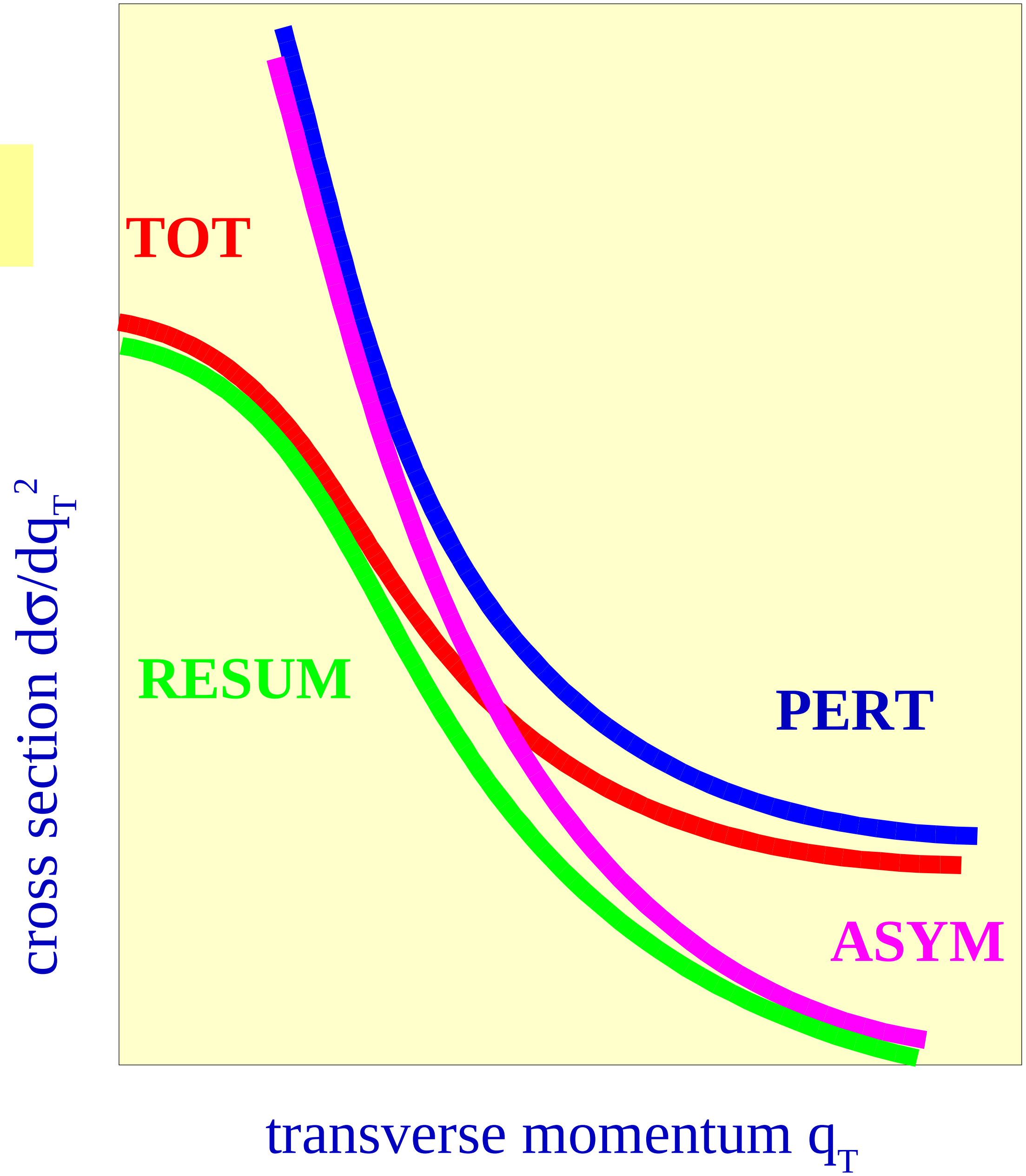
σ_{PERT} is a good representation for $q_T \sim M_W$

Putting it all together: $\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$

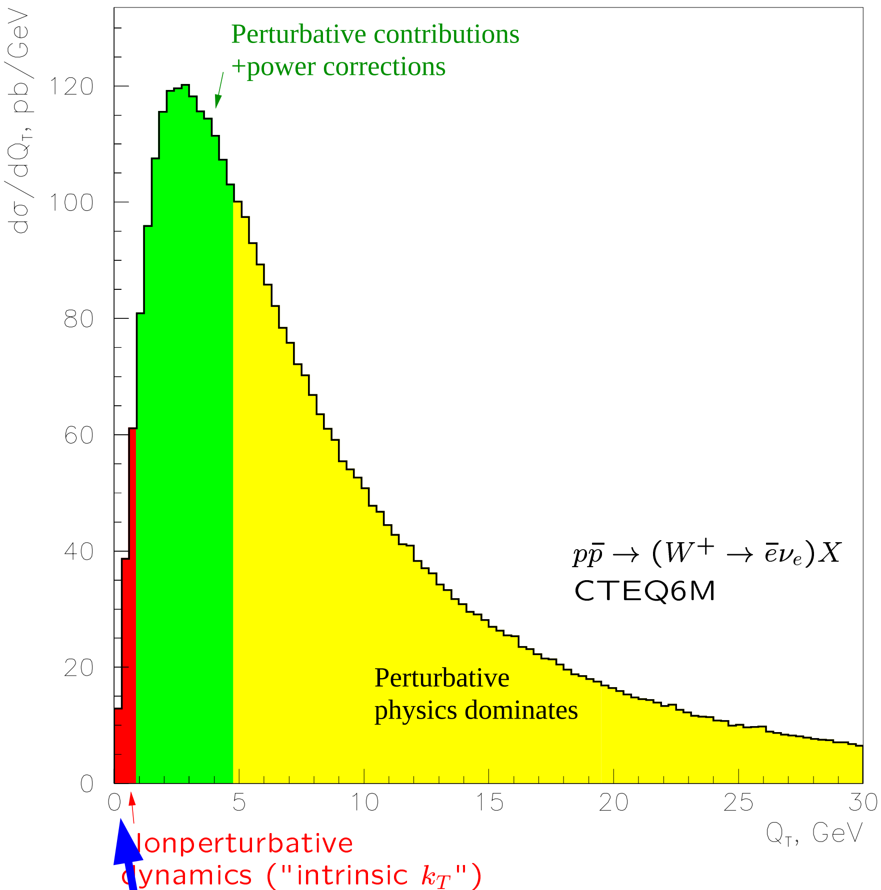
$$\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$$

σ_{RESUM} for $q_T \sim 0$

σ_{PERT} for $q_T \sim M_W$

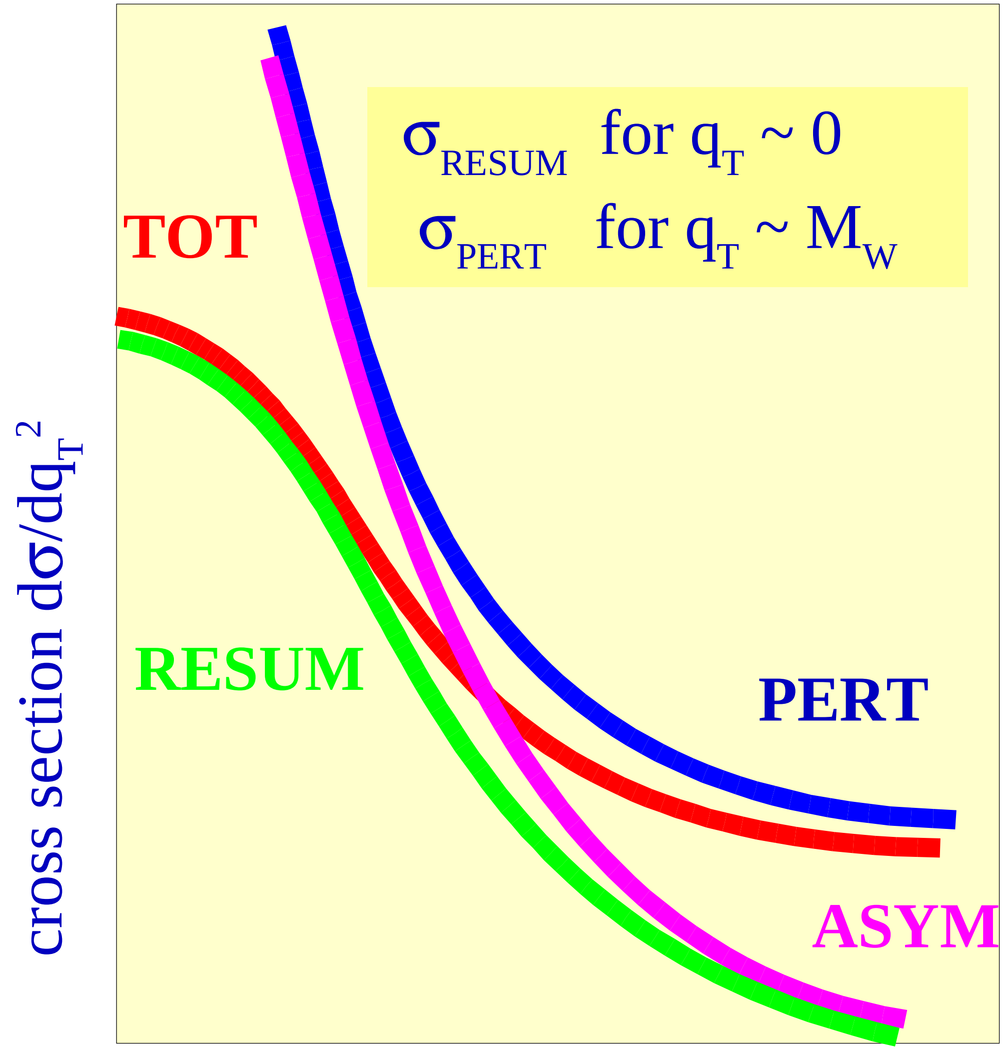


Putting it all together: $\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$



Extra power of q_T

$d\sigma/dq_T^{\,1}$

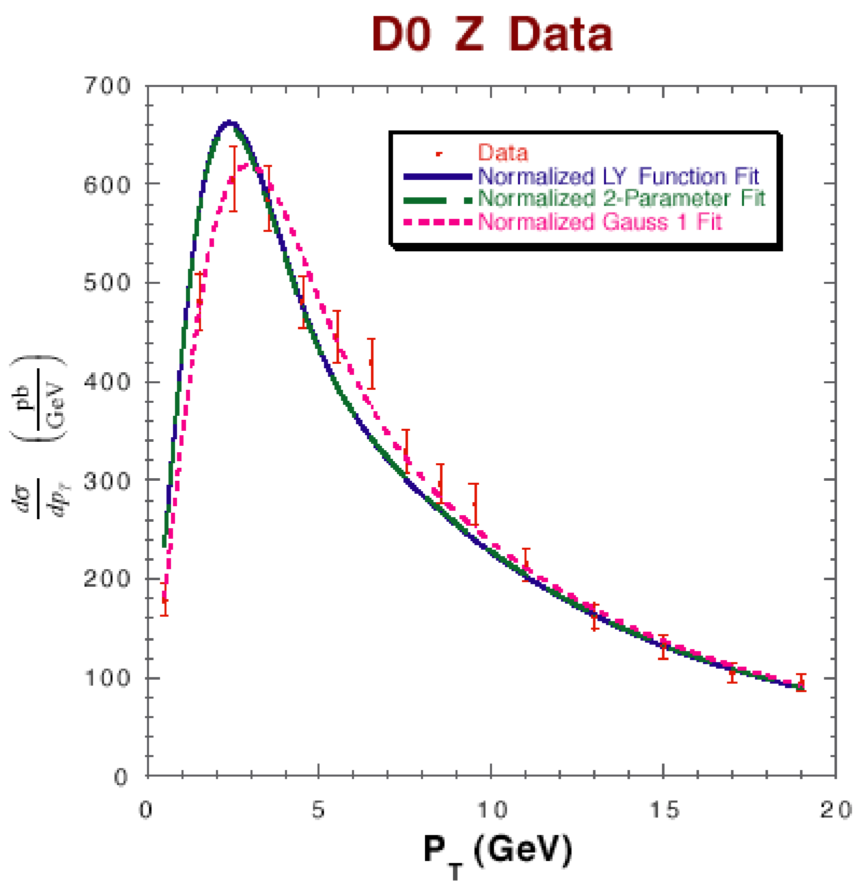


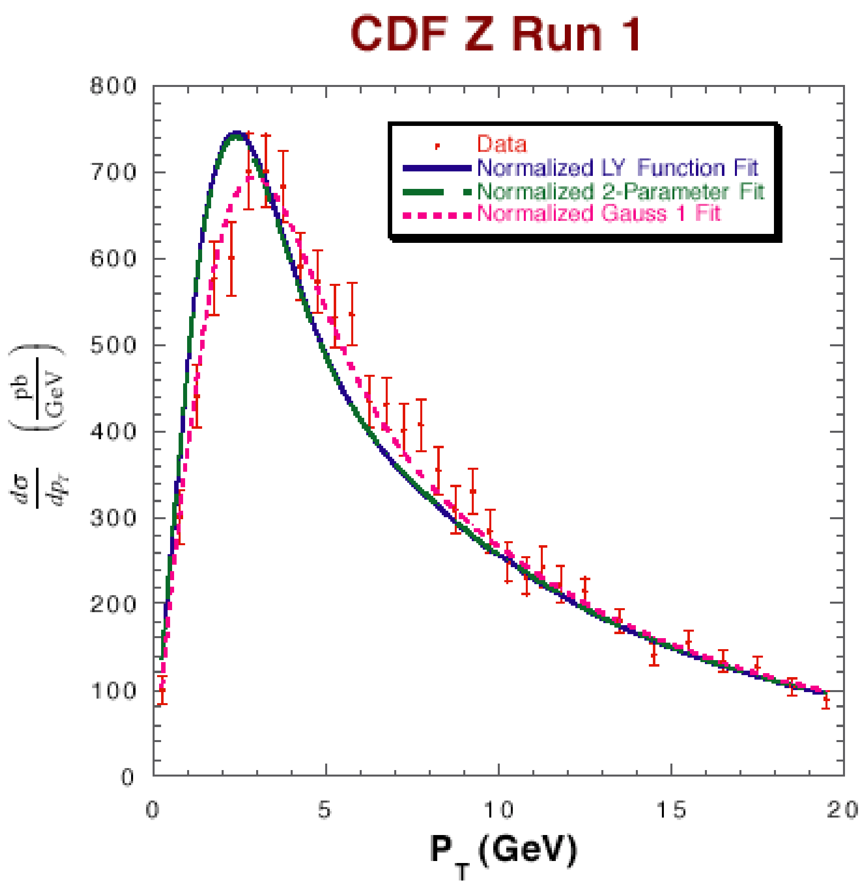
transverse momentum q_T

$\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$

Let's compare with some real results

We'll look at Z data where we can measure both leptons for $Z \rightarrow e^+e^-$





different $S_{NP}(b,Q)$ functions yield difference at small q_T.

Let's return

to the

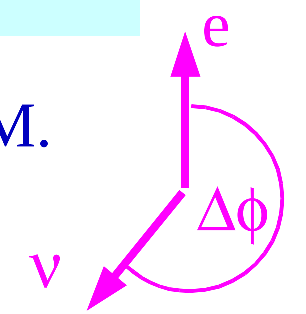
measurement

of M_W

Transverse Mass Distribution

We can measure $d\sigma/dp_T$ and look for the Jacobian peak.
However, there is another variable that is relatively insensitive to $p_T(W)$.

Transverse Mass	$M_T^2(e,\nu) = \left(\lvert\vec{p}_{eT}\rvert + \lvert\vec{p}_{\nu T}\rvert\right)^2 - \left(\vec{p}_{eT} + \vec{p}_{\nu T}\right)^2$
Invariant Mass	$M^2(e,\nu) = \left(\lvert\vec{p}_{e}\rvert + \lvert\vec{p}_{\nu}\rvert\right)^2 - \left(\vec{p}_{e} + \vec{p}_{\nu}\right)^2$



In the limit of vanishing longitudinal momentum, $M_T \sim M$.
M_T is invariant under longitudinal boosts.

M_T can also be expressed as: $$M_T^2(e,\nu) = 2\lvert\vec{p}_{eT}\rvert\lvert\vec{p}_{\nu T}\rvert\left(1 - \cos\Delta\phi_{e\nu}\right)$$

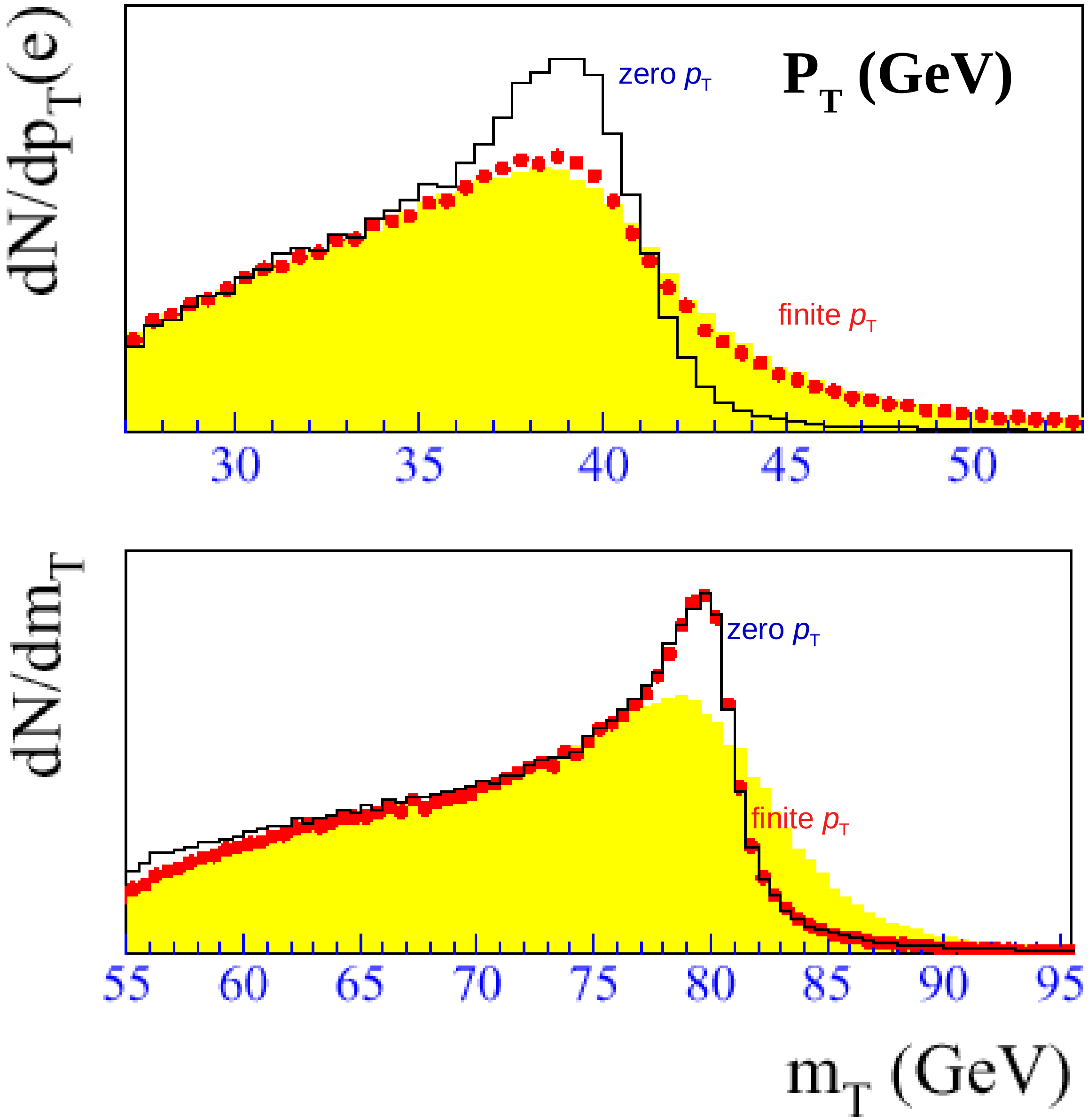
For small values of P_T^W, M_T is invariant to leading order.

Exercise:

a) Verify the above definitions of M_T are $\equiv$.

b) For $p_{Te} = +p^* + p_T^W/2$ and $p_{T\nu} = -p^* + p_T^W/2$; verify M_T is invariant to leading order in p_T^W.

Compare P_T and Transverse Mass Distribution



M_T distribution is much less sensitive to P_T of W

Still, we need P_T distribution of W to extract mass and width with precision

PDF and p_T(W) uncertainties will need to be controlled: currently uncertainty: ~10-15 & 5-10 MeV/c^2

Statistical precision in Run II will be miniscule…placing an enormous burden on control of modeling uncertainties.

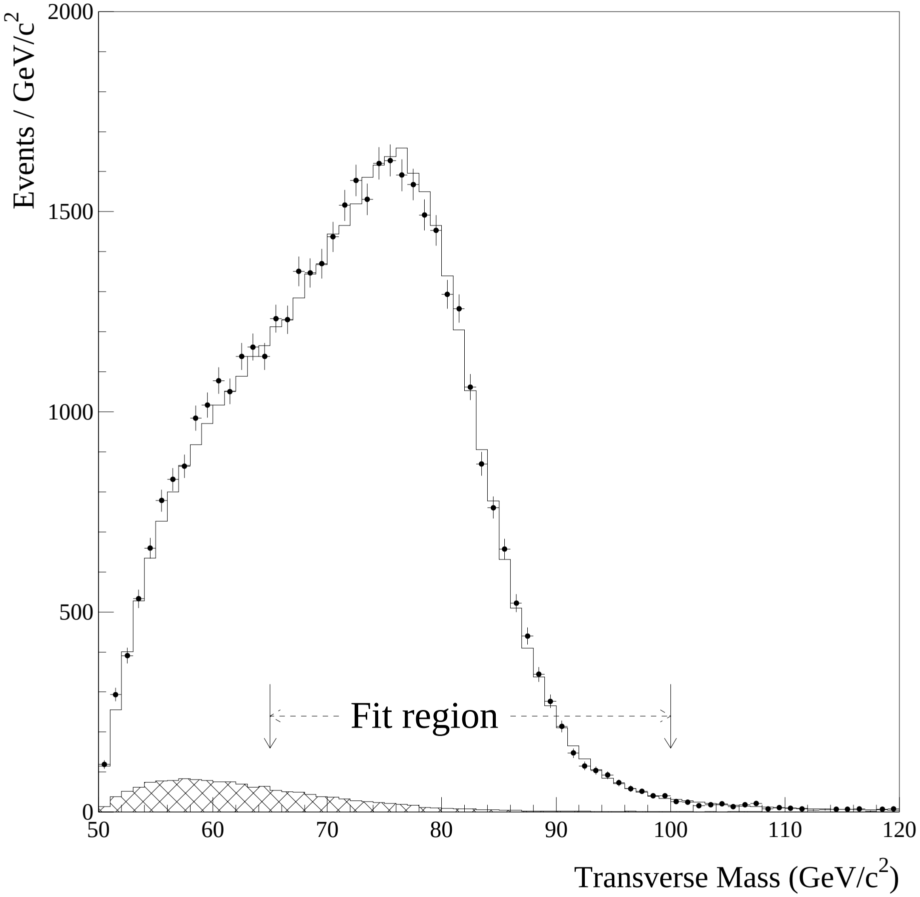
The Future:

Tevatron Run II ... *happening now*

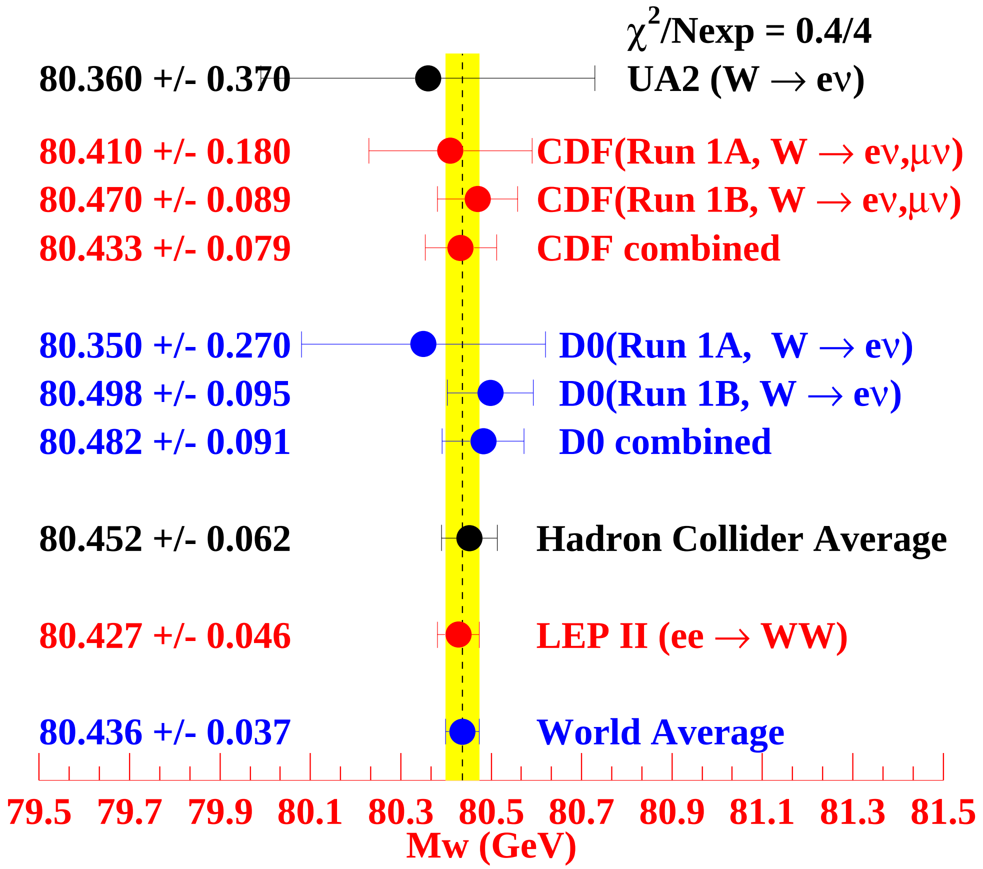
LHC ... *happening soon*

Transverse Mass Distribution and M_W Measurement

Transverse Mass Distribution from CDF



Combined World Measurements of M_W



T.Affolder, et al. [CDF Collaboration], PRD64, 052001 (2001)
Measurement of the W boson mass with the Collider Detector at Fermilab,

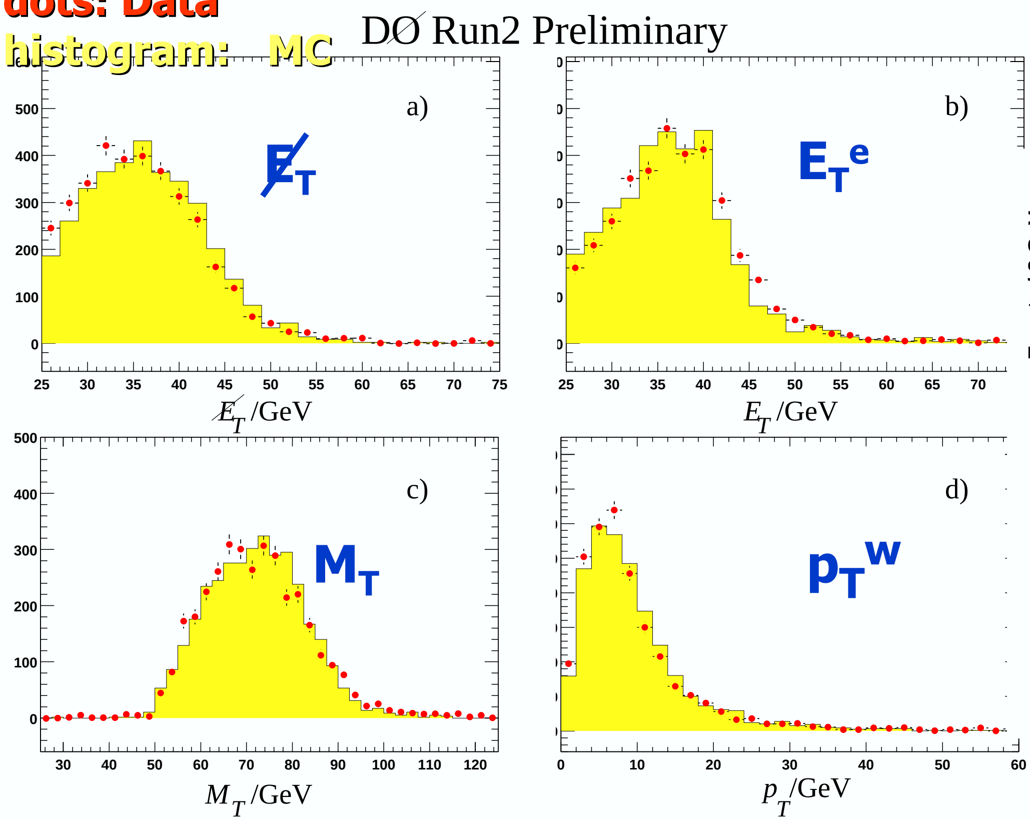
Preliminary
Run II
measurements

Electroweak Physics

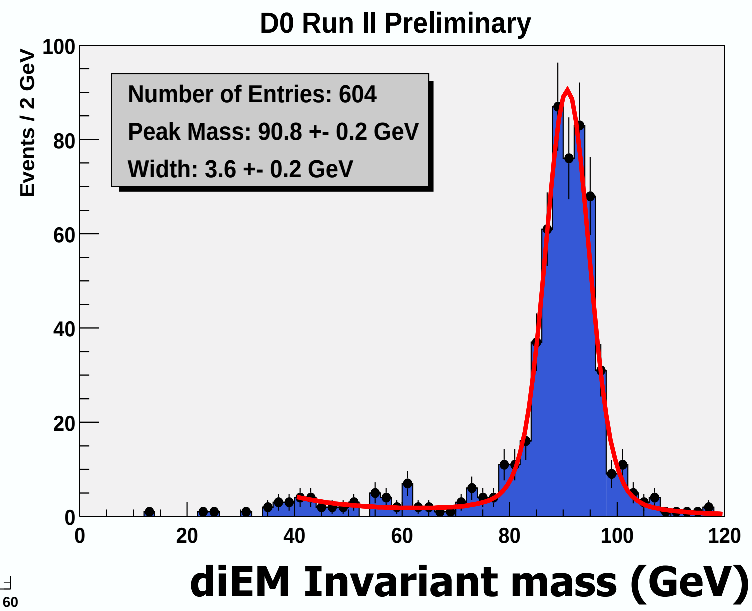
High priority measurements

- $W \to e\nu$ cross-section

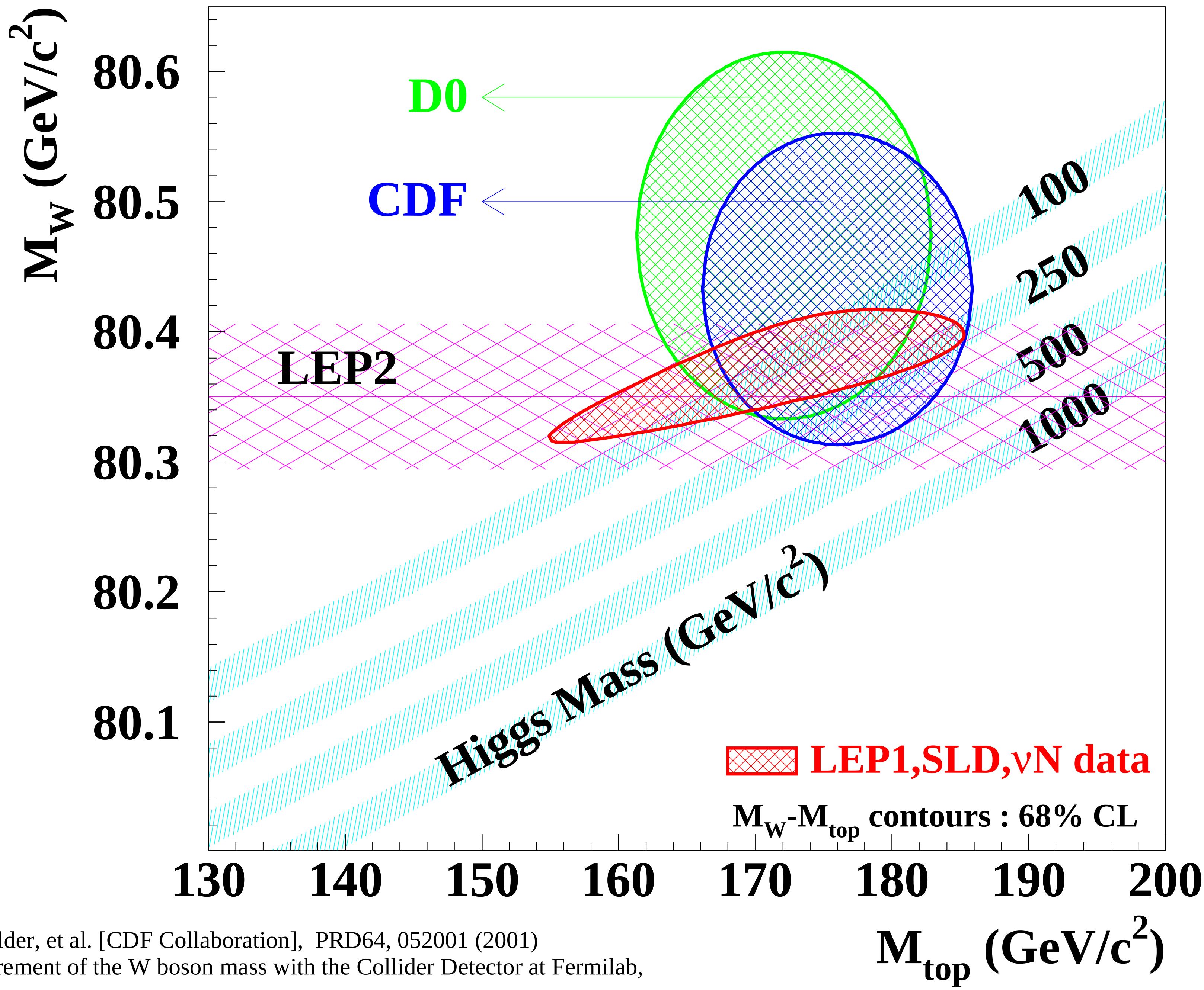
dots: Data
histogram: MC



- $Z \to e^+e^-$ cross-section



The W-Mass is an important fundamental quantity



T.Affolder, et al. [CDF Collaboration], PRD64, 052001 (2001)
Measurement of the W boson mass with the Collider Detector at Fermilab,

Part II: Drell-Yan Process: Where have we been???

Finding the W Boson Mass:

The Jacobian Peak, and the W Boson P_T

Multiple Soft Gluon Emissions

Single Hard Gluon Emission

Road map of Resummation

Summing 2 logs per loop: multi-scale problem (Q,q_T)

Correlated Gluon Emission

Non-Perturbative physics at small q_T.

Transverse Mass Distribution:

Improvement over P_T distribution

What can we expect in future?

Tevatron Run II

LHC

Thanks to ...

Jeff Owens

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C.P. Yuan

Pavel Nadolsky

Randy Scalise

Wu-Ki Tung

Steve Kuhlmann

Dave Soper

and my other CTEQ colleagues

and the many web pages where I borrowed my figures ...

References:

Ellis, Webber, Stirling

Barger & Phillips, 2nd Edition

Rick Field; Perturbative QCD

CTEQ Handbook

CTEQ Pedagogical Page:

CTEQ Lectures:

C.P. Yuan, 2002

Chip Brock, 2001

Jeff Owens, 2000

Attention:

You have reached the very last page of the internet.

We hope you enjoyed your browsing.

calculate

Now turn off your computer and go out and ~~play~~.

2003 CTEQ Summer School

Drell-Yan Process
Lecture #3

Fred Olness
(SMU)

For lecture 3, we will examine a NLO DY calculation. We will be using the notes from C.P. Yuan (beautifully typeset by Qing-Hong Cao), and supplement this with examples from FeynCalc and CompHEP

The sample files used will be posted after the school. In the meantime, refer to the attached lecture, and *Mathematica notebooks of FeynArts and FeynCalc sample calculations* which are on the web at cteq.org under the Miscellaneous heading.

A NLO Calculation of pQCD: Total Cross Section of $P\overline{P} \to W^+ + X$

C.–P. Yuan

Michigan State University

CTEQ Summer School, June 2002

Outline

1. **Parton Model**

⇒ **Born Cross Section**

2. **Factorization Theorem**

⇒ **How to organize a NLO calculation of pQCD**

3. **Feynman rules and Feynman diagrams**

⇒ **"Cut diagram" notation**

4. **Immediate Problems (Singularities)**

⇒ **Dimensional Regularization**

5. **Virtual Corrections**

6. **Real Emission Contribuation**

7. **Perturbative Parton Distribution Functions**

8. **Summary of NLO [$O(\alpha_s)$] Corrections**

Appendices:

A. γ-matrices in n dimensions

B. Some integrals and "special functions"

C. Angular integrals in n dimensions

D. Two-particle phase space in n dimensions

E. Explicit Calculations

(Typesetting: prepared by Qing-Hong Cao at MSU.)

A few references can be found in

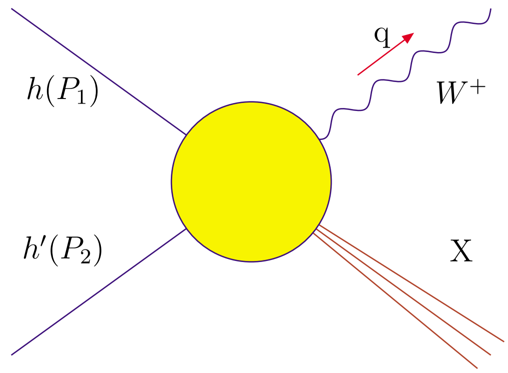
"Handbook of pQCD"

on CTEQ website

http://www.phys.psu.edu/~ cteq/

Parton Model

$\sigma_{hh'\to W^+X} =$



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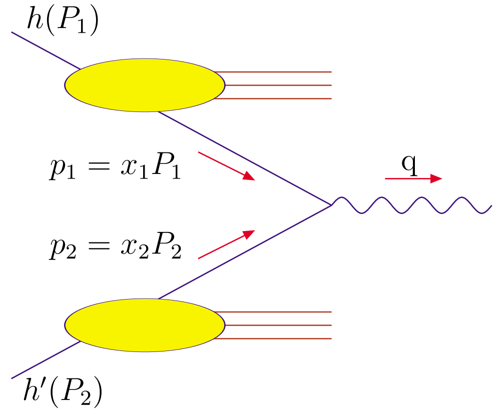


$$\sigma_{hh'\to W^+X} = \sum_{f,f'=q,\bar{q}} \int_0^1 dx_1 dx_2 \left\{ \phi_{f/h}(x_1)\, \hat{\sigma}_{ff'} \phi_{\bar{f}'/h'}(x_2) + (x_1 \leftrightarrow x_2) \right\}$$

Partonic "Born"
Cross Section of $f\overline{f}' \to W^+$

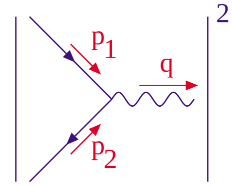
The probablility of finding a "parton" f with fraction x_1 of the hadron h momentum

Born Cross Section

$$\hat{\sigma}_{q\bar{q}'} = \frac{1}{2\hat{s}} \int \frac{d^3 q}{(2\pi)^3\, 2q_0} (2\pi)^4\, \delta^4 \left(p_1 + p_2 - q\right) \cdot \overline{|\mathcal{M}|^2}$$

where

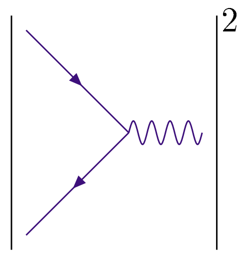
$$\overline{|\mathcal{M}|^2} = \underbrace{\left(\frac{1}{3}\cdot\frac{1}{3}\right)\left(\frac{1}{2}\cdot\frac{1}{2}\right)}_{\text{average color and spin}} \sum_{\substack{spin\\color}} \left| \right|^2$$



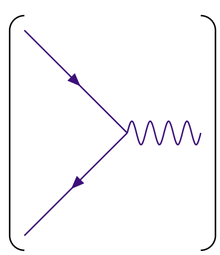
$$\left[\text{ Or, } \; - i\mathcal{M} = \bar{v}\,(p_2)\,\frac{ig_w}{\sqrt{2}}\gamma_\mu \frac{1}{2}\,(1-\gamma_5)\,u\,(p_1) \right]$$

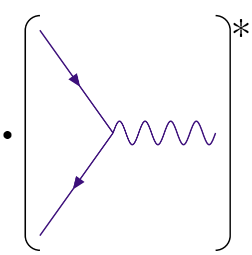
"Cut-diagram" notation

$\Sigma \left| \right|^2$



$= \Sigma \left(\right) \cdot \left(\right)^*$





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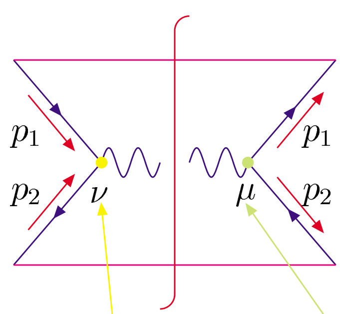


$\frac{ig_w}{\sqrt{2}}\gamma_\nu P_L$ $\qquad -\frac{ig_w}{\sqrt{2}}\gamma_\mu P_L \qquad P_L \equiv \frac{1}{2}(1-\gamma_5)$

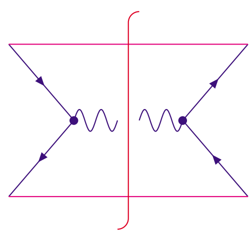
$$= \left(\frac{g_W}{\sqrt{2}}\right)^2 Tr\left[\not{p}_1\gamma_\mu P_L \;\not{p}_2\gamma_\nu P_L\right] \cdot \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{M^2}\right) \cdot Tr\, I_{3\times 3}$$

Doesn't contribute for $m_q = 0$, due to Ward identity

Color

$$
\begin{aligned}
& Tr\,[\not{p}_1\gamma_\mu P_L \;\not{p}_2\gamma^\mu P_L]\,(-1) \\
&= Tr\,[\not{p}_1\gamma_\mu \;\not{p}_2\gamma^\mu P_L]\,(-1) & P_L P_L = P_L = \frac{1}{2}(1-\gamma_5) \\
&= (-2)\,Tr\,[\not{p}_1 \;\not{p}_2 P_L]\,(-1) & \gamma_\mu \;\not{p}_2\gamma^\mu = -2\;\not{p}_2 \\
&= (-2)\cdot\frac{1}{2}\cdot 4\,(p_1\cdot p_2)\,(-1) & Tr\,(\not{p}_1\;\not{p}_2) = 4\,(p_1\cdot p_2) \\
&= +2\hat{s} & Tr\,(\not{p}_1\;\not{p}_2\gamma_5) = 0
\end{aligned}
$$

$Tr\,[I_{3\times 3}] = 3$ $\left(\hat{s} \equiv (p_1+p_2)^2 = q^2 \text{ and } p_1^2 = p_2^2 = 0\right)$

$\Rightarrow$  $= \left(\frac{g_w}{\sqrt{2}}\right)^2 \cdot (+2\hat{s})\,(3) = 3\,g_w^2\,\hat{s}$

$$
\begin{aligned}
\int \frac{d^3q}{2q_0}\delta^4\,(p_1+p_2-q) &= \int d^4q\,\delta^4\,(p_1+p_2-q)\,\delta^+\left(q^2-M^2\right) \\
&= \delta\left(q^2-M^2\right)
\end{aligned}
$$

where M is the mass of W-boson.

Thus,

$$
\begin{aligned}
\hat{\sigma}_{q\bar{q}'} &= \frac{1}{2\hat{s}}(2\pi)\cdot\delta\left(\hat{s}-M^2\right)\cdot\left(\frac{1}{3}\right)\left(\frac{1}{2}\cdot\frac{1}{2}\right)\cdot g_w^2\hat{s} \\
&= \frac{\pi}{12}g_w^2\delta\left(\hat{s}-M^2\right) \\
&= \frac{\pi}{12\hat{s}}g_w^2\delta\,(1-\hat{\tau})
\end{aligned}
$$

$$
\left(\begin{array}{c} \hat{\tau} = M^2/\hat{s}\,,\ \hat{s} = x_1x_2S \ \text{ for} \\ S = (P_1+P_2)^2 \ \text{ and } \ P_1^2 = P_2^2 = 0 \end{array}\right)
$$

Factorization Theorem

$$\sigma_{hh'} = \sum_{i,j} \int_0^1 dx_1 dx_2\, \phi_{i/h}\left(x, Q^2\right) H_{ij}\left(\frac{Q^2}{x_1 x_2 S}\right) \phi_{j/h'}\left(x_2, Q^2\right)$$

Nonperturbative, but universal, hence, measurable

IRS, Calculable in pQCD

Procedure:

(1) Compute σ_{kl} in pQCD with k, l partons

(not h, h' hadron)

$$\sigma_{kl} = \sum_{i,j} \int_0^1 dx_1 dx_2\, \phi_{i/k}\left(x, Q^2\right) H_{ij}\left(\frac{Q^2}{x_1 x_2 S}\right) \phi_{j/l}\left(x_2, Q^2\right)$$

(2) Compute $\phi_{i/k}, \phi_{j/l}$ in pQCD

(3) Extract H_{ij} in pQCD

H_{ij} IRS $\Rightarrow$ H_{ij} indepent of k, l

$\Rightarrow$ same H_{ij} with $(k \to h, l \to h')$

(4) Use H_{ij} in the above equation with $\phi_{i/h}, \phi_{j/h'}$

Extracting H_{ij} in pQCD

- Expansions in α_s:

$$\sigma_{kl} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sigma_{kl}^{(n)} \qquad\qquad \alpha_s = \frac{g^2}{4\pi}$$

$$H_{ij} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n H_{ij}^{(n)}$$

$$\phi_{i/k}(x) = \delta_{ik}\delta(1-x) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \phi_{i/k}^{(n)}$$

$\uparrow$

$\phi_{i/k}^{(0)}$ ($\alpha_s = 0 \Rightarrow$ Parton k " stays itself ")

- Consequences:

$$H_{ij}^{(0)} = \sigma_{ij}^{(0)} = \text{“Born”}$$

suppress "^" from now on

$$H_{ij}^{(1)} = \sigma_{ij}^{(1)} - \left[\ \sigma_{il}^{(0)}\, \phi_{l/j}^{(1)} + \phi_{k/i}^{(1)}\, \sigma_{kj}^{(0)}\ \right]$$

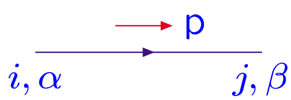
Computed from Feynman diagrams (process dependent)

Computed from the definition of perturbative parton distribution function (process independent, scheme dependent)

Feynman Rules

- Quark Propagator

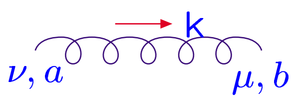
Take m=0 in
our calculation



$$\frac{i(\not p + m)_{\beta\alpha}}{p^2 - m^2 + i\epsilon}\delta_{ij}$$

(i,j=1,2,3)

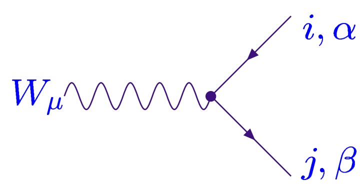
- Gluon Propagator



$$\frac{i(-g_{\mu\nu})}{k^2 + i\epsilon}\delta_{ab}$$

(a,b=1,2...,8)

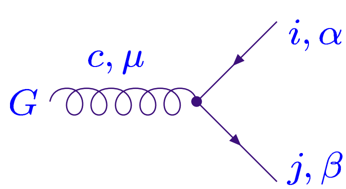
- Quark-W Vertex



$$i\frac{g_W}{\sqrt{2}}(\gamma_\mu)_{\beta\alpha}\frac{(1-\gamma_5)}{2}\delta_{ij}$$

$g_w = \frac{e}{\sin\theta_w}$, weak coupling

- Quark-Gluon Vertex



$$-ig\,(t_c)_{ji}\,(\gamma_\mu)_{\beta\alpha}$$

t_c is the $SU(N)_{N\times N}$ generator

- Quark Color Generators

$$[t_a, t_b] = i f_{abc} t_c$$

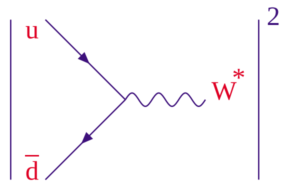
$$\sum_c t_c^2 = C_F\, I_{N\times N}$$

$$Tr(\sum_c t_c^2) = N\, C_F$$

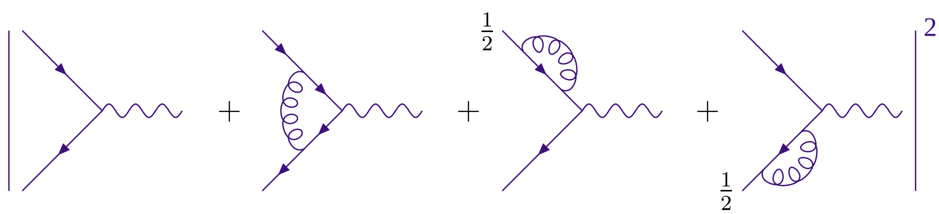
$$C_F = \frac{N^2-1}{2N} = \frac{4}{3}\ ,\ (N=3)$$

Feynman Diagrams

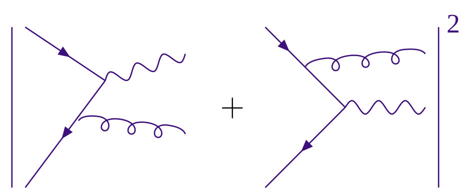
- Born level $\alpha_s^{(0)}$ $(q\overline{q'})_{Born}$



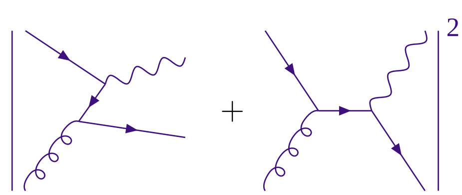
- NLO: $(\alpha_s^{(1)})$ virtual corrections $(q\overline{q'})_{virt}$



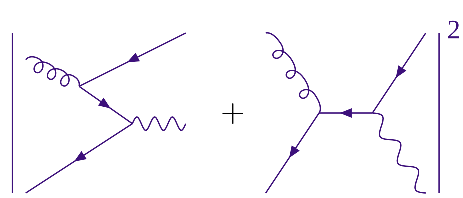
- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(q\overline{q'})_{real}$



- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(qG)_{real}$

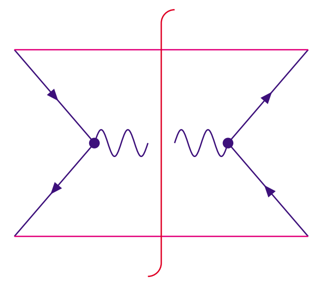


- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(G\overline{q'})_{real}$

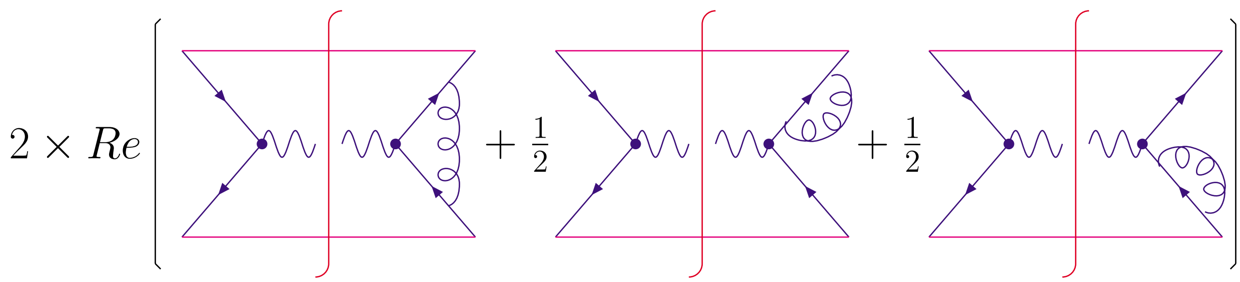


In "Cut-diagram" notation

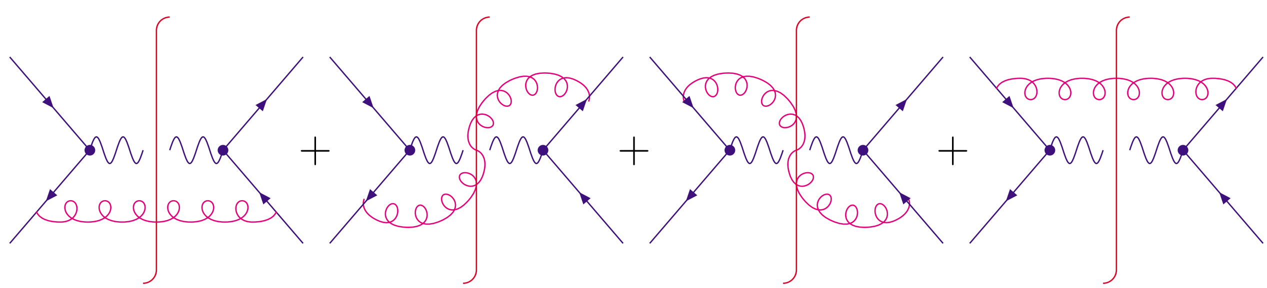
- $(q\overline{q'})_{Born}$



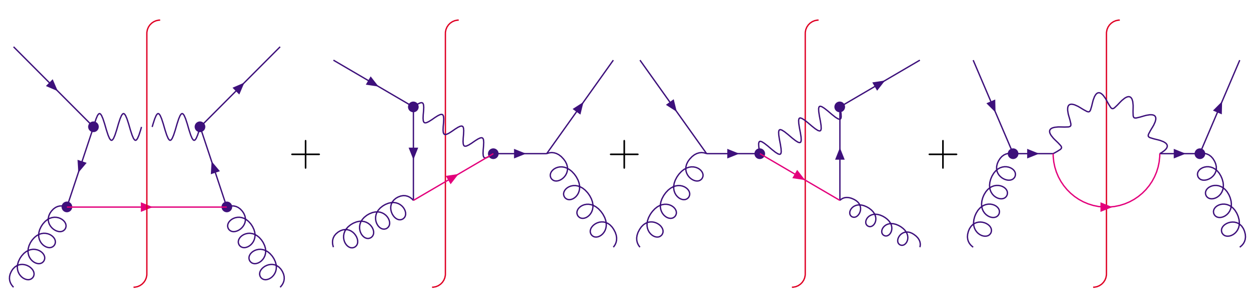
- $(q\overline{q'})_{virt}$



- $(q\overline{q'})_{real}$



- $(qG)_{real}$

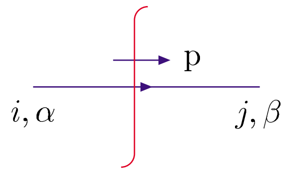


- $(G\overline{q'})_{real}$

Same as $(qG)_{real}$ after replacing q by $\overline{q'}$.

Feynman rules for cut-diagrams

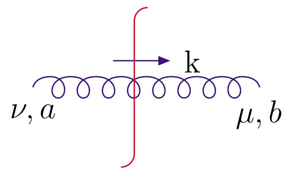
- quark line



$$(2\pi)\delta^+(p^2 - m^2)(\not{p} + m)_{\beta\alpha}\delta_{ij}$$

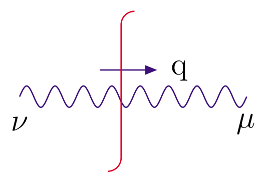
$\delta(p^2 - m^2)\theta(p_0)$

- gluon line



$$(2\pi)\delta^+(k^2)(-g_{\mu\nu})\delta_{ab}$$

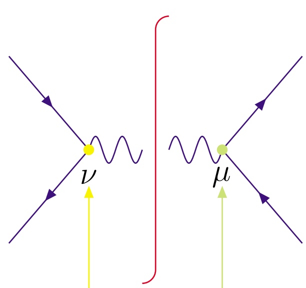
- W-boson line



$$(2\pi)\delta^+(q^2 - M^2)(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2})$$

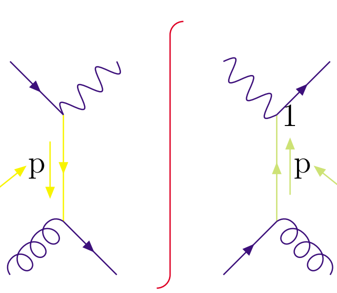
Doesn't contribute for $m_f = 0$ because of Ward identity

-



$i\frac{g_W}{\sqrt{2}}\gamma_\nu\frac{1}{2}(1 - \gamma_5)$ $-i\frac{g_W}{\sqrt{2}}\gamma_\mu\frac{1}{2}(1 - \gamma_5)$

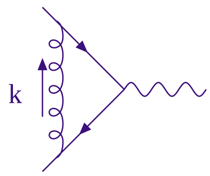
-



$\frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}$ $\frac{-i(\not{p}+m)}{p^2-m^2-i\epsilon}$

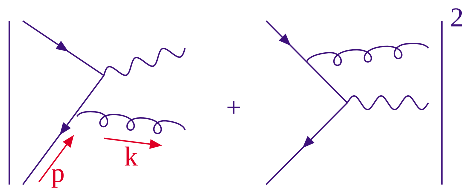
Immediate problems (Singularities)

- Ultraviolet singularity

(UV) 

$$\sim \quad \int^{\infty} d^4k \frac{\not{k}\ \not{k}}{(k^2)\,(k^2)\,(k^2)} \;\rightarrow\; \infty$$

- Infrared singularities

(IR) 

$$\rightarrow \quad \infty$$

as $k^\mu \rightarrow 0$ (soft divergence)

or $k^\mu \parallel p^\mu$ (collinear divergence)

(Similar singularities also exist in virtual diagrams.)

- Solutions

Compute H_{ij} in pQCD in $n = 4 - 2\varepsilon$ dimensions

(dimensional regularization)

(1) $n \neq 4 \Rightarrow$ UV & IR divergences appear as $\frac{1}{\varepsilon}$ poles
in $\sigma_{ij}^{(1)}$ (Feynman diagram calculation)

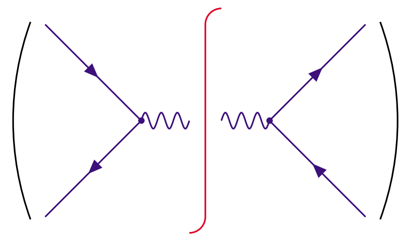
(2) H_{ij} is IR safe $\Rightarrow$ no $\frac{1}{\varepsilon}$ in H_{ij}
(H_{ij} is UV safe after "renormalization".)

Dimensional Regularization

(Revisit the Born Cross Section in n dimensions)

- $$\hat{\sigma}^{(0)}_{q\bar{q}'} = \frac{1}{2\hat{s}} \int \frac{d^{n-1}q}{(2\pi)^{n-1}\, 2q_0} (2\pi)^n \cdot \delta^n \left(p_1 + p_2 - q\right) \cdot \overline{|m|^2}$$

- $$\overline{|m|^2} = \left(\frac{1}{3} \cdot \frac{1}{3}\right) \left(\frac{1}{2} \cdot \frac{1}{2}\right) \cdot$$



In n-dim, the polarization degree of freedom is (2) for a quark, and (n-2) for a gluon.

- Using the Naive-γ^5 prescription:

$$Tr\left[\not{p}_1 \gamma_\mu P_L \not{p}_2 \gamma^\mu P_L\right](-1)$$
$$= Tr\left[\not{p}_1 \gamma_\mu \not{p}_2 \gamma^\mu P_L\right](-1) \qquad \gamma_\mu \not{p}_2 \gamma^\mu = -2\,(1-\varepsilon)\,\not{p}_2$$
$$= (-2)\,(1-\varepsilon)\, Tr\left[\not{p}_1 \not{p}_2 P_L\right](-1)$$
$$= (-2)\,(1-\varepsilon) \cdot \frac{1}{2} \cdot 4\,(p_1 \cdot p_2)\,(-1)$$
$$= 2\,(1-\varepsilon)\,\hat{s}$$

- In n dimensions

$$\hat{\sigma}^{(0)}_{q\bar{q}'} = \frac{\pi}{12\hat{s}} g_w^2 \cdot (1-\varepsilon) \cdot \delta\,(1-\hat{\tau})$$

Strong Coupling g in n dimensions

- In n dimensions

$$\int d^n x \mathcal{L}$$
$$\longrightarrow \int d^n x \left\{ \bar{\psi} i \not\partial \psi - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + g t^a \bar{\psi} \gamma^\mu G_\mu \psi + \cdots \right\}$$

The dimension in mass unit (μ)

$$[\psi] \sim \mu^{\frac{n-1}{2}}$$
$$[G] \sim \mu^{\frac{n-2}{2}}$$
$$\left[\bar{\psi} G \psi\right] \sim \mu^{\frac{n-1}{2} \times 2 + \frac{n-2}{2}} = \mu^{\frac{3n}{2} - 2}$$

Since $\left[g \bar{\psi} G \psi\right] \sim \mu^n$, so

$$[g] \sim \mu^{\frac{-n}{2} + 2} \qquad n = 4 - 2\varepsilon$$
$$= \mu^{\varepsilon}$$

$\Rightarrow$ g has a dimension in mass when $\varepsilon \neq 0$

$\Rightarrow$ Feynman rules should read $g \to g\mu^{\varepsilon}$

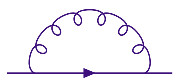
Calculations

- Tools needed for a NLO calculation are collected in Appendices A-D

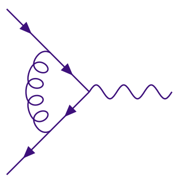
- The detailed calculation for each subprocess can be found in Appendices E

- In the following, I shall summarize the result for each subprocess

Virtual Corrections $(q\overline{q}')_{virt}$
(in Feynman Gauge)

-  $= 0$

  $\frac{1}{\varepsilon_{IR}}$ and $\frac{1}{\varepsilon_{UV}}$ poles cancel when $\varepsilon_{UV} = -\varepsilon_{IR} \equiv \varepsilon$

- 

  $\frac{1}{\varepsilon_{UV}}$ cancel $\Rightarrow$ Electroweak coupling is not renormalized by QCD interactions at one-loop order (Ward identity, a renormalizable theory)

  $\frac{1}{\varepsilon_{IR}}$ poles remain

- $\sigma_{virt}^{(1)}$ is free of ultraviolent singularity.

$$\sigma_{virt}^{(1)} = \sigma^{(0)} \frac{\alpha_s}{2\pi} \delta(1-\hat{\tau}) \left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \cdot \left\{ -\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \frac{2\pi^2}{3} \right\} \cdot (C_F)$$

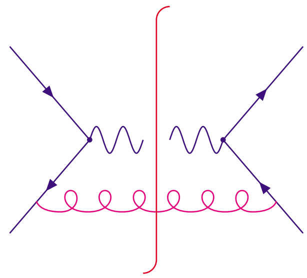
$-\frac{2}{\varepsilon^2}$: soft and collinear singularities

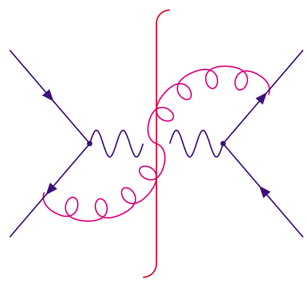
$-\frac{3}{\varepsilon}$: soft or collinear singularities

C_F: color factor

$\sigma^{(0)} \equiv \frac{\pi}{12\hat{s}} g_w^2$

Real Emission Contribution $(q\overline{q}')_{real}$

-  $\sim \dfrac{1}{\varepsilon}$ Collinear

   $\sim \dfrac{1}{\varepsilon^2}$ Soft and Collinear

- $$\sigma^{(1)}_{\text{real}}\left(q\overline{q}'\right) = \sigma^{(0)}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\cdot C_F$$
$$\cdot\left\{\frac{2}{\varepsilon^2}\delta(1-\hat{\tau}) - \frac{2}{\varepsilon}\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + 4\left(1+\hat{\tau}^2\right)\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ - 2\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\ln\hat{\tau}\right\}$$

Note: $[\cdots]_+$ is a distribution,

$$\int_0^1 dz\, f(z)\left[\frac{1}{1-z}\right]_+ = \int_0^1 dz\frac{f(z)-f(1)}{1-z},$$ which is finite.

- In the soft limit, $\hat{\tau}\to 1$ $\left(\hat{\tau} = \frac{M^2}{\hat{s}}\right)$,

$$\sigma^{(1)}_{\text{real}}\left(q\overline{q}'\right) \longrightarrow \sigma^{(0)}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1+\varepsilon)}\cdot C_F$$
$$\cdot\left\{\frac{2}{\varepsilon^2}\delta(1-\hat{\tau}) - \frac{4}{\varepsilon}\frac{1}{(1-\hat{\tau})_+} + 8\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+\right\}$$

$\left(q\bar{q}'\right)_{virt} + \left(q\bar{q}'\right)_{\text{real}}$ at NLO

- $$\begin{aligned}\sigma^{(1)}_{q\bar{q}'} &= \sigma^{(1)}_{virt}\left(q\bar{q}'\right) + \sigma^{(1)}_{\text{real}}\left(q\bar{q}'\right) \\ &= \sigma^{(0)}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\cdot C_F \\ &\quad \cdot\left\{\frac{-2}{\varepsilon}\left(\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\right)_+ - 2\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\ln\hat{\tau} + 4\left(1+\hat{\tau}^2\right)\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ \right. \\ &\qquad \left. + \left(\frac{2\pi^2}{3} - 8\right)\delta(1-\hat{\tau})\right\}\end{aligned}$$

  Where we have used

  $$\frac{-2}{\varepsilon}\left[\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + \frac{3}{2}\delta(1-\hat{\tau})\right] = \frac{-2}{\varepsilon}\left(\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\right)_+$$

- All the soft singularities $\left(\frac{1}{\varepsilon^2}, \frac{1}{\varepsilon}\right)$ cancel in $\sigma^{(1)}_{q\bar{q}'}$

  $\Rightarrow$ KLN theorem

  (Kinoshita-Lee-Navenberg)

- $$\sigma^{(1)}_{q\bar{q}'} \sim \underset{\substack{\uparrow \\ \text{Collinear Singularity}}}{\frac{1}{\varepsilon}}(\text{term}) + \text{finite (terms)}$$

Factorization Theorem

- Perturbative PDF

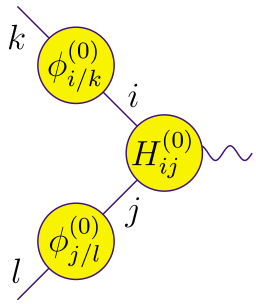
$\phi_{i/k}^{(0)} = \delta_{ik}\delta\left(1 - x\right)$

$\frac{\alpha_s}{\pi}\phi_{i/k}^{(1)}$ can be calculated from the definition of PDF.

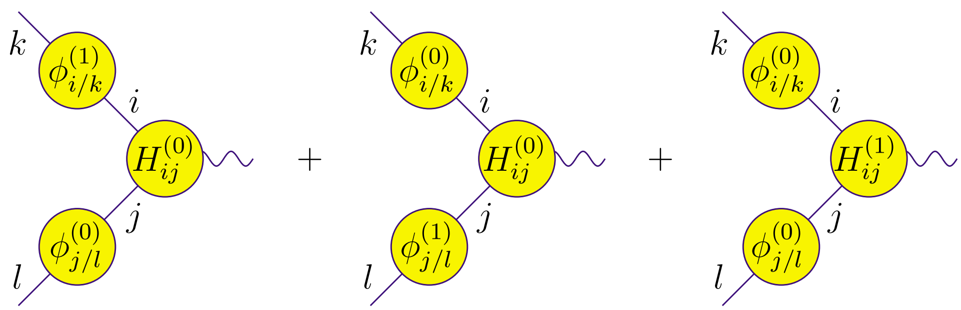
(Process independent,but factorization scheme dependent)

-

(1)

$\sigma_{kl}^{(0)} =$  $\Rightarrow H_{kl}^{(0)} = \sigma_{kl}^{(0)}$

(2)

$\sigma_{kl}^{(1)} =$ 

Factorization scheme dependent

$$\Rightarrow \quad H_{kl}^{(1)} = \sigma_{kl}^{(1)} - \left[\phi_{i/k}^{(1)} H_{il}^{(0)} + H_{kj}^{(0)} \phi_{j/l}^{(1)}\right]$$

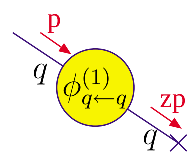
Finite

Divergent

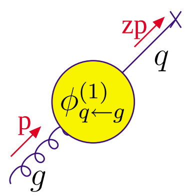
Perturbative PDF

- In $\overline{MS}$-scheme (modified minimal subtraction)

$$\phi^{(1)}_{q/q}(z) = \phi^{(1)}_{\bar{q}/\bar{q}}(z) = \frac{-1}{\varepsilon}\frac{1}{2}\left(4\pi e^{-\gamma_E}\right)^{\varepsilon} P^{(1)}_{q\leftarrow q}(z)$$
$$\phi^{(1)}_{q/g}(z) = \phi^{(1)}_{\bar{q}/g}(z) = \frac{-1}{\varepsilon}\frac{1}{2}\left(4\pi e^{-\gamma_E}\right)^{\varepsilon} P^{(1)}_{q\leftarrow g}(z)$$

where the splitting kernel for  is

$$P^{(1)}_{q\leftarrow q}(z) = C_F\left(\frac{1+z^2}{1-z}\right)_+$$
$$= C_F\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right)$$

and for  :

$$P^{(1)}_{q\leftarrow g}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

(Note: The Pole part in the $\overline{MS}$ scheme is

$\frac{1}{\hat{\varepsilon}} = \frac{1}{\varepsilon}(4\pi e^{-\gamma_E})^{\varepsilon} = \frac{1}{\varepsilon} + \ln 4\pi - \gamma_E$

In the MS scheme, the pole part is just $\frac{1}{\varepsilon}$)

Find $H^{(1)}_{q\bar{q}'}$ (in the $\overline{MS}$ scheme)

- Take off the factor $\left(\frac{\alpha_s}{\pi}\right)$

$$\sigma^{(1)}_{q\bar{q}'} = \sigma^{(0)} \left\{ P^{(1)}_{q\leftarrow q}(\hat{\tau}) \left[\ln\left(\frac{M^2}{\mu^2}\right) - \frac{1}{\varepsilon} + \gamma_E - \ln 4\pi \right] + C_F \left[-\frac{1+\hat{\tau}^2}{1-\hat{\tau}} \ln\hat{\tau} + 2\left(1+\tau^2\right) \left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ + \left(\frac{\pi^2}{3} - 4\right) \delta(1-\hat{\tau}) \right] \right\}$$

-

$$\begin{aligned} H^{(1)}_{q\bar{q}'}(\hat{\tau}) &= \sigma^{(1)}_{q\bar{q}'} - \left[2\phi^{(1)}_{q\leftarrow q}\sigma^{(0)}_{q\bar{q}'}\right] \\ &= \sigma^{(0)} \cdot \left\{ P^{(1)}_{q\leftarrow q}(\hat{\tau}) \ln\left(\frac{M^2}{\mu^2}\right) \right. \\ &\left. + C_F \left[-\frac{1+\hat{\tau}^2}{1-\hat{\tau}} \ln\hat{\tau} + 2\left(1+\tau^2\right) \left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ + \left(\frac{\pi^2}{3} - 4\right) \delta(1-\hat{\tau}) \right] \right\} \end{aligned}$$

where

$$\hat{\tau} = \frac{M^2}{\hat{s}} = \frac{M^2}{x_1 x_2 S},$$

$$\sigma^{(0)} = \frac{\pi}{12\hat{s}} g_w^2 = \frac{\pi g_w^2}{12S} \frac{1}{x_1 x_2}$$

- pQCD prediction

$$\begin{aligned} \sigma_{hh'} &= \sum_{f=q,\bar{q}'} \int dx_1 dx_2 \phi_{f/h}\left(x_1,\mu^2\right) \left[\sigma^{(0)}\delta(1-\hat{\tau})\right] \phi_{\bar{f}/h'}\left(x_2,\mu^2\right) \\ &+ \sum_{f=q,\bar{q}'} \int dx_1 dx_2 \phi_{f/h}\left(x_1,\mu^2\right) \left[\frac{\alpha_s\left(\mu^2\right)}{\pi} H^{(1)}_{f\bar{f}}(\hat{\tau})\right] \phi_{\bar{f}/h'}\left(x_2,\mu^2\right) \\ &+ \left\{ \sum_{f=q,\bar{q}'} \int dx_1 dx_2 \phi_{f/h}\left(x_1,\mu^2\right) \left[\frac{\alpha_s\left(\mu^2\right)}{\pi} H^{(1)}_{fG}(\hat{\tau})\right] \phi_{G/h'}\left(x_2,\mu^2\right) + (x_1 \leftrightarrow x_2) \right\} \end{aligned}$$

Find $H_{qG}^{(1)}$ (in the $\overline{MS}$ scheme)

- Take off the factor $\left(\frac{\alpha_s}{\pi}\right)$

$$\sigma_{qG}^{(1)} = \sigma^{(0)} \cdot \frac{1}{4} \cdot \left\{ 2P_{q\leftarrow g}^{(1)}(\hat{\tau}) \left[\frac{-1}{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} + \ln \frac{M^2 (1-\hat{\tau})^2}{4\pi\mu^2 \hat{\tau}} \right] + \frac{3}{2} + \hat{\tau} - \frac{3}{2}\hat{\tau}^2 \right\}$$

-

$$\begin{aligned} H_{qG}^{(1)}(\hat{\tau}) &= \sigma_{qG}^{(1)} - \left[\sigma_{q\bar{q}'}^{(0)} \phi_{\bar{q}'\leftarrow G}^{(1)} \right] \\ &= \frac{\sigma^{(0)}}{2} \cdot \left\{ P_{q\leftarrow g}^{(1)}(\hat{\tau}) \left[\ln\left(\frac{M^2}{\mu^2}\right) + \ln\left(\frac{(1-\hat{\tau})^2}{\hat{\tau}}\right) \right] + \frac{3}{4} + \frac{\hat{\tau}}{2} - \frac{3}{4}\hat{\tau}^2 \right\} \end{aligned}$$

- Similarly,

$$\begin{aligned} H_{G\bar{q}'}^{(1)} &= \sigma_{G\bar{q}'}^{(1)} - \left[\phi_{q\leftarrow G}^{(1)} \sigma_{q\bar{q}'}^{(0)} \right] \\ &= H_{qG}^{(1)} \end{aligned}$$

(Note: If we choose the renormalization scale $\mu^2 = M^2$, then $\ln\left(\frac{M^2}{\mu^2}\right) = 0$)

Summary

- $\phi_{f/h}(x, \mu^2)$ depends on scheme ($\overline{MS}$,DIS,...)
  $\Rightarrow H_{ij}$ **scheme dependent**

- Evolution equations allow us to perdict
  q^2–**dependent of** $\phi(x, q^2)$

- Essentially identical procedure for
  $hh' \to jets$, inclusive $Q\overline{Q}$,...
  But, when the Born level process involves
  strong interaction (eg. $q\bar{q} \to t\bar{t}$),
  it is also necessary to renormalize the
  strong coupling α_s, etc, to eliminate
  ultraviolate singularities

Appendix A
γ-matrices in n dimensions

- Dirac algebra

$$\{\gamma^\mu, \gamma^\nu\}_+ \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$$
$$\mu, \nu = 1, 2, \cdots, n \qquad g^{\mu\nu} = diag\,(1, -1, \cdots, -1)$$
$$g^{\mu\nu} g_{\mu\nu} = n$$
$$\left\{\gamma^\mu, \gamma^5\right\}_+ = 0 \qquad \left(\text{Naive-}\gamma^5\text{prescription}\right)$$

This works in calculating the inclusive rate of W-boson , but fails in the differential distributions of the leptons from the W-boson decay.

- Matrix identities

$n = 4 - 2\varepsilon$

$$\gamma_\mu \not{a} \gamma^\mu = -2\,(1-\varepsilon)\not{a}$$
$$\gamma_\mu \not{a} \not{b} \gamma^\mu = 4a\cdot b - 2\varepsilon \not{a} \not{b}$$
$$\gamma_\mu \not{a} \not{b} \not{c} \gamma^\mu = -2 \not{c} \not{b} \not{a} + 2\varepsilon \not{a} \not{b} \not{c}$$

- Traces

$$Tr\,[\not{a} \not{b}] = 4\,(a\cdot b)$$
$$Tr\,[\not{a} \not{b} \not{c} \not{d}] = 4\,\{(a\cdot b)\,(c\cdot d) - (a\cdot c)\,(b\cdot d) + (a\cdot d)\,(b\cdot c)\}$$
$$Tr\,[\gamma_5 \not{a} \not{b}] = 0$$

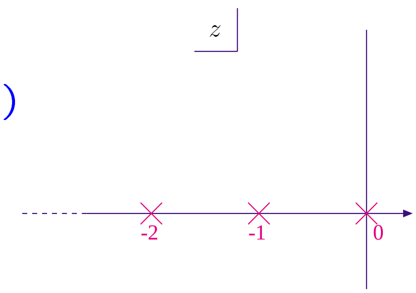
Appendix B
Some integrals and "special functions"

- The "Gamma function"

$$\Gamma(z) = \int_0^\infty dx x^{z-1} e^{-x} \qquad (\text{Re}(z) > 0)$$

$$\Gamma(z-1) = \frac{\Gamma(z)}{z-1} \quad (\text{for all z})$$

$\Rightarrow$ Poles in $\Gamma(z)$



$$\Gamma(n) = (n-1)! \qquad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma_E + \frac{\varepsilon}{2}\left(\gamma_E^2 - \frac{\pi^2}{6}\right) + \cdots$$

$$(\gamma_E = 0.5772\cdots, \text{ Euler constant})$$

$$\Gamma(1-\varepsilon) = -\varepsilon\Gamma(\varepsilon) = 1 + \varepsilon\gamma_E + \frac{1}{2}\varepsilon^2\left(\frac{\pi^2}{6} + \gamma_E^2\right) + O\left(\varepsilon^3\right)$$

$$\Gamma(1-\varepsilon)\Gamma(1+\varepsilon) = 1 + \varepsilon^2\frac{\pi^2}{6} + O\left(\varepsilon^4\right)$$

$$z^\varepsilon = e^{\ln z^\varepsilon} = e^{\varepsilon \ln z} = 1 + \varepsilon \ln z + \cdots$$

- The "Beta function"

$$B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$B(\alpha,\beta) = \int_0^1 dy\, y^{\alpha-1}(1-y)^{\beta-1} = \int_0^\infty dy\, y^{\alpha-1}(1+y)^{-\alpha-\beta}$$

$$= 2\int_0^{\frac{\pi}{2}} d\theta\, (\sin\theta)^{2\alpha-1}(\cos\theta)^{2\beta-1}$$

- Feynman trick

$$\frac{1}{ab} = \int_0^1 dx \frac{1}{[ax + b(1-x)]^2}$$

$$\frac{1}{a^\alpha b^\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dx \frac{x^{\alpha-1}(1-x)^{\beta-1}}{[ax + b(1-x)]^{\alpha+\beta}}$$

- n-dimension integrals

$$\int d^n l \frac{l_\mu}{(l^2 - M^2)^\alpha} = 0$$

$$\int d^n l \frac{l_\mu l_\nu}{(l^2 - M^2)^\alpha} = \int d^n l \frac{\left(\frac{l^2 g_{\mu\nu}}{n}\right)}{(l^2 - M^2)^\alpha}$$

$$\int \frac{d^n l}{(2\pi)^n} \frac{1}{(l^2 - M^2)^\alpha} = \frac{(-1)^\alpha}{(4\pi)^{n/2}} \frac{\Gamma\left(\alpha - \frac{n}{2}\right)}{\Gamma(\alpha)} \left(\frac{1}{M^2}\right)^{\alpha - \frac{n}{2}}$$

$$\int d^n l \frac{l^2}{(l^2 - M^2)^\alpha} = \int d^n l \frac{(l^2 - M^2) + M^2}{(l^2 - M^2)^\alpha}$$

- $$\mathrm{Re}\left[(-1)^\varepsilon\right] = 1 - \varepsilon^2 \frac{\pi^2}{2} + O\left(\varepsilon^4\right)$$

- "plus distribution" — to isolate $\frac{1}{\varepsilon}$ poles

Consider $\dfrac{1}{(1-z)^{1+2\varepsilon}}$

$$= \frac{1}{(1-z)^{1+2\varepsilon}} - \left[\delta(1-z)\int_0^1 \frac{dz'}{(1-z')^{1+2\varepsilon}} + \frac{1}{2\varepsilon}\delta(1-z)\right]$$

cancel

because $\displaystyle\int_0^1 \frac{dz'}{(1-z')^{1+2\varepsilon}} = \frac{-1}{2\varepsilon}$ for $\varepsilon \to 0^-$

$$\equiv \left[\frac{1}{(1-z)^{1+2\varepsilon}}\right]_+ - \frac{1}{2\varepsilon}\delta(1-z)$$

$$= \frac{1}{(1-z)_+} - 2\varepsilon\left[\frac{\ln(1-z)}{1-z}\right]_+ + O\left(\varepsilon^2\right) - \frac{1}{2\varepsilon}\delta(1-z)$$

because $\displaystyle\frac{1}{(1-z)^{2\varepsilon}} = (1-z)^{-2\varepsilon}$

$\displaystyle = 1 - 2\varepsilon\ln(1-z) + \cdots$

- $[\cdots]_+$ is a distribution

$$\int_0^1 dz\, f(z)\left[\frac{1}{1-z}\right]_+$$

$$\equiv \int_0^1 dz\,\frac{f(z)}{1-z} - \int_0^1 dz\, f(z)\,\delta(1-z)\int_0^1 \frac{dz'}{(1-z')}$$

$$= \int_0^1 dz \frac{f(z)-f(1)}{1-z},$$ which is finite.

Appendix C
Angular integrals in *n* dimensions

- In *n* dimensions

$$\int d^n x = \int r^{n-1} d\Omega_{n-1}$$

-

$$\int d\Omega_n = \int_0^\pi d\theta_{n-1} \sin^{n-1}\theta_{n-1} \int_0^\pi d\theta_{n-2} \sin^{n-2}\theta_{n-2} \cdots \int_0^\pi d\theta_1 \sin\theta_1 \int_0^{2\pi} d\phi$$

$$\Rightarrow \int d\Omega_1 = \int_0^{2\pi} d\phi \qquad \longrightarrow \Omega_1 = 2\pi$$

$$\int d\Omega_2 = \int_0^\pi d\theta_1 \sin\theta_1 \int d\Omega_1 \qquad \longrightarrow \Omega_2 = 4\pi$$

$$\vdots$$

$$\int d\Omega_n = \int_0^\pi d\theta_{n-1} \sin^{n-1}\theta_{n-1} \int d\Omega_{n-1}$$

$$\Rightarrow \quad \Omega_n = \frac{2^n \pi^{\frac{n}{2}} \Gamma\left(\frac{n}{2}\right)}{\Gamma(n)}$$ from repeated use of $B(\alpha, \beta)$

$$= \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)}$$ because $\Gamma(n) = \dfrac{2^{n-1}\Gamma\left(\frac{n}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}$

Appendix D
Two-particle phase space in n dimensions

- $$\int_{PS_2(p)} dk\, dq = \int \frac{d^{n-1}\vec{k}}{(2\pi)^{n-1}\, 2k_0} \frac{d^{n-1}\vec{q}}{(2\pi)^{n-1}\, 2q_0} \cdot (2\pi)^n\, \delta^n\,(p-q-k)$$

  with $k^\mu = \left(k_0, \vec{k}\right)$, etc.

  Use $\frac{d^{n-1}\vec{q}}{2q_0} = \int d^n q \delta^+\left(q^2 - Q^2\right)$, we get

- $$\int_{PS_2(p)} dk\, dq = \frac{1}{(2\pi)^{n-2}} \int \frac{d^{n-1}\vec{k}}{2k_0} \delta^+\left((p-k)^2 - Q^2\right)$$
  $$= \frac{1}{(2\pi)^{n-2}} \int \frac{dk\, k^{n-3}}{2} \int d\Omega_{n-2} \delta\left(\hat{s} - 2k\sqrt{\hat{s}} - Q^2\right)$$
  $$\left(p^2 \equiv \hat{s}, k^2 = 0, k = \left|\vec{k}\right|\right)$$

  Use $n = 4 - 2\varepsilon$, then in the c.m. frame $\left(p^\mu = \left(\sqrt{\hat{s}}, \vec{0}\right)\right)$,

- $$\int_{PS_2(p)} dk\, dq = \frac{\Omega_{n-3}}{(2\pi)^{2(1-\varepsilon)}} \int \frac{dk\, k^{1-2\varepsilon}}{4\sqrt{\hat{s}}} \int_0^\pi d\theta\, (\sin\theta)^{1-2\varepsilon} \cdot \delta\left(k - \frac{\hat{s} - Q^2}{2\sqrt{\hat{s}}}\right)$$

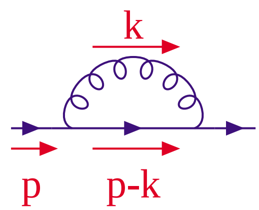
  Use new variables:

  $$z = \frac{Q^2}{\hat{s}}, y = \frac{1}{2}(1 + \cos\theta) \Rightarrow k = \frac{\sqrt{\hat{s}}}{2}(1-z),$$

- $$\int_{PS_2(p)} dk\, dq = \frac{1}{8\pi} \left(\frac{4\pi}{Q^2}\right)^\varepsilon \frac{z^\varepsilon\,(1-z)^{1-2\varepsilon}}{\Gamma\,(1-\varepsilon)} \int_0^1 dy\, [y\,(1-y)]^{-\varepsilon}$$

Appendix E
Explicit Calculations

- Consider



$$\int \frac{d^n k}{(2\pi)^n} \frac{\gamma_\mu \left(\not{p} - \not{k}\right) \gamma^\mu}{\left(k^2 + i\epsilon\right)\left((p-k)^2 + i\epsilon\right)}$$

$$\to \int \frac{d^n k}{(2\pi)^n} \int_0^1 dx \frac{(2-n)\left(\not{p} - \not{k}\right)}{\left[k^2 - 2k \cdot xp\right]^2} \qquad (l \equiv k - xp)$$

$$= \int \frac{d^n l}{(2\pi)^n} \int_0^1 dx \frac{(2-n)\left[(1-x)\ \not{p} - \not{l}\right]}{\left[l^2 + i\epsilon\right]^2}$$

$$= \left[\left(1 - \frac{n}{2}\right)\ \not{p}\right] \cdot \underbrace{\int \frac{d^n l}{(2\pi)^n} \frac{1}{\left[l^2 + i\epsilon\right]^2}}$$

$$\downarrow$$

$$= 0 \quad \left(\begin{array}{c}\text{Because there is}\\ \text{no mass scale}\end{array}\right)$$

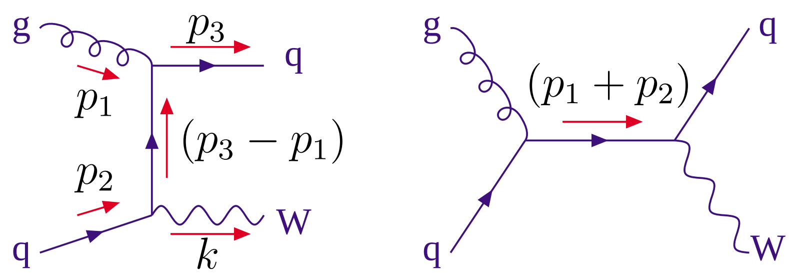
$$\uparrow$$

$$\left(\begin{array}{c}\text{Due to cancellation}\\ \text{of } \frac{1}{\varepsilon_{UV}} \text{ and } \frac{1}{\varepsilon_{IR}}\\ \text{Trick:} A = A - B + B\end{array}\right)$$

$$= \int \frac{d^n l}{(2\pi)^n} \left\{ \underbrace{\left[\frac{1}{(l^2)^2} - \frac{1}{(l^2 - \Lambda^2)^2}\right]}_{\text{IR div.}} + \underbrace{\left[\frac{1}{(l^2 - \Lambda^2)^2}\right]}_{\text{UV div.}} \right\}$$

$$= \frac{i}{16\pi^2}\left(\frac{1}{\varepsilon_{IR}}\right) + \frac{i}{16\pi}\left(\frac{1}{\varepsilon_{UV}}\right), \qquad \left(\begin{array}{c} n - 4 = 2\varepsilon_{IR} \\ 4 - n = 2\varepsilon_{UV}\end{array}\right)$$

- consider the real emission process



Define the Mandelstam variables

$$\hat{s} = (p_1 + p_2)^2 = \ 2p_1 \cdot p_2$$
$$\hat{t} = (p_1 - p_3)^2 = -2p_1 \cdot p_3$$
$$\hat{u} = (p_2 - p_3)^2 = -2p_2 \cdot p_3$$

After averaging over colors and spins

$$\overline{|\mathcal{M}|^2} = \underbrace{\left(\frac{1}{2\,(1-\varepsilon)}\frac{1}{2}\right)}_{\text{Spin}} \cdot \underbrace{\left(\frac{1}{3}\cdot\frac{1}{8}\right) \cdot Tr\,(t^a t^a)}_{\text{Color}} \cdot (g\mu^\varepsilon)^2$$
$$\cdot\, g_w^2 \cdot 2\,(1-\varepsilon)$$
$$\cdot \left[(1-\varepsilon)\left(-\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}}\right) - \frac{2\hat{u}M^2}{\hat{t}\hat{s}} + 2\varepsilon\right]$$

Note: The d.o.f. of gluon polarization is $2\,(1-\varepsilon)$, and that of quark polarization is 2.

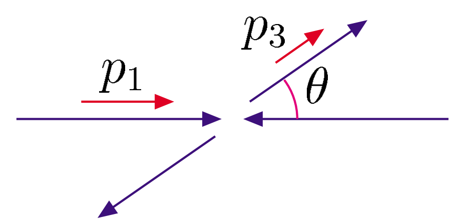
- In the parton c.m. frame, the constituent cross section

$$\hat{\sigma} = \frac{1}{2\hat{s}}\overline{|\mathcal{M}|^2}\cdot(PS_2)$$

$$= \frac{1}{2\hat{s}}\cdot\left\{\frac{1}{4}\cdot\frac{1}{6}\cdot 2g_s^2\mu^{2\varepsilon}g_w^2\,(1-\varepsilon)\cdot\left[(1-\varepsilon)\left(-\frac{\hat{s}}{\hat{t}}-\frac{\hat{t}}{\hat{s}}\right)-\frac{2\hat{u}M^2}{\hat{t}\hat{s}}+2\varepsilon\right]\right\}$$

$$\cdot\left\{\frac{1}{8\pi}\left(\frac{4\pi}{M^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\hat{\tau}^{\varepsilon}(1-\hat{\tau})^{1-2\varepsilon}\int_0^1 dy\,[y\,(1-y)]^{-\varepsilon}\right\}$$

where $\quad y \equiv \frac{1}{2}(1+\cos\theta)$



Using $\quad \hat{t} = -\hat{s}\left(1-\frac{M^2}{\hat{s}}\right)(1-y)$

$$\hat{u} = -\hat{s}\left(1-\frac{M^2}{\hat{s}}\right)y$$

and

$$\int_0^1 dy\,y^{\alpha}\,(1-y)^{\beta} = \frac{\Gamma(1+\alpha)\,\Gamma(1+\beta)}{\Gamma(2+\alpha+\beta)}\ ,$$

we get

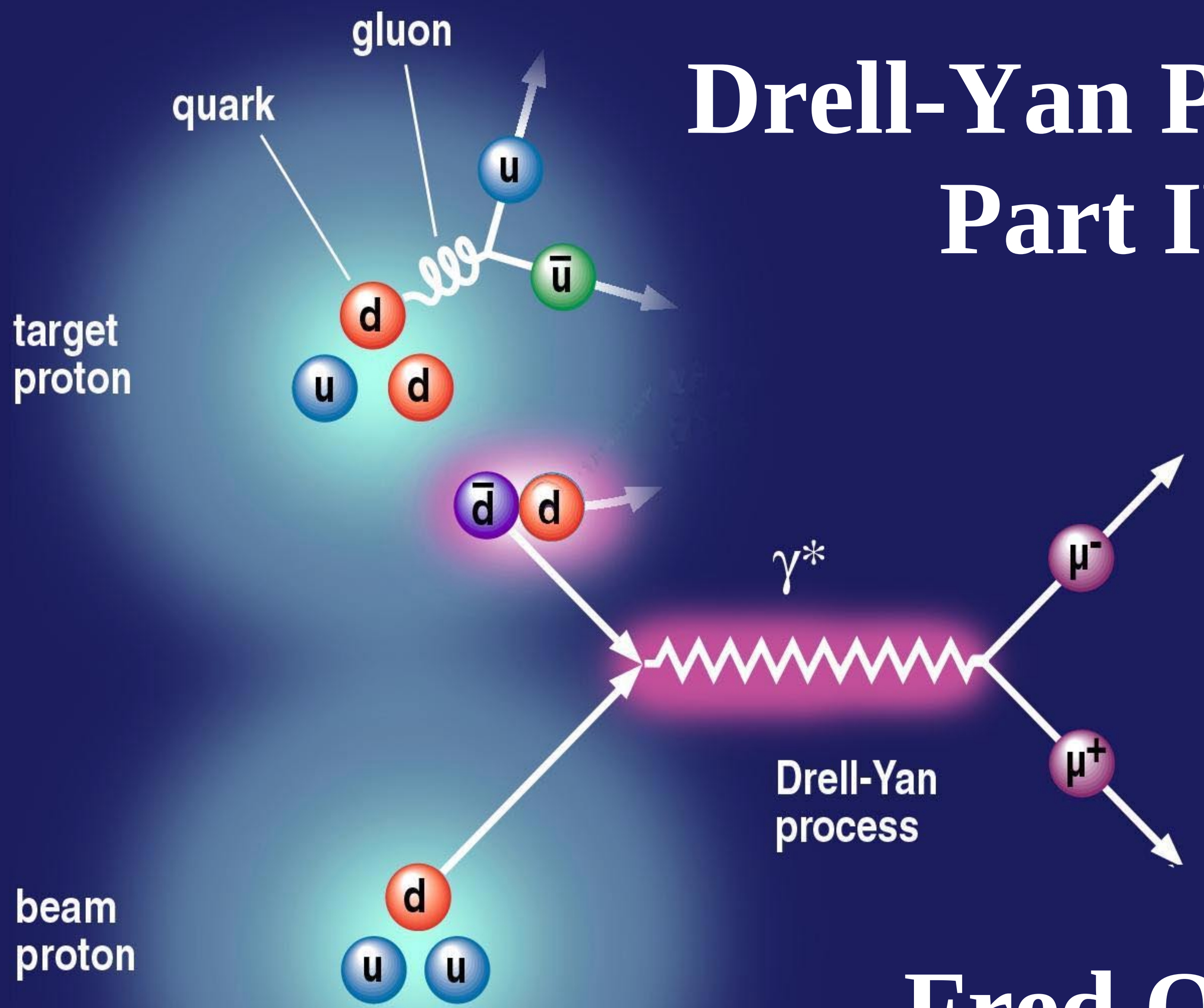
$$\hat{\sigma}_{qG} = \sigma^{(0)}\frac{\alpha_s}{4\pi}\cdot\left\{2P^{(1)}_{q\leftarrow g}(\hat{\tau})\left[\frac{-1}{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}+\ln\frac{M^2(1-\hat{\tau})^2}{4\pi\mu^2\hat{\tau}}\right]+\frac{3}{2}+\hat{\tau}-\frac{3}{2}\hat{\tau}^2\right\},$$

with

$$P^{(1)}_{q\leftarrow g}(\hat{\tau}) = \frac{1}{2}\left[\hat{\tau}^2+(1-\hat{\tau})^2\right]$$

$$\sigma^{(0)} \equiv \frac{\pi}{12}g_w^2\frac{1}{\hat{s}}$$

Drell-Yan Process: Part IV



Fred Olness
SMU

There is a rigorous factorization proof ...

Ingredients of Factorization

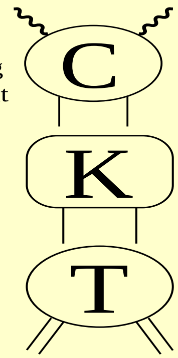
Decompose into (t-channel) 2PI amplitudes:

$$\sigma = \Sigma_{N=1}^{\infty} \; C \; (K)^N \; T \; + \textit{Non-leading}$$

Hard scattering coefficient

Kernel

Target



Collins, Soper, Sterman. Perturbative QCD, World Scientific (1989). Collins, in preparation

After reorganization of the infinite sum:

$$\sigma \approx \overbrace{\underbrace{C\,[1-(1-Z)\,K\,]^{-1}}_{\text{Wilson Coefficient (Hard Scatt. }\hat{\sigma}\text{)}}\; Z \;\underbrace{[1-K\,]^{-1}\,T}_{\text{Parton Distribution}}}^{\text{Parton Model}} + \overbrace{C\,[1-(1-Z)\,K\,]^{-1}\,(1-Z)\,T}^{\text{Remainder}}$$

Z: collinear projection

Power Suppressed

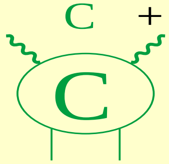
Wilson Coefficient:

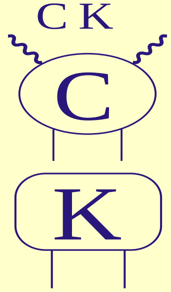
Leading Order

Next to Leading Order

$$C\,[1-(1-Z)\,K\,]^{-1} \approx C + \overbrace{C\,K - C\,Z\,K}^{\text{Next to Leading Order}} + \ldots$$

All orders result





Wilson Coefficient:
IR safe "hard"
scattering cross section

C Z K is the subtraction

A formal proof was constructed by numerous groups.

This proof was explicitly extended to the case of massive quarks

(Collins,1998)

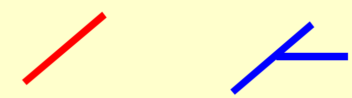
THOUGH EXPERIMENT
To keep things simple, let's consider scattering off a parton target.

Basic Factorization Formula

$$\boxed{\sigma = f \otimes \omega \otimes d \ \ + O(\Lambda^2/Q^2)}$$

At Zeroth Order:

$$\sigma^0 = f^0 \otimes \omega^0 \otimes d^0 \ \ + O(\Lambda^2/Q^2)$$



f^0 f^1

for parton target

Use: $f^0 = \delta$ and $d^0 = \delta$ for a parton target.

Therefore:

$$\sigma^0 = f^0 \otimes \omega^0 \otimes d^0 = \delta \otimes \omega^0 \otimes \delta = \omega^0$$

$$\boxed{\sigma^0 = \omega^0}$$

Warning: This trivial result leads to many misconceptions at higher orders

Basic Factorization Formula

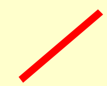
$$\sigma = f \otimes \omega \otimes d \;\; + O(\Lambda^2/Q^2)$$

At First Order:

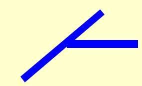
$$\sigma^1 = f^1 \otimes \omega^0 \otimes d^0 + f^0 \otimes \omega^1 \otimes d^0 + f^0 \otimes \omega^0 \otimes d^1$$

$$\sigma^1 = \quad f^1 \otimes \sigma^0 + \omega^1 + \sigma^0 \otimes d^1$$

We used: $f^0 = \delta$ and $d^0 = \delta$ for a parton target.



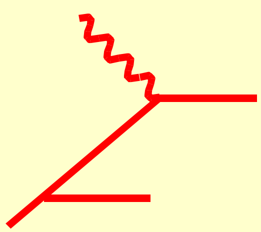
f^0



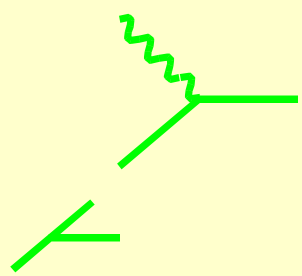
f^1

Therefore:

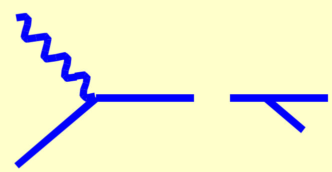
$$\omega^1 = \sigma^1 - f^1 \otimes \sigma^0 - \sigma^0 \otimes d^1$$



σ^1



$f^1 \otimes \sigma^0$



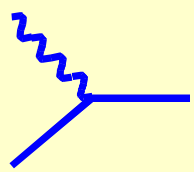
$\sigma^0 \otimes d^1$

Combined Result:

$$\omega^0 + \omega^1 = \sigma^0 + \sigma^1 - \underbrace{\left\{ f^1 \otimes \sigma^0 + \sigma^0 \otimes d^1 \right\}}$$

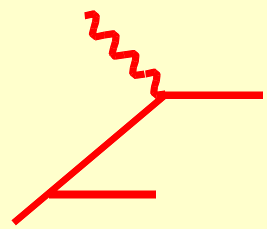
TOT

HE



Heavy Excitation

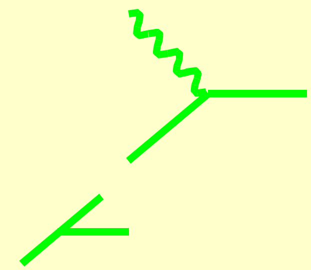
HC



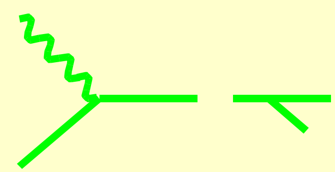
Heavy Excitation

SUB

Subtraction



$f^1 \otimes \sigma^0$



$\sigma^0 \otimes d^1$

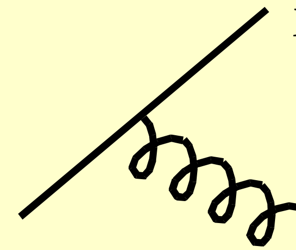
TOT = HE + HC – SUB

Splitting Kernel to α_s^1 order

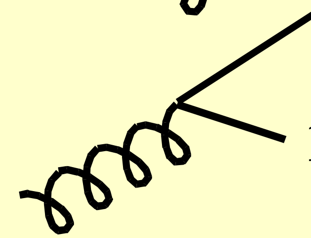
$$\phi_{i\leftarrow j}(x,\epsilon) = \delta(1-x)\,\delta_{ij} + \frac{\alpha_s}{2\pi}\left(-\frac{1}{\epsilon}\right)\left[\frac{\mu^2}{M^2}\right]^{\epsilon} P^{(1)}_{i\leftarrow j}(x)$$

Splitting Kernel to α_s^1 order

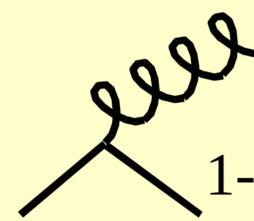
$$P^{(1)}_{q\leftarrow q}(x) \;=\; C_F \left[\frac{1+x^2}{1-x}\right]_+$$



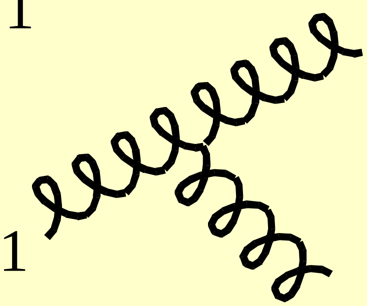
$$P^{(1)}_{q\leftarrow g}(x) \;=\; T_F \left[(1-x)^2+x^2\right]$$



$$P^{(1)}_{g\leftarrow q}(x) \;=\; C_F \,\frac{(1-x)^2+1}{x}$$



$$P^{(1)}_{g\leftarrow g}(x) \;=\; 2\,C_F \left[\frac{x}{(1-x_+)}+\frac{1-x}{x}+x(1-x)\right] + \;\delta(1-x)\left[\frac{11}{6}C_A-\frac{2}{3}T_F N_F\right]$$



$$C_F=\frac{4}{3} \qquad C_A=3 \qquad T_F=\frac{1}{2}$$

Use the Basic Factorization Formula

$$\sigma = f \otimes \omega \otimes d \;\; + O(\Lambda^2/Q^2)$$

At Second Order:

$$\sigma^2 = f^2 \otimes \omega^0 \otimes d^0 + ...$$
$$f^1 \otimes \omega^1 \otimes d^0 + ...$$

Therefore:

$$\omega^2 = ???$$

- Compute ω^2 at second order.
- Make a diagrammatic representation of each term.

HOMEWORK PROBLEM: CONVOLUTIONS

Part 1) Show these 3 definitions are equivalent; work out the limits of integration.

$$f \otimes g = \int f(x)\, g(y)\, \delta(z - x * y)\, dx\, dy$$

$$f \otimes g = \int f(x)\, g\left(\frac{z}{x}\right) \frac{dx}{x}$$

$$f \otimes g = \int f\left(\frac{z}{y}\right) g(y)\, \frac{dy}{y}$$

Part 2) Show convolutions are the ``natural" way to multiply probabilities.

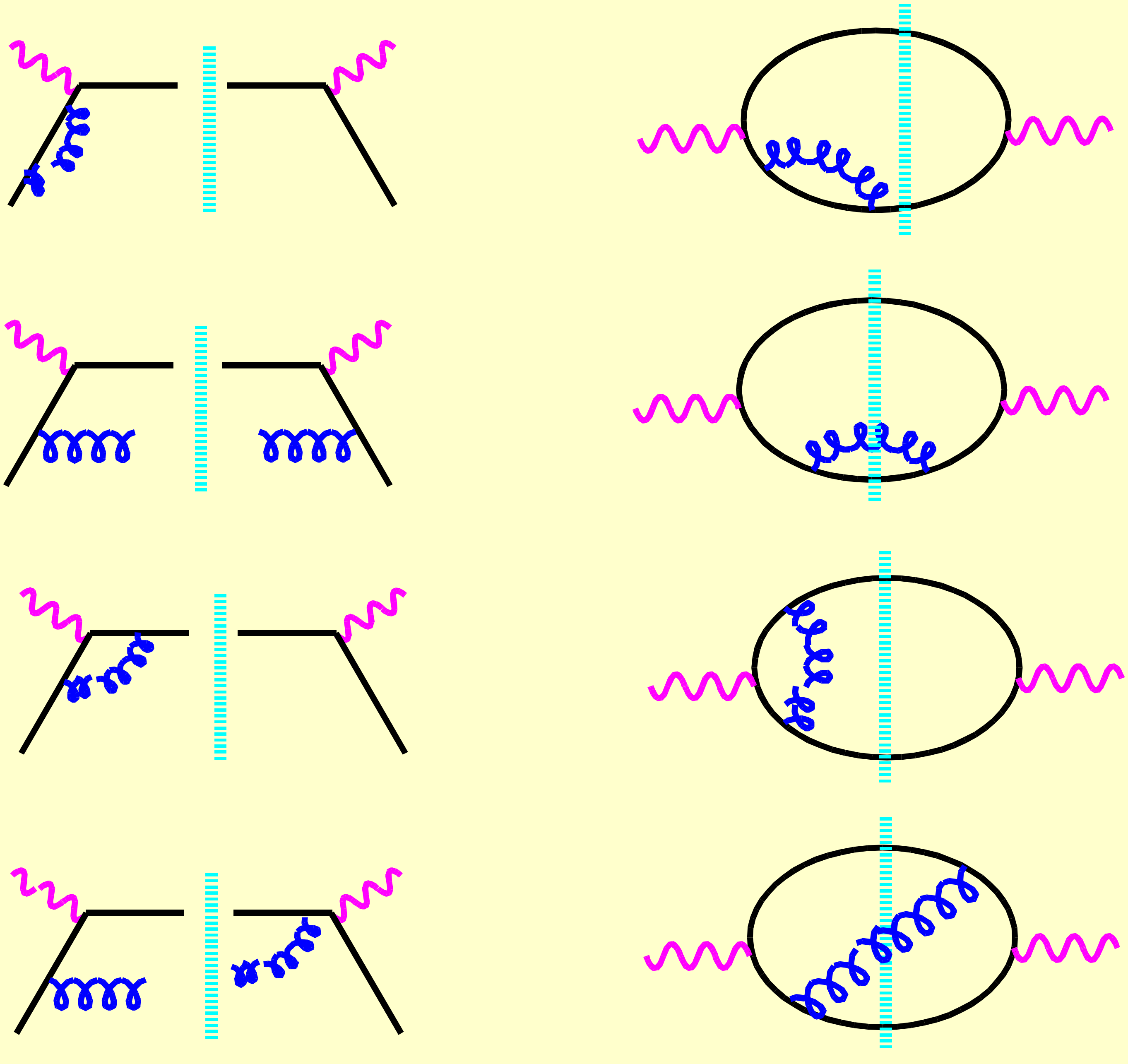
If f represents the heads/tails probability distribution for a single coin flip, show that the distribution of 2 coins is f⊕f and 3 coins is f⊕f⊕f.

$$f \oplus g = \int f(x)\, g(y)\, \delta(z - (x + y))\, dx\, dy$$

$$f(x) = \frac{1}{2}\left(\delta(1 - x) + \delta(1 + x)\right)$$

BONUS: How many processes can you think of that don't factorize?

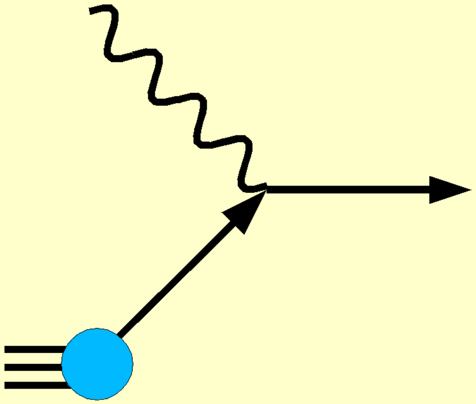
KLN Theorem: cancellations of soft singularities

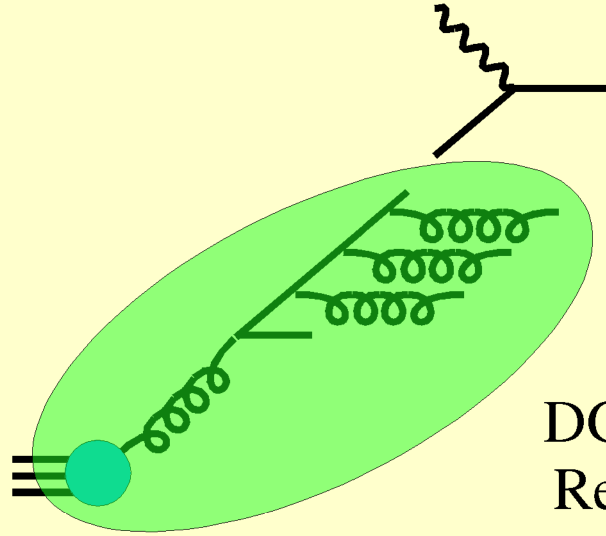


Mass-Independent Evolution.

Why is it valid?

DGLAP Equation and the Heavy Quark PDF





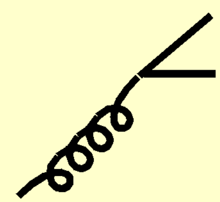
DGLAP equation
Resums iterative
splittings inside
the proton

$$\mathrm{HE} = \int f(P \to a) \otimes \sigma(a \to c)$$

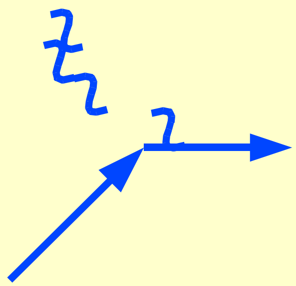
DGLAP Equation
$$\frac{df_i}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\ {}^{1}P_{j\to i} \otimes f_j + \ldots$$

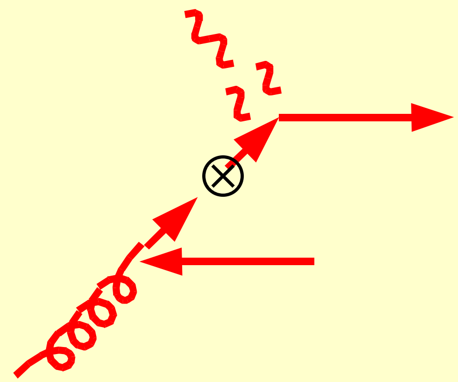
Splitting Function

$$ {}^{1}P_{g\to q} = \frac{1}{2}\left[x^2 + (1-x)^2\right] + \left(\frac{M_H^2}{\mu^2}\right)\left[x(1-x)\right]$$



Effect of Heavy Quark Mass in the Calculation





$$\mathrm{HE} = \int \underbrace{f(P\to a)}\otimes\sigma(a\to c)$$

$$\mathrm{SUB} = \int f(P\to g)\otimes{}^{1}P(g\to a)\otimes\sigma(a\to c)$$

$$\approx \quad f(P\to g)\otimes{}^{1}P(g\to a)$$

${}^{1}P$ splittings must match

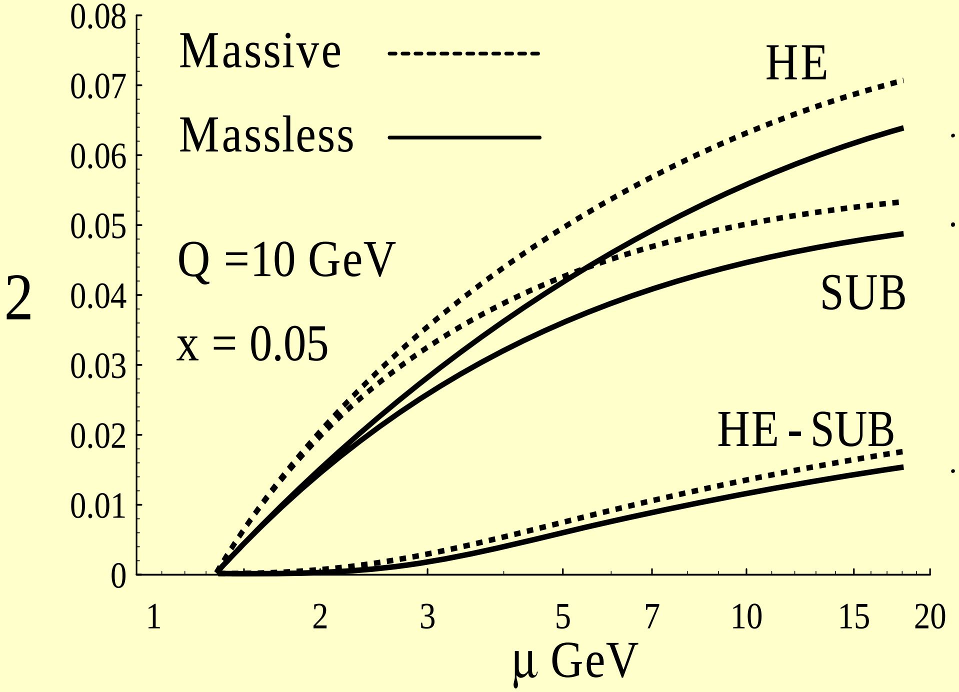
valid near threshold (M_H~Q)

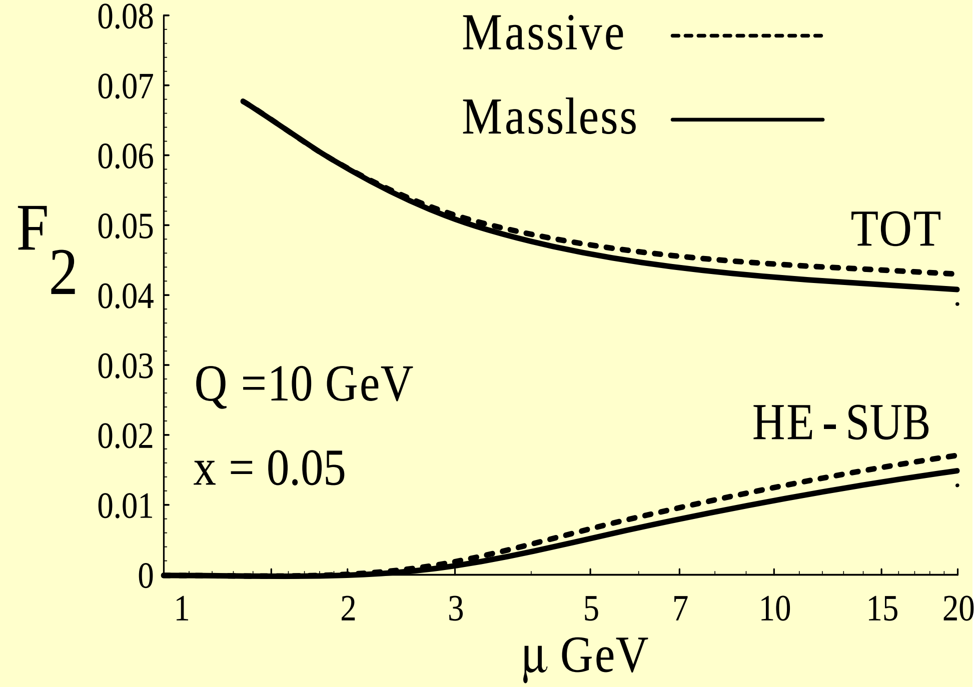
In Summary:

Near threshold(M_H~Q), mass effects cancel between HE and SUB

Above threshold(M_H<<Q), mass effects can be ignored

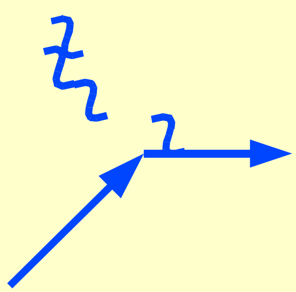
Effect of Heavy Quark Mass in the Calculation is Trivial

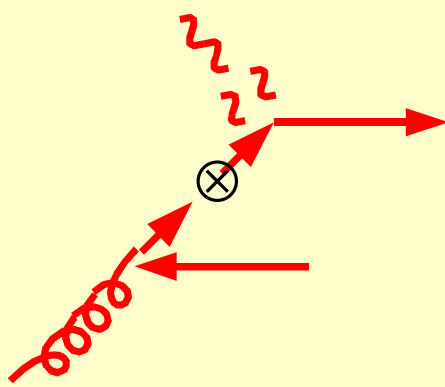


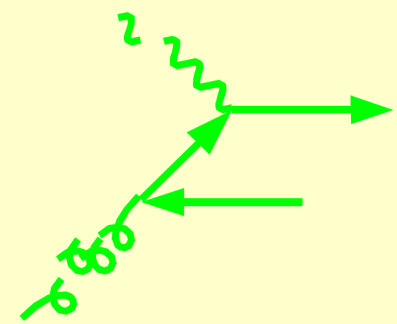


$$\mathrm{HE} = \int f(P \to a) \otimes \sigma(a \to c)$$

$$\mathrm{HC} = \int f(P \to g) \otimes \sigma(g \to c)$$







$$\mathrm{SUB} = \int f(P \to g) \otimes {}^{1}P(g \to a) \otimes \sigma(a \to c)$$