

#### **History:**

Discovery of J/ $\psi$ , Upsilon, W/Z, and "New Physics" ???

#### **Calculation of** $q q \rightarrow \mu^+ \mu^-$ in the Parton Model

Scaling form of the cross section

Rapidity, longitudinal momentum, and x<sub>F</sub>

#### **Comparison with data:**

NLO QCD corrections essential (the K-factor)  $\sigma(pd)/\sigma(pp)$  important for d-bar/ubar W Rapidity Asymmetry important for slope of d/u at large x Where are we going? P<sub>T</sub> Distribution W-mass measurement Resummation of soft gluons

## Historical

# Background

#### **Our story begins in the late 1960's at CERN**



#### **Brookhaven National Lab Alternating Gradient Synchrotron**





#### **An Early Experiment:**



with the decay of the W into muone as the signature 1/2. Failure to observe a muon signal from any

#### What is the explanation???

In DIS, we have two choices for an interpretation:





The Parton Model

#### **Discovery of the J/Psi Particle**



(Received 12 November 1974)

We report the observation of a heavy particle J, with mass m = 3.1 GeV and width approximately zero. The observation was made from the reaction  $p + \text{Be} \rightarrow e^+ + e^- + x$  by measuring the  $e^+e^-$  mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

This experiment is part of a large program to

daily with a thin Al foil. The beam spot

very narrow width  $\Rightarrow$  long lifetime





#### **The November Revolution**



currents. The run at reduced current was taken two months later than the normal run.

**More Discoveries with Drell-Yan** 

1974: The J/Psi (charm) discovery

 $p{+}N \rightarrow J/\psi$ 

... 1976 Nobel Prize

1977: The Upsilon (bottom) discovery

$$p+N \rightarrow \Upsilon$$

1983: The W and Z discovery

 $p + \overline{p} \rightarrow W/Z$ 

... 1984 Nobel Prize



### W/Z in the electron channel



- 1139 Z $\rightarrow$ ee candidates . |η<sup>e</sup>|<1.1, E-25 GeV, no</li>
  - track match required
- ε(Z)≈8%, bkgd ~ 18%

 $\sigma(Z)Br(Z \rightarrow ee) = 294 \pm 11(N_z) \pm 8(sys) \pm 29(lumi)$  pb

UIC

#### **The Future of Drell-Yan**

Where do we find

### **New Physics??**

- New Higgs Bosons
- New W' or Z'
- SUSY
- ... unknown...







- High Mass Dileptons
  - electrons & muons used
- Sensitive to Z' and Randall-Sundrum Graviton
- No excess observed



## Let's

## Calculate

First, we'll compute the partonic  $\hat{\sigma}$  in the partonic CMS



Gathering factors and contracting  $g^{\mu\nu}$ , we obtain:

$$-iM = iQ_i \frac{e^2}{q^2} \{\overline{v}(p_2) \gamma^{\mu} u(p_1)\} \{\overline{u}(p_3) \gamma_{\mu} v(p_4)\}$$

Squaring, and averaging over spin and color, ....

$$\overline{|M|^2} = \left(\frac{1}{2}\right)^2 3\left(\frac{1}{3}\right)^2 Q_i^2 \frac{e^4}{q^4} Tr\left[p_{\overline{2}}\gamma^{\mu}p_{\overline{1}}\gamma^{\nu}\right] Tr\left[p_{\overline{3}}\gamma_{\mu}p_{\overline{4}}\gamma_{\nu}\right]$$

#### Let's work out some parton level kinematics



$$p_{1} = \frac{\sqrt{\hat{s}}}{2} (1,0,0,+1)$$

$$p_{2} = \frac{\sqrt{\hat{s}}}{2} (1,0,0,-1)$$

$$p_{3} = \frac{\sqrt{\hat{s}}}{2} (1,+\sin(\theta),0,+\cos(\theta))$$

$$p_{4} = \frac{\sqrt{\hat{s}}}{2} (1,-\sin(\theta),0,-\cos(\theta))$$

#### Defining the Mandelstam variables ...

$$\begin{aligned} \hat{s} &= (p_1 + p_2)^2 = (p_3 + p_4)^2 & \hat{t} &= -\frac{\hat{s}}{2} \left(1 - \cos(\theta)\right) \\ \hat{t} &= (p_1 - p_3)^2 = (p_2 - p_4)^2 & \hat{u} &= -\frac{\hat{s}}{2} \left(1 + \cos(\theta)\right) \\ \hat{u} &= (p_1 - p_4)^2 = (p_2 - p_3)^2 & \hat{u} &= -\frac{\hat{s}}{2} \left(1 + \cos(\theta)\right) \end{aligned}$$

Manipulating the traces, we find ...

$$Tr\left[p_{\overline{2}}\gamma^{\mu}p_{\overline{1}}\gamma^{\nu}\right] Tr\left[p_{\overline{3}}\gamma_{\mu}p_{\overline{4}}\gamma_{\nu}\right] = 4\left[p_{1}^{\mu}p_{2}^{\nu}+p_{2}^{\mu}p_{1}^{\nu}-g^{\mu\nu}(p_{1}\cdot p_{2})\right] \times 4\left[p_{3}^{\mu}p_{4}^{\nu}+p_{4}^{\mu}p_{3}^{\nu}-g^{\mu\nu}(p_{3}\cdot p_{4})\right] = 2^{5}\left[(p_{1}\cdot p_{3})(p_{2}\cdot p_{4})+(p_{1}\cdot p_{4})(p_{2}\cdot p_{3})\right] = 2^{3}\left[\hat{t}^{2}+\hat{u}^{2}\right]$$

Where we have used:

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$$

$$\hat{s} = 2(p_1 \cdot p_2) = 2(p_3 \cdot p_4)$$
$$\hat{t} = 2(p_1 \cdot p_3) = 2(p_2 \cdot p_4)$$
$$\hat{u} = 2(p_1 \cdot p_4) = 2(p_2 \cdot p_3)$$

Putting all the pieces together, we have:

$$\overline{|M|^{2}} = Q_{i}^{2} \alpha^{2} \frac{2^{5} \pi^{2}}{3} \left(\frac{\hat{t}^{2} + \hat{u}^{2}}{\hat{s}^{2}}\right) \qquad \forall$$

with

$$q^{2} = (p_{1} + p_{2})^{2} = \hat{s}$$
$$\alpha = \frac{e^{2}}{4\pi}$$

#### ... and put it together to find the cross section

$$d\hat{\sigma} \simeq \frac{1}{2\hat{s}} \overline{|M|^2} d\Gamma$$
 In the partonic CMS system

$$d\Gamma = \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) = \frac{d\cos(\theta)}{16\pi}$$

Recall,

$$\hat{t} = \frac{-\hat{s}}{2} \left( 1 - \cos(\theta) \right) \quad and \quad \hat{u} = \frac{-\hat{s}}{2} \left( 1 + \cos(\theta) \right)$$

so, the differential cross section is ...

$$\frac{d\,\widehat{\sigma}}{d\cos(\theta)} = Q_i^2 \,\alpha^2 \,\frac{\pi}{6} \,\frac{1}{\widehat{s}} \left(1 + \cos^2(\theta)\right)$$

and the total cross section is ...

$$\widehat{\sigma} = Q_i^2 \alpha^2 \frac{\pi}{6} \frac{1}{\widehat{s}} \int_{-1}^{1} d\cos(\theta) \left(1 + \cos^2(\theta)\right) = \frac{4\pi \alpha^2}{9\widehat{s}} Q_i^2 \equiv \widehat{\sigma}_0$$

#### **Some Homework:**

#1) Show:

$$\frac{d^{3}p}{(2\pi)^{3}2E} = \frac{d^{4}p}{(2\pi)^{4}} (2\pi) \delta^{+}(p^{2}-m^{2})$$

This relation is often useful as the RHS is manifestly Lorentz invariant

#### #2) Show that the 2-body phase space can be expressed as:

$$d\Gamma = \frac{d^{3}p_{3}}{(2\pi)^{3}2E_{3}} \frac{d^{3}p_{4}}{(2\pi)^{3}2E_{4}} (2\pi)^{4} \delta(p_{1}+p_{2}-p_{3}-p_{4}) = \frac{d\cos(\theta)}{16\pi}$$

Note, we are working with massless partons, and  $\theta$  is in the partonic CMS frame

#### #3) Let's work out the general $2\rightarrow 2$ kinematics for general masses.



a) Start with the incoming particles.

Show that these can be written in the general form:

$$p_1 = (E_1, 0, 0, +p) \qquad p_1^2 = m_1^2$$
$$p_2 = (E_2, 0, 0, -p) \qquad p_2^2 = m_2^2$$

... with the following definitions:

$$E_{1,2} = \frac{\hat{s} \pm m_1^2 \mp m_2^2}{2\sqrt{\hat{s}}} \quad p = \frac{\Delta(\hat{s}, m_1^2, m_2^2)}{2\sqrt{\hat{s}}}$$
$$\Delta(a, b, c) = \sqrt{a^2 + b^2 + c^2 - 2(ab + bc + ca)}$$

Note that  $\Delta(a,b,c)$  is symmetric with respect to its arguments, and involves the only invariants of the initial state: s,  $m_1^2$ ,  $m_2^2$ .

b) Next, compute the general form for the final state particles,  $p_3$  and  $p_4$ . Do this by first aligning  $p_3$  and  $p_4$  along the z-axis (as  $p_1$  and  $p_2$  are), and then rotate about the y-axis by angle  $\theta$ .

#### What does the angular dependence tell us?

Observe, the angular dependence:  $q + \overline{q} \rightarrow e^+ + e^-$ 

$$\frac{d\,\widehat{\sigma}}{d\cos\left(\theta\right)} = Q_i^2 \,\alpha^2 \,\frac{\pi}{6} \,\frac{1}{\widehat{s}} \left(1 + \cos^2(\theta)\right)$$

#### Characteristic of scattering of spin 1/2 constitutients by a spin 1 vector



Note, for the photon, the mirror image of the above is also valid; hence the symmetric distribution. The W has V-A couplings, so we'll find:  $(1+\cos\theta)^2$ 

### Next, we'll compute the hadronic CMS

#### **Kinematics in the Hadronic Frame**



$$P_{1} = \frac{\sqrt{s}}{2} (1,0,0,+1) \qquad P_{1}^{2} = 0$$
$$P_{2} = \frac{\sqrt{s}}{2} (1,0,0,-1) \qquad P_{2}^{2} = 0$$

$$s = (P_1 + P_2)^2 = \frac{\hat{s}}{x_1 x_2} = \frac{\hat{s}}{\tau}$$

$$\tau = x_1 x_2 = \frac{\hat{s}}{s} \equiv \frac{Q^2}{s}$$

• Fractional energy<sup>2</sup> between partonic and hadronic system

$$\frac{d\sigma}{dQ^2} = \sum_{q,\overline{q}} \int dx_1 \int dx_2 \left\{ q(x_1)\overline{q}(x_2) + \overline{q}(x_1)q(x_2) \right\} \widehat{\sigma}_0 \,\delta(Q^2 - \hat{s})$$
Hadronic Parton Partonic cross distribution cross section functions section

**Scaling form of the Drell-Yan Cross Section** 

Using: 
$$\widehat{\sigma}_0 = \frac{4\pi\alpha^2}{9\widehat{s}}Q_i^2$$
 and  $\delta(Q^2 - \widehat{s}) = \frac{1}{sx_1}\delta(x_2 - \frac{\tau}{x_1})$ 

we can write the cross section in the scaling form:

$$Q^{4} \frac{d\sigma}{dQ^{2}} = \frac{4\pi\alpha^{2}}{9} \sum_{q,\bar{q}} Q_{i}^{2} \int_{\tau}^{1} \frac{dx_{1}}{x_{1}} \tau \left\{ q(x_{1})\overline{q}(\tau/x_{1}) + \overline{q}(x_{1})q(\tau/x_{1}) \right\}$$



Notice the RHS is a function of only  $\tau$ , not Q.

This quantity should lie on a universal scaling curve.

Cf., DIS case, & scattering of point-like constituents Partonic CMS has longitudinal momentum w.r.t. the hadron frame

$$p_1 = x_1 P_1 \qquad p_2 = x_2 P_2$$

$$p_{12}$$

$$p_{12} = (p_1 + p_2) = (E_{12}, 0, 0, p_L)$$
$$E_{12} = \frac{\sqrt{s}}{2}(x_1 + x_2)$$
$$p_L = \frac{\sqrt{s}}{2}(x_1 - x_2) \equiv \frac{\sqrt{s}}{2}x_F$$

 $x_F$  is a measure of the longitudinal momentum

The rapidity is defined as:  $x_{1,2} = \sqrt{\tau} e^{\pm y}$   $y = \frac{1}{2} \ln \left\{ \frac{E_{12} + p_L}{E_{12} - p_L} \right\} = \frac{1}{2} \ln \left\{ \frac{x_1}{x_2} \right\}$   $dx_1 dx_2 = d\tau dy$   $dQ^2 dx_F = dy d\tau s \sqrt{x_F^2 + 4\tau}$ 

$$\frac{d\sigma}{dQ^{2} dx_{F}} = \frac{4\pi\alpha^{2}}{9Q^{4}} \frac{1}{\sqrt{x_{F}^{2} + 4\tau}} \tau \sum_{q,\bar{q}} Q_{i}^{2} \{q(x_{1})\bar{q}(\tau/x_{1}) + \bar{q}(x_{1})q(\tau/x_{1})\}$$

So, we're ready to compare with data

(or so we think...)

#### Let's compare data and theory



Table 1.2:	Experimental	K-factors.
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Expe	eriment	Interaction	Beam Momentum	$K = \sigma_{\rm meas.}/\sigma_{\rm DY}$
E288	[Kap 78]	p P t	$300/400~{ m GeV}$	$\sim 1.7$
WA39	[Cor 80]	$\pi^{\pm} W$	$39.5~{ m GeV}$	$\sim 2.5$
E439	[Smi 81]	p W	$400~{ m GeV}$	$1.6 \pm 0.3$
NA3	[Bad 83]	$(\bar{p} - p)Pt$	$150  { m GeV}$	$2.3 \pm 0.4$
		$p \ Pt$	$400~{ m GeV}$	$3.1\pm0.5\pm0.3$
		$\pi^{\pm} Pt$	$200~{ m GeV}$	$2.3\pm0.5$
		$\pi^- Pt$	$150  { m GeV}$	$2.49\pm0.37$
		$\pi^- Pt$	$280  { m GeV}$	$2.22\pm0.33$
NA10	[Bet 85]	$\pi^- W$	$194~{ m GeV}$	$\sim 2.77 \pm 0.12$
E326	[Gre 85]	$\pi^- W$	$225~{ m GeV}$	$2.70 \pm 0.08 \pm 0.40$
E537	[Ana 88]	$\bar{p} W$	$125~{ m GeV}$	$2.45 \pm 0.12 \pm 0.20$
E615	[Con 89]	$\pi^- W$	$252~{ m GeV}$	$1.78\pm0.06$



Oooops,

we need the

QCD corrections

$$K = 1 + \frac{2\pi\alpha}{3}(...) + ... = ? = e^{2\pi\alpha}/3$$

p + Cu at 800 GeV

p + d at 800 GeV



pp & pN processes sensitive to anti-quark distributions

A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne,
Eur. Phys. J. C23, 73 (2002);
Eur. Phys. J. C14, 133 (2000);
Eur. Phys. J. C4, 463 (1998)

Drell-Yan can give us unique and detailed information about PDF's.

We'll now examine two examples:

1) Ratio of pp/pd cross section

2) W Rapidity Asymmetry

#### A measurement of $\overline{d}(x)/\overline{u}(x)$ Antiquark asymmetry in the Nucleon Sea FNAL E866/NuSea



ACU, ANL, FNAL, GSU, IIT, LANL, LSU, NMSU, UNM, ORNL, TAMU, Valpo.

800 GeV 
$$p + p$$
 and  $p + d \rightarrow \mu^+ \mu^- X$ 



 $u \Leftrightarrow d$ Obtain the neutron PDF via isospin symmetry:  $\overline{u} \Leftrightarrow \overline{d}$  $\sigma^{pp} \propto \frac{4}{9} u(x_1) \overline{u}(x_2) + \frac{1}{9} d(x_1) \overline{d}(x_2)$ In the limit  $x_1 >> x_2$ :  $\sigma^{pn} \propto \frac{4}{9} u(x_1) \overline{d}(x_2) + \frac{1}{9} d(x_1) \overline{u}(x_2)$ For the ratio we have:  $\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2} \frac{\left(1 + \frac{1}{4}\frac{d_1}{u_1}\right)}{\left(1 + \frac{1}{4}\frac{d_1}{u_1}\frac{\overline{d}_2}{\overline{u}_2}\right)} \quad \left(1 + \frac{\overline{d}_2}{\overline{u}_2}\right) \approx \frac{1}{2} \left(1 + \frac{\overline{d}_2}{\overline{u}_2}\right)$ 

As promised, this provides information about the sea-quark distributions

$$\frac{\sigma^{pd}}{2\,\sigma^{pp}} \approx \frac{1}{2} \left( 1 + \frac{\overline{d}_2}{\overline{u}_2} \right)$$

EXERCISE: Verify the above.

#### **Does the theory match the data???**





#### **E866** required significant changes in the hi-x sea distributions

With increased flexibility in the parameterization of the sea-quark distributions, good fits are obtained

E.A. Hawker, et al. [FNAL E866/NuSea Collaboration], Measurement of the light antiquark flavor asymmetry in the nucleon sea, PRL 80, 3715 (1998)

H. L. Lai, et al. } [CTEQ Collaboration], Global {QCD} analysis of parton structure of the nucleon: CTEQ5 parton distributions, EPJ C12, 375 (2000)



### 2) W Rapidity Asymmetry

#### Where do the W's and Z's come from ???

$$\frac{d\sigma}{dy}(W^{\pm}) = \frac{2\pi}{3} \frac{G_F}{\sqrt{2}} \sum_{q\bar{q}} |V_{q\bar{q}}|^2 \left[q(x_a) \bar{q}(x_b) + q(x_b) \bar{q}(x_a)\right]$$
  
Havour decomposition of W cross sections  

$$\frac{u(x_a)}{proton} \frac{d(x_b)}{W^+} \text{ anti-proton}$$
For anti-proton:  

$$u(x) \Leftrightarrow \bar{u}(x) \quad d(x) \Leftrightarrow \bar{d}(x)$$
Therefore  

$$\frac{d\sigma}{dy}(W^+) \approx \frac{2\pi}{3} \frac{G_F}{\sqrt{2}} \left[u(x_a) d(x_b)\right]$$

$$\frac{d\sigma}{dy}(W^-) \approx \frac{2\pi}{3} \frac{G_F}{\sqrt{2}} \left[d(x_a) u(x_b)\right]$$
A.D. Martin, R.G. Roberts, W.J. Strong, W.S. There, Strong, Str

Eur. Phys. J. C23, 73 (2002); Eur. Phys. J. C4, 463 (1998)

#### A bit of calculation



$$A(y) = \frac{\frac{d\sigma}{dy}(W^{+}) - \frac{d\sigma}{dy}(W^{-})}{\frac{d\sigma}{dy}(W^{+}) + \frac{d\sigma}{dy}(W^{-})}$$

With the previous approximation,

$$A \approx \frac{u(x_a)d(x_b) - d(x_a)u(x_b)}{u(x_a)d(x_b) + d(x_a)u(x_b)} =$$
  
where 
$$R_{du}(x) = \frac{d(x)}{u(x)}$$

We can make Taylor expansions:

#### Thus, the asymmetry is:

EXERCISE: Verify the above.

$$\frac{R_{du}(x_b) - R_{du}(x_a)}{R_{du}(x_b) + R_{du}(x_a)}$$

$$x_{1,2} = x_0 e^{\pm y} \simeq x_0 (1 \pm y)$$
$$R_{du}(x_{1,2}) \approx R_{du}(x_0) \pm y x_0 R'_{du}(\sqrt{\tau})$$
$$A(y) = -y x_0 \frac{R'_{du}(x_0)}{R_{du}(x_0)}$$

#### **Charged Lepton Asymmetry**

Unfortunately, we don't measure the W directly since W→ev.

Still the lepton contains important information



$$A(y) = \frac{\frac{d\sigma}{dy}(l^{+}) - \frac{d\sigma}{dy}(l^{-})}{\frac{d\sigma}{dy}(l^{+}) + \frac{d\sigma}{dy}(l^{-})}$$

#### d/u Ratio at High-x

The form of the d/u ratio at large x as a function of

1) Parameterization

2) Nuclear Corrections



S. Kuhlmann, et al., Large-x parton distributions, PL B476, 291 (2000)

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#### **Comparison with data:**

NLO QCD corrections essential (the K-factor)  $\sigma(pd)/\sigma(pp)$  important for d-bar/ubar W Rapidity Asymmetry important for slope of d/u at large x Where are we going?  $P_T$  Distribution W-mass measurement Resummation of soft gluons