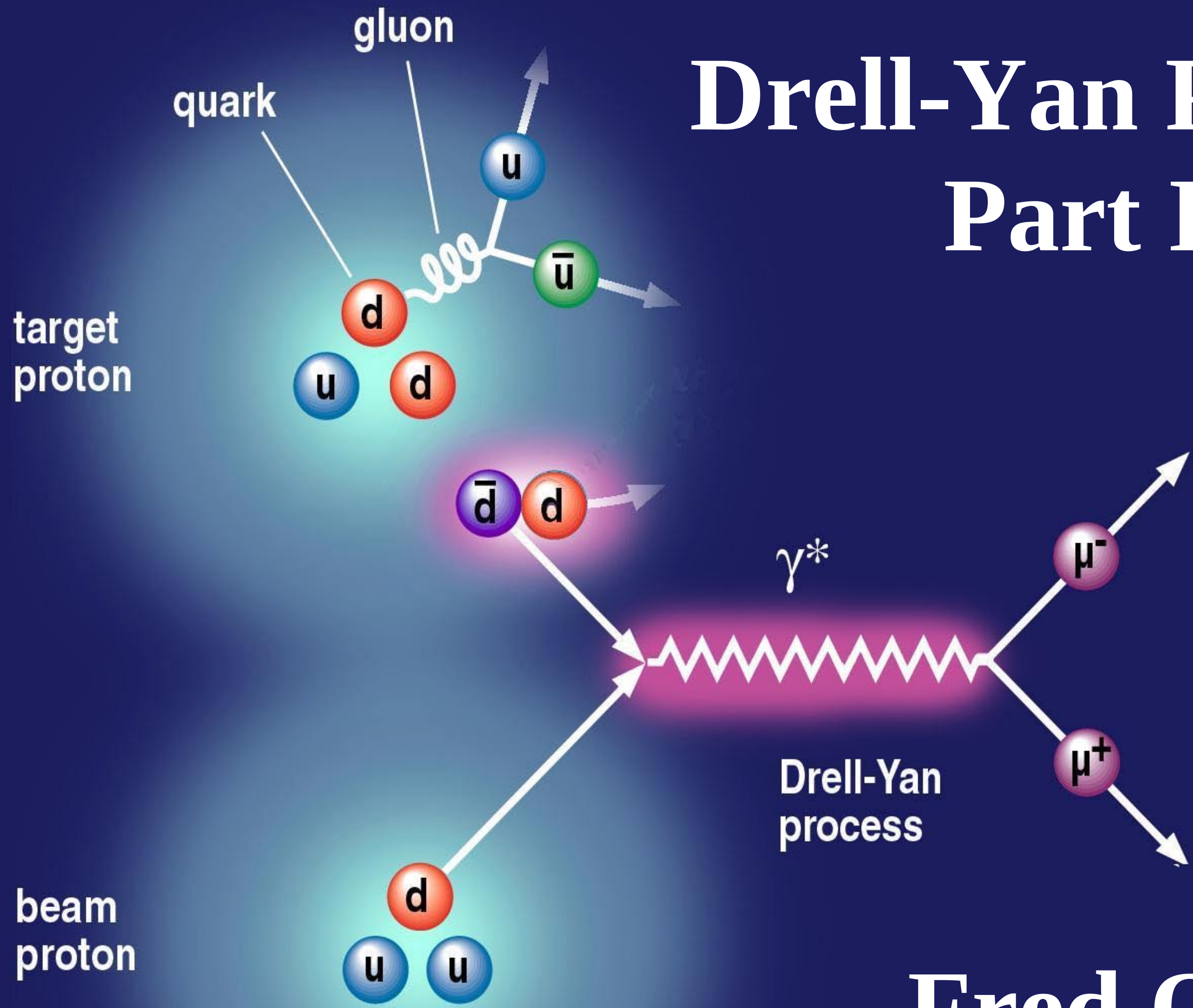


Drell-Yan Process: Part IV

Fred Olness
SMU

OpenOffice.org Source Project

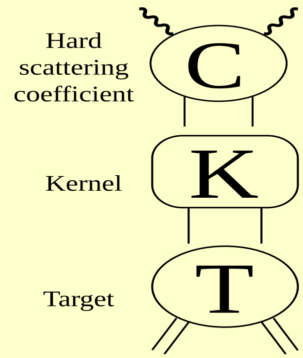
Power Point

There is a rigorous factorization proof ...

Ingredients of Factorization

Decompose into (t-channel) 2PI amplitudes:

$$\sigma = \Sigma_{N=1}^{\infty} \; C \; (K)^N \; T \; + \textit{Non-leading}$$



Collins, Soper, Sterman. Perturbative QCD, World Scientific (1989). Collins, in preparation

After reorganization of the infinite sum:

$$\sigma \approx \underbrace{\overbrace{C\,[1-(1-Z)\,K\,]^{-1}}^{}\; Z \;\; [1-K\,]^{-1}\, T}_{\text{Parton Model}} + \underbrace{C\,[1-(1-Z)\,K\,]^{-1}\,(1-Z)T}_{\text{Remainder}}$$

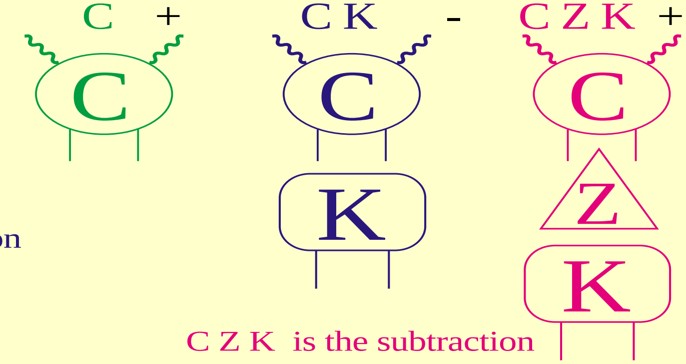
- $C\,[1-(1-Z)\,K\,]^{-1}$: Wilson Coefficient (Hard Scatt. $\hat{\sigma}$)
- Z: collinear projection
- $[1-K\,]^{-1}\,T$: Parton Distribution
- $(1-Z)$: Power Suppressed

Wilson Coefficient:

$$C\,[1-(1-Z)\,K\,]^{-1} \approx \underbrace{C}_{\text{Leading Order}} + \underbrace{C\,K - C\,Z\,K}_{\text{Next to Leading Order}} + \ldots$$

All orders result

Wilson Coefficient: IR safe "hard" scattering cross section



C Z K is the subtraction

A formal proof was constructed by numerous groups.

This proof was explicitly extended to the case of massive quarks

(Collins,1998)

THOUGH EXPERIMENT
To keep things simple, let's consider scattering off a parton target.

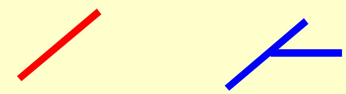
Application of Factorization Formula at Leading Order (LO)

Basic Factorization Formula

$$\sigma = f \otimes \omega \otimes d \;\; + O(\Lambda^2/Q^2)$$

At Zeroth Order:

$$\sigma^0 = f^0 \otimes \omega^0 \otimes d^0 \;\; + O(\Lambda^2/Q^2)$$



f^0 f^1

for parton target

Use: $f^0 = \delta$ and $d^0 = \delta$ for a parton target.

Therefore:

$$\sigma^0 = f^0 \otimes \omega^0 \otimes d^0 = \delta \otimes \omega^0 \otimes \delta = \omega^0$$

$$\sigma^0 = \omega^0$$

Warning: This trivial result leads to many misconceptions at higher orders

Basic Factorization Formula

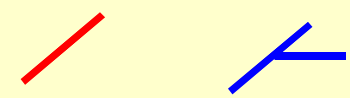
$$\sigma = f \otimes \omega \otimes d \; + O(\Lambda^2/Q^2)$$

At First Order:

$$\sigma^1 = f^1 \otimes \omega^0 \otimes d^0 + f^0 \otimes \omega^1 \otimes d^0 + f^0 \otimes \omega^0 \otimes d^1$$

$$\sigma^1 = \quad f^1 \otimes \sigma^0 + \omega^1 + \sigma^0 \otimes d^1$$

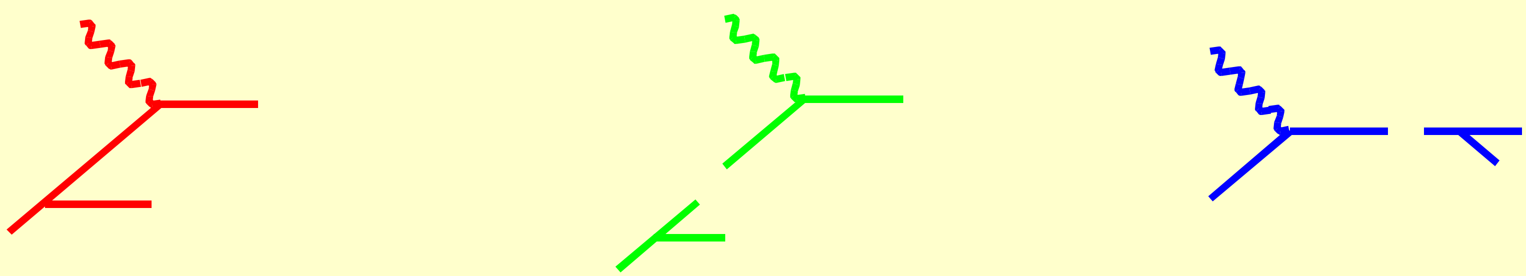
We used: $f^0 = \delta$ and $d^0 = \delta$ for a parton target.



f^0 f^1

Therefore:

$$\omega^1 = \sigma^1 - f^1 \otimes \sigma^0 - \sigma^0 \otimes d^1$$



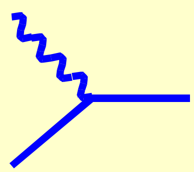
σ^1 $f^1 \otimes \sigma^0$ $\sigma^0 \otimes d^1$

Combined Result:

$$\omega^{0}+\omega^{1}=\sigma^{0}+\sigma^{1}-\left\{f^{1}\otimes\sigma^{0}+\sigma^{0}\otimes d^{1}\right\}$$

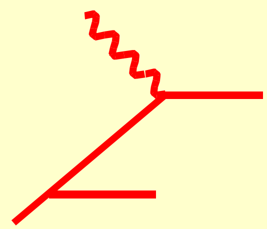
TOT

HE



Heavy Excitation

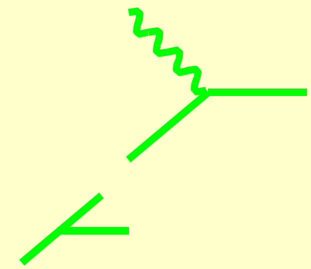
HC



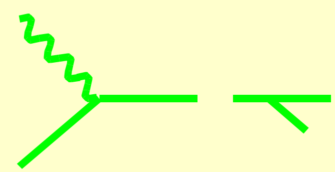
Heavy Excitation

SUB

Subtraction



$f^{1}\otimes\sigma^{0}$



$\sigma^{0}\otimes d^{1}$

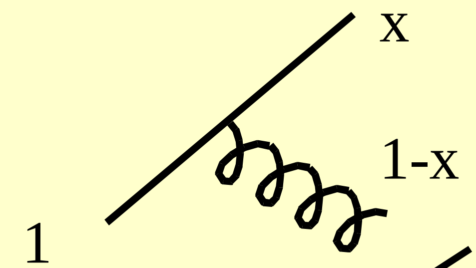
TOT = HE + HC – SUB

Splitting Kernel to α_s^1 order

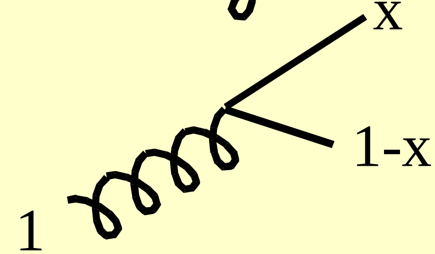
$$\phi_{i\leftarrow j}(x,\epsilon) = \delta(1-x)\,\delta_{ij} + \frac{\alpha_s}{2\pi}\left(-\frac{1}{\epsilon}\right)\left[\frac{\mu^2}{M^2}\right]^{\epsilon} P^{(1)}_{i\leftarrow j}(x)$$

Splitting Kernel to α_s^1 order

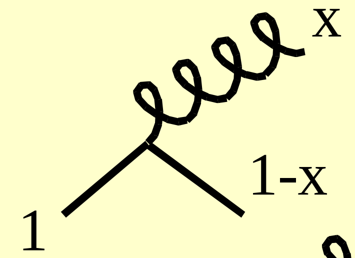
$$P^{(1)}_{q\leftarrow q}(x) = C_F\left[\frac{1+x^2}{1-x}\right]_+$$



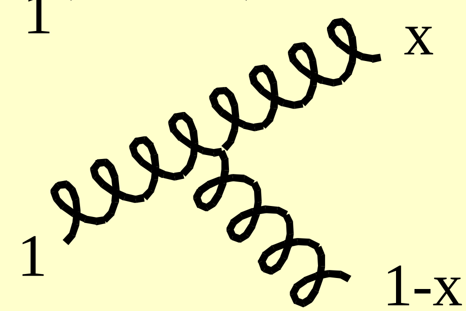
$$P^{(1)}_{q\leftarrow g}(x) = T_F\left[(1-x)^2+x^2\right]$$



$$P^{(1)}_{g\leftarrow q}(x) = C_F\,\frac{(1-x)^2+1}{x}$$



$$P^{(1)}_{g\leftarrow g}(x) = 2\,C_F\left[\frac{x}{(1-x_+)}+\frac{1-x}{x}+x(1-x)\right] + \delta(1-x)\left[\frac{11}{6}C_A-\frac{2}{3}T_F N_F\right]$$



$$C_F=\frac{4}{3} \qquad C_A=3 \qquad T_F=\frac{1}{2}$$

Use the Basic Factorization Formula

$$\sigma = f \otimes \omega \otimes d \;\; + O(\Lambda^2/Q^2)$$

At Second Order:

$$\sigma^2 = f^2 \otimes \omega^0 \otimes d^0 + \ldots$$
$$f^1 \otimes \omega^1 \otimes d^0 + \ldots.$$

Therefore:

$$\omega^2 = ???$$

- Compute ω^2 at second order.
- Make a diagrammatic representation of each term.

HOMEWORK PROBLEM: CONVOLUTIONS

Part 1) Show these 3 definitions are equivalent; work out the limits of integration.

$$f \otimes g = \int f(x)\, g(y)\, \delta(z - x * y)\, dx\, dy$$

$$f \otimes g = \int f(x)\, g\left(\frac{z}{x}\right) \frac{dx}{x}$$

$$f \otimes g = \int f\left(\frac{z}{y}\right) g(y) \frac{dy}{y}$$

Part 2) Show convolutions are the ``natural" way to multiply probabilities.

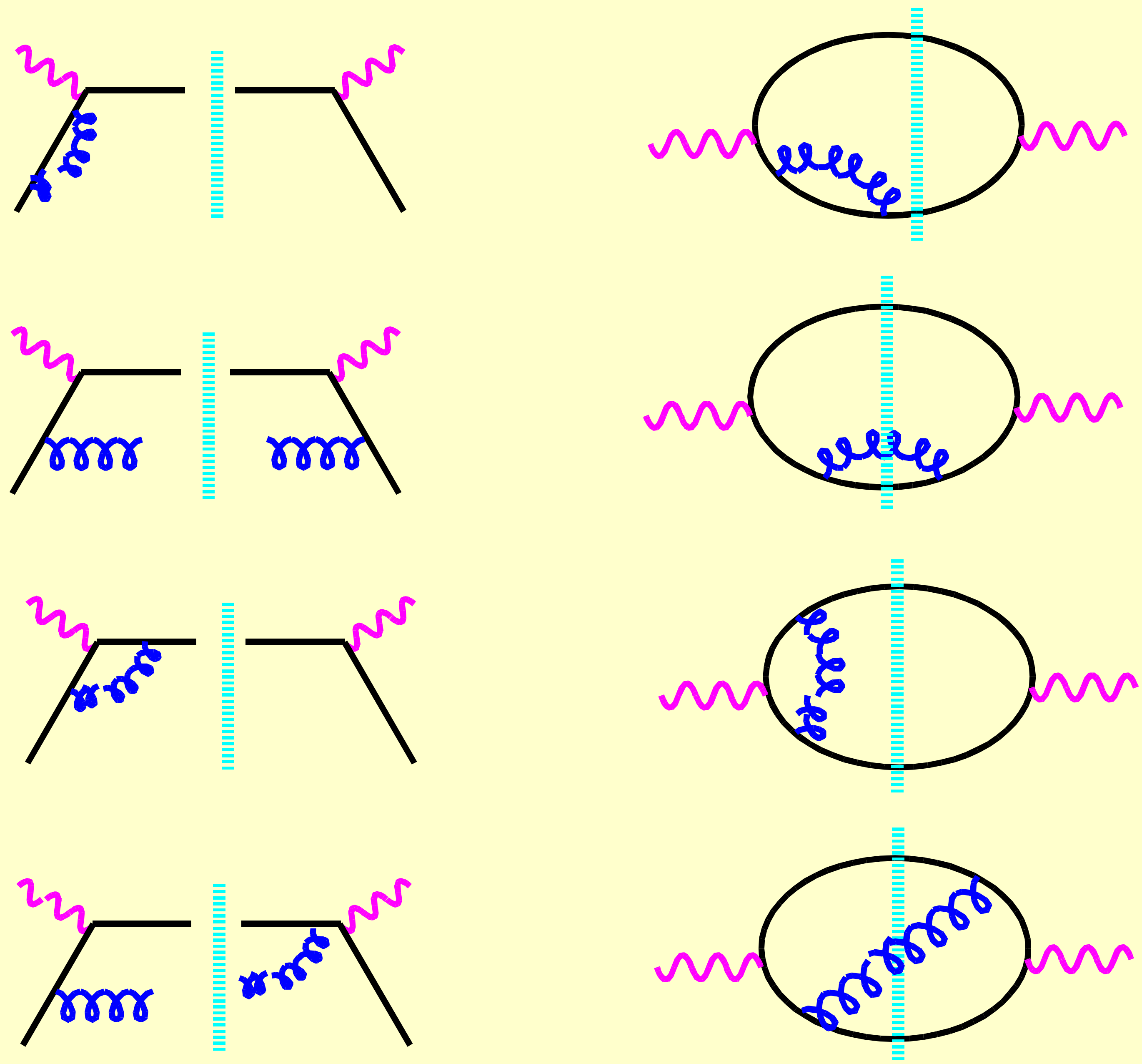
If f represents the heads/tails probability distribution for a single coin flip, show that the distribution of 2 coins is f⊕f and 3 coins is f⊕f⊕f.

$$f \oplus g = \int f(x)\, g(y)\, \delta(z - (x + y))\, dx\, dy$$

$$f(x) = \frac{1}{2}(\delta(1 - x) + \delta(1 + x))$$

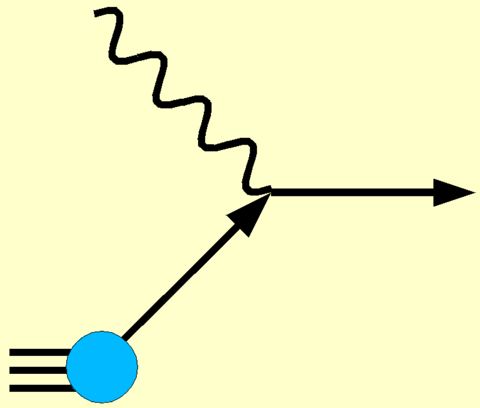
BONUS: How many processes can you think of that don't factorize?

KLN Theorem: cancellations of soft singularities

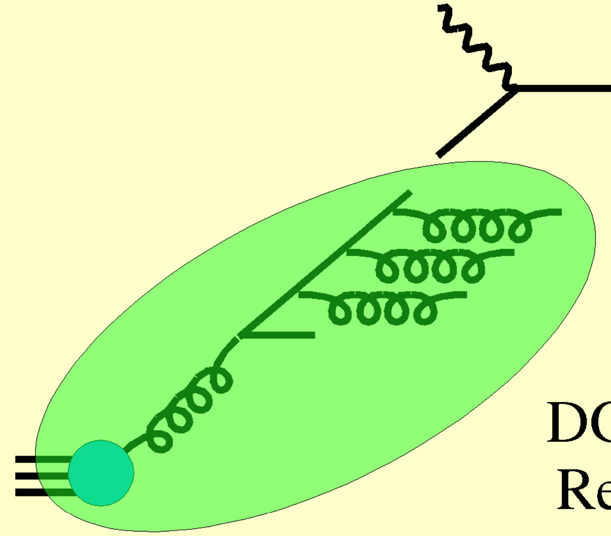


Mass-Independent Evolution.

Why is it valid?



$$HE = \int f(P \to a) \otimes \sigma(a \to c)$$



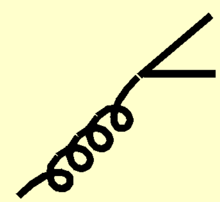
DGLAP equation
Resums iterative
splittings inside
the proton

DGLAP Equation

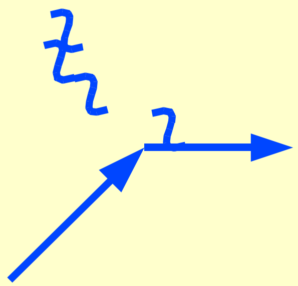
$$\frac{df_i}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\, {}^{1}P_{j\to i} \otimes f_j + \ldots$$

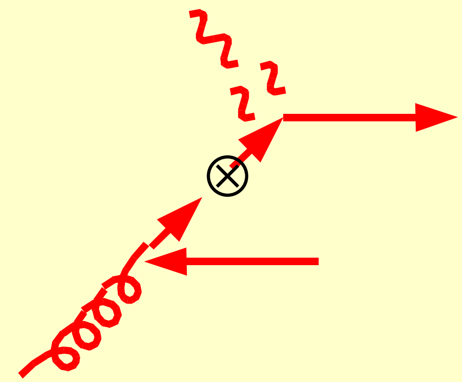
Splitting Function

$${}^{1}P_{g\to q} = \frac{1}{2}\left[x^2 + (1-x)^2\right] + \left(\frac{M_H^2}{\mu^2}\right)\left[x(1-x)\right]$$



Effect of Heavy Quark Mass in the Calculation





$$\mathrm{HE} = \int \underbrace{f(P \to a)} \otimes \sigma(a \to c)$$

$$\approx \quad f(P \to g) \otimes {}^{1}P(g \to a)$$

valid near threshold (M_H~Q)

$$\mathrm{SUB} = \int f(P \to g) \otimes {}^{1}P(g \to a) \otimes \sigma(a \to c)$$

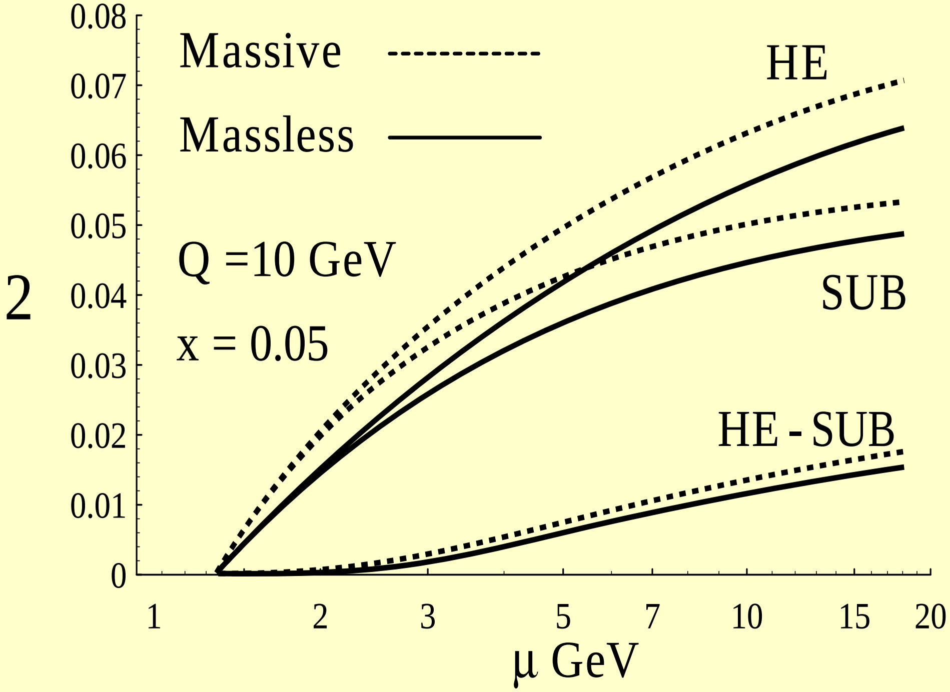
${}^{1}P$ splittings must match

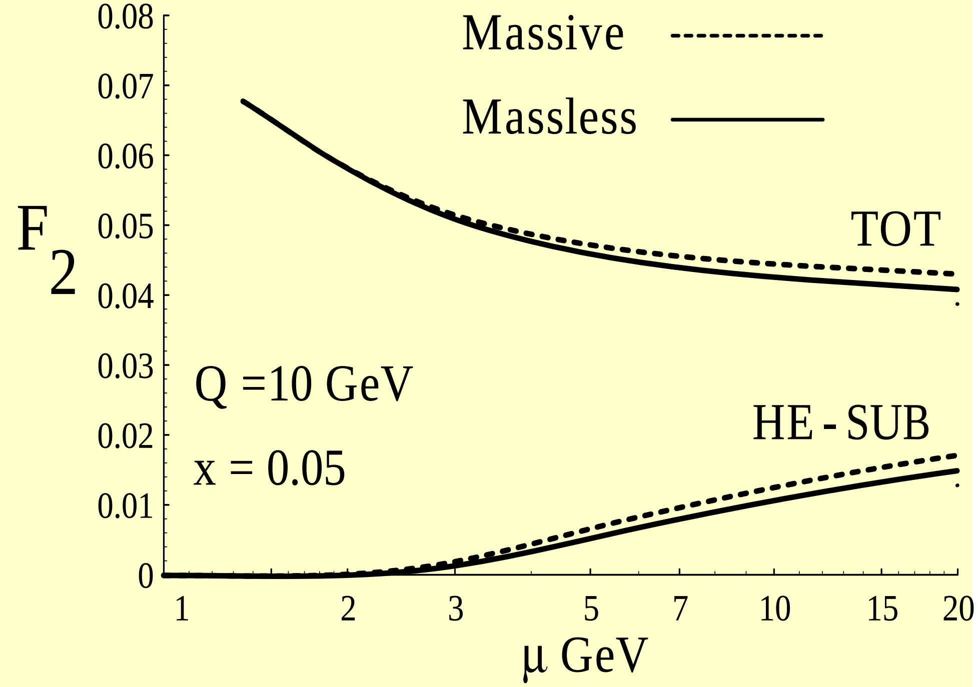
In Summary:

Near threshold(M_H~Q), mass effects cancel between HE and SUB

Above threshold(M_H<<Q), mass effects can be ignored

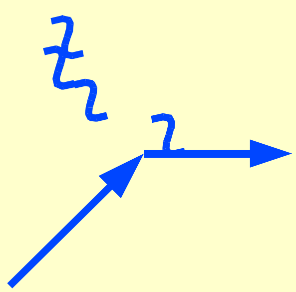
Effect of Heavy Quark Mass in the Calculation is Trivial

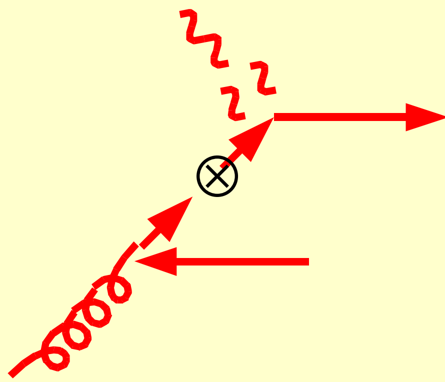


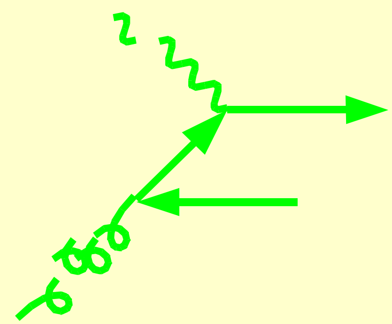


$$\mathrm{HE} = \int f(P \to a) \otimes \sigma(a \to c)$$

$$\mathrm{HC} = \int f(P \to g) \otimes \sigma(g \to c)$$







$$\mathrm{SUB} = \int f(P \to g) \otimes {}^{1}P(g \to a) \otimes \sigma(a \to c)$$