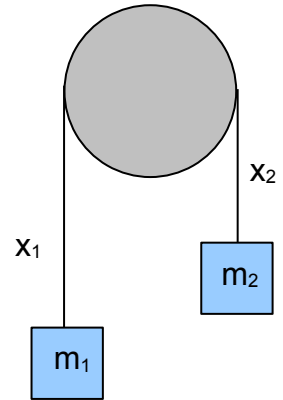


1,2,3) Consider the 2-body Atwood machine given as an example in Chapter 1 (problem 2, page 27),  
We are going to solve this many different ways.

- 1) Solve this problem using Newton's 2<sup>nd</sup> equation:  $F=ma$  for  $\{a, T\}$ , the acceleration of the masses and the tension  $T$ .
- 2) Solve this again, but use the Lagrangian method (as illustrated in your book) with ONE independent variable by using  $x_1+x_2=L$ , therefore  $x_2=L-x_1$ .
- 3) Solve this again, but use the Lagrangian method (as illustrated in your book) with TWO independent variables,  $\{x_1,x_2\}$ . This time you will use a constraint equation  $x_1+x_2=L$  and introduce a Lagrange multiplier  $\lambda$ . Solve for  $\{a_1,a_2,\lambda\}$



- 4) An Atwood machine is built as follows: A string of length  $L_1$  passes over a fixed light pulley supporting a mass  $m_1$  on one end and a pulley of mass  $m_2$  (negligible moment of inertia) on the other. Over this second pulley passes a string of length  $L_2$  supporting a mass  $m_3$  on one end and  $m_4$  on the other where  $m_3 \neq m_4$ . Set up Lagrange's equations for this system using the method of Lagrange multipliers. Use as coordinates the (not independent) distances of the four masses from the fixed pulley ( $x_1, x_2, x_3, x_4$ ). Solve for the two forces of constraint and show that the mass  $m_1$  remains in equilibrium if:

$$m_1 = m_2 + m_3 + m_4 - \frac{(m_3 - m_4)^2}{(m_3 + m_4)}$$

- 5) Given that the kinetic energy is  $T = \frac{1}{2} m v^2$  (where velocity  $v=x'$ ) and the potential energy is  $U = e\phi - \frac{e}{c} \vec{A} \circ \vec{v}$  where  $\phi$  is a scalar potential and  $A$  is a vector potential which depends on  $x$ . Compute the equations of motion.

