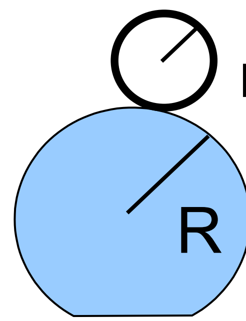
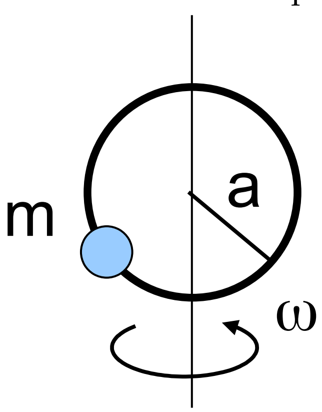
Physics 6321 — Professor Olness
Homework 3 — Due: Thursday February 1, 2024

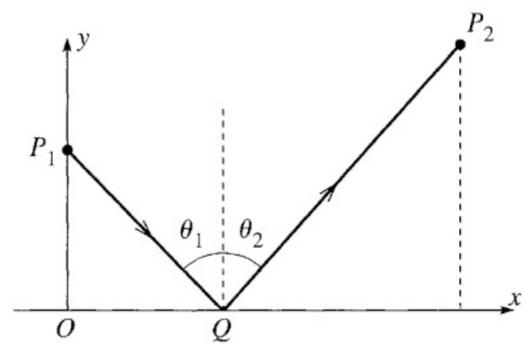
1) Complete the solution of the brachistochrone problem begun in the text and show the desired curve is a cycloid with a cusp at the initial point at which the particle is released. Show also that if the particle is projected with an initial kinetic energy ($\frac{1}{2}$)mv_0^2 that the solution is still a cycloid passing through the two points with a cusp at a height z above the initial point given by $v_0^2=2gz$.



2) A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R. The only external force is gravity. If the smaller cylinder starts rolling from rest on top of the larger cylinder, find (using Lagrange multipliers) the point at which the hoop falls off the cylinder.

3) A point mass is constrained to move on a massless hoop of radius a fixed in a vertical plane that is rotating about the vertical with constant angular speed ω. Obtain the Lagrange equations of motion assuming the only external forces arise from gravity. What are the constants of motion? Show that if ω is greater than a critical value ω_0, there can be a solution in which the particle remains stationary on the hoop at a point other than at the bottom, but that if $\omega<\omega_0$, the only stationary point for the particle is at the bottom of the hoop. What is the value of ω_0? [FIGURE BELOW]





4 Consider a ray of light traveling in a vacuum from point P_1 to P_2 by way of the point Q on a plane mirror, as in Figure 6.8. Show that Fermat's principle implies that, on the actual path followed, Q lies in the same vertical plane as P_1 and P_2 and obeys the law of reflection, that $\theta_1 = \theta_2$. [*Hints:* Let the mirror lie in the xz plane, and let P_1 lie on the y axis at $(0, y_1, 0)$ and P_2 in the xy plane at $(x_2, y_2, 0)$. Finally let $Q = (x, 0, z)$. Calculate the time for the light to traverse the path P_1QP_2 and show that it is minimum when Q has $z = 0$ and satisfies the law of reflection.]

5) a) For the following equation $r(\phi) = \dfrac{1}{1+\epsilon\cos(\phi)}$, make a POLAR PLOT of r(ϕ) for a selection of ε. Describe the features of the plot, and the relevant range of ε.

b) Repeat, for the following equation $r(\phi) = \dfrac{1}{1+\epsilon\cos(\alpha\phi)}$, make a POLAR PLOT of r(ϕ) for a selection of ε<1 and choices of α . Describe the features of the plot, and the relevant range of α.