Physics 6321 | Professor Olness
Homework 4 | Due: February 8, 2024

1. Goldstein, 2nd edition, Chapter 3, Exercise 6

2. Goldstein, 2nd edition, Chapter 3, Exercise 14

3. Goldstein, 2nd edition, Chapter 3, Exercise 13

4. Goldstein, 2nd edition, Chapter 3, Exercise 16

5. For the repulsive central potential

$$V = +V_o \quad \text{for } r \leq R$$
$$V = 0 \quad \text{for } r > R$$

draws the trajectory and compute the differential and total scattering cross sections for the two cases:

a) $E < V_o$
b) $E > V_o$

6. Start with a potential of the form

a) attractive: $V(r) = -k / r$.
b) repulsive: $V(r) = +(½)\, k\, r^2$.

a) Analytically compute the equation for the orbit, $dr/d\theta$. (If you can not do the integral analytically, do not worry, but give it a try.)

b) Compute the turning points. Does this effective potential have a stable minimum? Are there both elliptical and hyperbolic orbits?

c) Plot the potential V(r), the centrifugal term, and the effective potential.

d) Using my Mathematica program for numerically computing the orbits of planets. choose some values, and plot the orbit for your particular potential. Try this for both $E<0$ and $E>0$. Comment as the accuracy of the program, and if the results are reasonable.

e) For the two potentials above, estimate the number of turning points encountered per revolution.

6. a) Show that if a particle describes a circular orbit under the influence of an attractive central force directed toward a point on the circle, then the force varies as the inverse fifth power of the distance.

b) Show that for the orbit described the total energy of the particle is zero.

c) Find the period of the motion.

d) Find $\dot{x}$, $\dot{y}$, and v as a function of angle around the circle and show that all three quantities are infinite as the particle goes through the center of force.

14. Show that the motion of a particle in the potential field

$$V(r) = -\frac{k}{r} + \frac{h}{r^2}$$

is the same as that of the motion under the Kepler potential alone when expressed in terms of a coordinate system rotating or precessing around the center of force.

For negative total energy show that if the additional potential term is very small compared to the Kepler potential, then the angular speed of precession of the elliptical orbit is

$$\dot{\Omega} = \frac{2\pi mh}{l^2\tau}.$$

The perihelion of Mercury is observed to precess (after correction for known planetary perturbations) at the rate of about 40″ of arc per century. Show that this precession could be accounted for classically if the dimensionless quantity

$$\eta = \frac{h}{ka}$$

(which is a measure of the perturbing inverse square potential relative to the gravitational potential) were as small as 7×10^{-8}. (The eccentricity of Mercury's orbit is 0.206, and its period is 0.24 year.)

13. A uniform distribution of dust in the solar system adds to the gravitational attraction of the sun on a planet an additional force

$$\mathbf{F} = -mC\mathbf{r},$$

where m is the mass of the planet, C is a constant proportional to the gravitational constant and the density of the dust, and $\mathbf{r}$ is the radius vector from the sun to the planet (both considered as points). This additional force is very small compared to the direct sun–planet gravitational force.

a) Calculate the period for a circular orbit of radius r_0 of the planet in this combined field.

b) Calculate the period of radial oscillations for slight disturbances from this circular orbit.

c) Show that nearly circular orbits can be approximated by a precessing ellipse and find the precession frequency. Is the precession in the same or opposite direction to the orbital angular velocity?

16. Evaluate approximately the ratio of the mass of the sun to that of the earth, using only the lengths of the year and of the lunar month (27.3 days), and the mean radii of the earth's orbit (1.49×10^8 km) and of the moon's orbit (3.8×10^5 km).