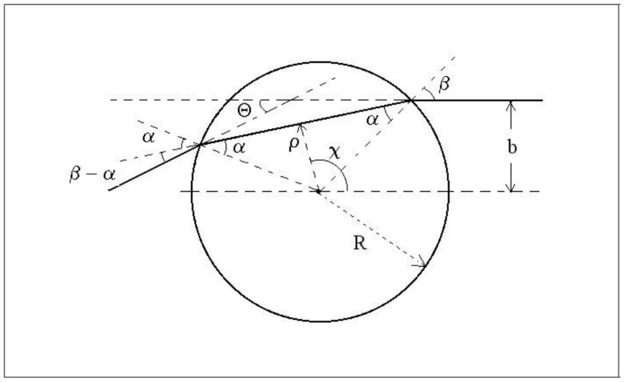
Scattering by a soft sphere:



In[1]:=
```
eq1 = θ == 2 (β - α);
```

In[2]:=
```
(*  Initial  Momentum : p0
     Inside  Momentum : p1
     n:  index  of  refraction :  n= √((E+U0)/E)
        n>1  for  attractive  potential
*)
eq2 = p1 == n p0;
```

In[3]:=
```
(* Angular  Momentum  Conservation :*)
eq3 = b p0  == ρ p1  /. {p1 → n  p0}
```

Out[3]= $b\,p0 == n\,p0\,\rho$

In[4]:=
```
eq3 = eq3  /. p0 → 1
```

Out[4]= $b == n\,\rho$

In[5]:=
```
eq4 = R Sin[α] == ρ
```

Out[5]= $R\,\mathrm{Sin}[\alpha] == \rho$

In[6]:=
```
eq5 =  R Sin[β] == b
```

Out[6]= $R\,\mathrm{Sin}[\beta] == b$

In[7]:=
```
βSol  = Solve[eq1 , β][[1]]
```

Out[7]= $\left\{\beta \to \frac{1}{2}(2\,\alpha + \theta)\right\}$

In[8]:=
```
αSol  = Solve[eq4 , α][[2]]  /. {C[_] → 0} // Normal
```

Out[8]= $\left\{\alpha \to \mathrm{ArcSin}\left[\frac{\rho}{R}\right]\right\}$

In[9]:=
```
ρSol = Solve[eq3, ρ][[1]]
```

Out[9]= $\left\{\rho \to \frac{b}{n}\right\}$

In[10]:=
```
eq5 /. βSol // Simplify
```

Out[10]= $b == R\,\text{Sin}\left[\alpha + \frac{\theta}{2}\right]$

In[11]:=
```
eq5 /. βSol /. αSol // Simplify
```

Out[11]= $b == R\,\text{Sin}\left[\frac{\theta}{2} + \text{ArcSin}\left[\frac{\rho}{R}\right]\right]$

In[12]:=
```
eq6 = eq5 /. βSol /. αSol /. ρSol // Simplify
```

Out[12]= $b == R\,\text{Sin}\left[\frac{\theta}{2} + \text{ArcSin}\left[\frac{b}{n\,R}\right]\right]$

In[14]:=
```
(* Attractive Potential *)
bSol = Solve[eq6, b][[2]] // FullSimplify
```

Out[14]= $\left\{b \to \frac{n\,R\,\text{Sin}\left[\frac{\theta}{2}\right]}{\sqrt{1 + n^2 - 2\,n\,\text{Cos}\left[\frac{\theta}{2}\right]}}\right\}$

In[15]:=
```
db = D[b /. bSol, θ] // FullSimplify
```

Out[15]= $$\frac{n\,R\left(2\left(1 + n^2\right)\text{Cos}\left[\frac{\theta}{2}\right] - n\,(3 + \text{Cos}[\theta])\right)}{4\left(1 + n^2 - 2\,n\,\text{Cos}\left[\frac{\theta}{2}\right]\right)^{3/2}}$$

In[16]:=
```
integrand = db b/Sin[θ] 2 π Sin[θ] /. bSol
```

Out[16]= $$\frac{n^2\,\pi\,R^2\left(2\left(1 + n^2\right)\text{Cos}\left[\frac{\theta}{2}\right] - n\,(3 + \text{Cos}[\theta])\right)\text{Sin}\left[\frac{\theta}{2}\right]}{2\left(1 + n^2 - 2\,n\,\text{Cos}\left[\frac{\theta}{2}\right]\right)^{2}}$$

In[17]:=
```
s[θ_] = Integrate[integrand, θ] // FullSimplify
```

Out[17]= $$\frac{n\,\pi\,R^2\left(1 + n + n^2 - n\left(2\,\text{Cos}\left[\frac{\theta}{2}\right] + \text{Cos}[\theta]\right)\right)}{2\left(1 + n^2 - 2\,n\,\text{Cos}\left[\frac{\theta}{2}\right]\right)}$$

Let' s find the limits for the integration

In[18]:=
```
tmp1 = db /. {R → 1} // Numerator
```

Out[18]= $n\left(2\left(1 + n^2\right)\text{Cos}\left[\frac{\theta}{2}\right] - n\,(3 + \text{Cos}[\theta])\right)$

In[19]:=
```
maxθ = Solve[tmp1 == 0, θ] /. {C[1] → 0}
```

Out[19]=
$$\left\{\left\{\theta \to 2\,\text{ArcTan}\left[\frac{1}{n}, -\frac{\sqrt{-1+n^2}}{n}\right]\right\}, \left\{\theta \to 2\,\text{ArcTan}\left[\frac{1}{n}, \frac{\sqrt{-1+n^2}}{n}\right]\right\}, \left\{\theta \to 2\,\text{ArcTan}\left[n, -\sqrt{1-n^2}\right]\right\}, \left\{\theta \to 2\,\text{ArcTan}\left[n, \sqrt{1-n^2}\right]\right\}\right\}$$

In[20]:=
```
s[θ] - s[0] /. maxθ // FullSimplify
```

Out[20]=
$$\{\pi R^2, \pi R^2, n^2 \pi R^2, n^2 \pi R^2\}$$

Repulsive potential: n → 1 /n

In[21]:=
```
(* Attractive Potential *)
bSol = Solve[eq6, b][[2]] // FullSimplify
```

Out[21]=
$$\left\{b \to \frac{n\,R\,\text{Sin}\left[\frac{\theta}{2}\right]}{\sqrt{1+n^2-2\,n\,\text{Cos}\left[\frac{\theta}{2}\right]}}\right\}$$

In[22]:=
```
(* Repulsive Potential *)
bRepulsive = b^2 /. bSol /. n → 1/n // FullSimplify  // Sqrt // PowerExpand
```

Out[22]=
$$\frac{R\,\text{Sin}\left[\frac{\theta}{2}\right]}{\sqrt{1+n^2-2\,n\,\text{Cos}\left[\frac{\theta}{2}\right]}}$$

In[23]:=
```
dbRep = D[bRepulsive, θ] // FullSimplify
```

Out[23]=
$$-\frac{R\left(-2\left(1+n^2\right)\text{Cos}\left[\frac{\theta}{2}\right]+n\left(3+\text{Cos}[\theta]\right)\right)}{4\left(1+n^2-2\,n\,\text{Cos}\left[\frac{\theta}{2}\right]\right)^{3/2}}$$

In[24]:=
$$\texttt{integrand2 = dbRep}\ \frac{\texttt{bRepulsive}}{\texttt{Sin}[\theta]}\ 2\,\pi\,\texttt{Sin}[\theta]$$

Out[24]=
$$-\frac{\pi R^2\left(-2\left(1+n^2\right)\text{Cos}\left[\frac{\theta}{2}\right]+n\left(3+\text{Cos}[\theta]\right)\right)\text{Sin}\left[\frac{\theta}{2}\right]}{2\left(1+n^2-2\,n\,\text{Cos}\left[\frac{\theta}{2}\right]\right)^2}$$

In[25]:=
```
s2[θ_] = Integrate[integrand2, θ] // FullSimplify
```

Out[25]=
$$\frac{\pi R^2\left(1+n+n^2-n\left(2\,\text{Cos}\left[\frac{\theta}{2}\right]+\text{Cos}[\theta]\right)\right)}{2\,n\left(1+n^2-2\,n\,\text{Cos}\left[\frac{\theta}{2}\right]\right)}$$

Let' s find the limits for the integration

In[26]:=
```
tmp2 = dbRep /. {R → 1} // Numerator
```

Out[26]=

$$2\left(1+n^2\right)\mathrm{Cos}\left[\frac{\theta}{2}\right]-n\,(3+\mathrm{Cos}[\theta])$$

In[27]:=
```
maxθ2 = Solve[tmp2 == 0, θ] /. {C[1] → 0}
```

Out[27]=

$$\left\{\left\{\theta\to 2\,\mathrm{ArcTan}\left[\frac{1}{n},-\frac{\sqrt{-1+n^2}}{n}\right]\right\},\left\{\theta\to 2\,\mathrm{ArcTan}\left[\frac{1}{n},\frac{\sqrt{-1+n^2}}{n}\right]\right\},\left\{\theta\to 2\,\mathrm{ArcTan}\left[n,-\sqrt{1-n^2}\right]\right\},\left\{\theta\to 2\,\mathrm{ArcTan}\left[n,\sqrt{1-n^2}\right]\right\}\right\}$$

In[28]:=
```
s2[θ] - s2[0] /. maxθ // FullSimplify
```

Out[28]=

$$\left\{\frac{\pi R^2}{n^2},\frac{\pi R^2}{n^2},\pi R^2,\pi R^2\right\}$$

In[29]:=
```
Plot[{integrand, integrand2} /. {n → 3/2, R → 1},
 {θ, 0, π/2}, Ticks → {Table[i π, {i, 0, 1, 1/8}], Automatic}]
```

Out[29]=