Physics 6321 Professor Olness
Homework 5

1. The matrix A=

$1/\sqrt{2}$	$1/\sqrt{2}$	0
0	0	1
$1/\sqrt{2}$	$-1/\sqrt{2}$	0

represents a finite rotation about a certain axis. Find the direction cosines of the axis and the angle of rotation.

2. Consider the Foucault Pendulum. Compute the rotation frequency of the pendulum as a function of the co-latitude θ.

3. The matrix L represents a rotation by an angle ϕ around some axis. The eigenvalues of L are $\lambda 1 = +1$, $\lambda 2 = (\sqrt{3}+i)/2$. $\lambda 3 = (\sqrt{3}-i)/2$. Find the angle ϕ.

4. Consider a tensor of third rank whose components t_{ijk} are antisymmetric under exchange of any two indices. Show that a single number is sufficient to characterize this tensor and that this number transforms like a pseudoscalar.

5. We want to demonstrate that the 3x3 and 2x2 representation of rotations are identical for infinitesimal rotations on a vector v={x,y,z}.
The 3x3 rotations can be represented as Exp[$\theta\, n \bullet M$] where n is the axis vector, and M are the matrix generators of the rotation, and the rotated vector is Exp[$\theta\, n \bullet M$]•v.
The 2X2 rotations can be represented as Exp[-I $\theta/2\ n \bullet \sigma$] where n is the axis vector, and σ are the matrix generators of the rotation, and the rotated vector is Exp[-I $\theta/2\ n \bullet \sigma$]•v.
a) Perform an infinitesimal rotation about the x-axis with both representations, and show the components transform identically.
b) Repeat for the y-axis.
c) Repeat for the z-axis.