4) 9.8 Coriolis and Centrifugal Forces

In[ ]:=
```
Clear["Global`*"]
```

a) moving south near north pole

In[ ]:=
```
(* a: moving south near north pole *)
Ω = Ω0 {0, 0, 1};
r = {0, Sin[θ], Cos[θ]};
v = v0 {0, +Cos[θ], -Sin[θ]};
```

In[ ]:=
```
(* Direction : west: +x direction *)
coriolis = 2 m Cross[v, Ω]
```

Out[ ]= $\{2\, m\, v0\, \Omega0\, \mathrm{Cos}[\theta],\ 0,\ 0\}$

In[ ]:=
```
(* Direction : up and south : +y direction *)
centrifugal = m Cross[Cross[Ω, r], Ω]
```

Out[ ]= $\{0,\ m\, \Omega0^2\, \mathrm{Sin}[\theta],\ 0\}$

b) moving east near equator

In[ ]:=
```
(* a: moving south near north pole *)
Ω = Ω0 {0, 0, 1};
r = {0, Sin[θ], Cos[θ]};
v = v0 {-1, 0, 0};
```

In[ ]:=
```
(* Direction : up: +y direction *)
coriolis = 2 m Cross[v, Ω]
```

Out[ ]= $\{0,\ 2\, m\, v0\, \Omega0,\ 0\}$

In[ ]:=
```
(* Direction : up: +y direction *)
centrifugal = m Cross[Cross[Ω, r], Ω]
```

Out[ ]= $\{0,\ m\, \Omega0^2\, \mathrm{Sin}[\theta],\ 0\}$

In[ ]:=
```
centrifugal /. {θ → π / 2}
```

Out[ ]= $\{0,\ m\, \Omega0^2,\ 0\}$

c) moving south near equator

In[ ]:=
```
(* a: moving south near north pole *)
Ω = Ω0 {0, 0, 1};
r = {0, Sin[θ], Cos[θ]};
v = v0 {0, 0, -1};
```

In[ ]:=
```
(* Direction : up: +y direction *)
coriolis = 2 m Cross[v, Ω]
```

Out[ ]= {0, 0, 0}

In[ ]:=
```
(* Direction : up: +y direction *)
centrifugal = m Cross[Cross[Ω, r], Ω]
```

Out[ ]= $\{0,\ m\,\Omega 0^2\,\mathrm{Sin}[\theta],\ 0\}$

In[ ]:=
```
centrifugal /. {θ → π / 2}
```

Out[ ]= $\{0,\ m\,\Omega 0^2,\ 0\}$

5) 9.9 Bullet problem

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
Ω = Ω0 {0, 0, 1};
r = {0, Sin[θ], Cos[θ]};
v = v0 {0, -Cos[θ], Sin[θ]};
```

In[ ]:=
```
(* Direction :east:-x direction *)
coriolis = 2 m Cross[v, Ω]
```

Out[ ]= {-2 m v0 Ω0 Cos[θ], 0, 0}

In[ ]:= values = $\left\{\theta \to 40\ \mathrm{Degree},\ v0 \to 1000,\ \Omega 0 \to \frac{2\pi}{86400},\ g \to 9.8\right\}$;

In[ ]:=
```
tmp1 = coriolis[[1]] /. values
```

Out[ ]= $-\frac{5}{108}\, m\, \pi\, \mathrm{Cos}[40\,°]$

In[ ]:= $\frac{tmp1}{m\, g}$ //. values

Out[ ]= -0.011369

6) 9.13 Plumb line problem

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
Ω = Ω0 {0, 0, 1};
r = re {0, Sin[θ], Cos[θ]};
```

In[ ]:=
```
centrifugal = m Cross[Cross[Ω, r], Ω]
```

Out[ ]= $\{0, m\, re\, \Omega0^2 \operatorname{Sin}[\theta], 0\}$

In[ ]:=
```
(* Take part perpendicular to gravity:Cos[θ] *)
tmp1 = centrifugal [[2]] Cos[θ]
```

Out[ ]= $m\, re\, \Omega0^2 \operatorname{Cos}[\theta] \operatorname{Sin}[\theta]$

In[ ]:= $\tan\alpha = \frac{tmp1}{m\, g}$

Out[ ]= $\frac{re\, \Omega0^2 \operatorname{Cos}[\theta] \operatorname{Sin}[\theta]}{g}$

In[ ]:=
```
tanα // TrigReduce
```

Out[ ]= $\frac{re\, \Omega0^2 \operatorname{Sin}[2\, \theta]}{2\, g}$

Extra) Foucault Pendulum

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
Ωz = Ω Cos[θ]
```

Out[ ]= $\Omega \operatorname{Cos}[\theta]$

In[ ]:=
```
eta = amp Exp[-I Ωz t] Cos[ω0 t]
```

Out[ ]= $amp\, e^{-i\, t\, \Omega \operatorname{Cos}[\theta]} \operatorname{Cos}[t\, \omega0]$

In[ ]:=
```
values = {ω0 → 16 Ω, amp → 1, Ω → 2 π}
```

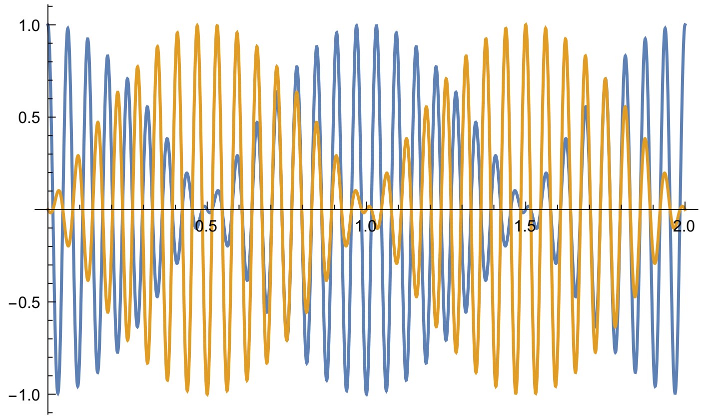
Out[ ]= $\{\omega0 \to 16\, \Omega, amp \to 1, \Omega \to 2\, \pi\}$

In[ ]:=
```
f[t_, θ_] = eta //. values
```

Out[ ]= $e^{-2\, i\, \pi\, t \operatorname{Cos}[\theta]} \operatorname{Cos}[32\, \pi\, t]$

In[ ]:=
```
Plot[{f[t, π / 3] // Re, f[t, π / 3] // Im}, {t, 0, 2}]
```

Out[ ]=



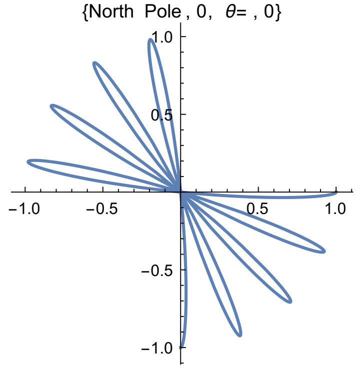
In[ ]:=
```
titles = {"North Pole", "Dallas", "Equator", "South America", "South Pole"};
```

In[ ]:=
```
ff[x_] := ParametricPlot[{f[t, x π/4] // Re, f[t, x π/4] // Im},
  {t, 0, 1 / 4},
  PlotLabel → {titles[[x + 1]], x, " θ= ", x π/4}]
```

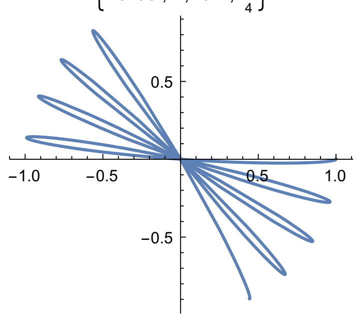
In[ ]:=
```
Table[ff[i], {i, 0, 4, 1}] // TableForm
```

Out[ ]//TableForm=

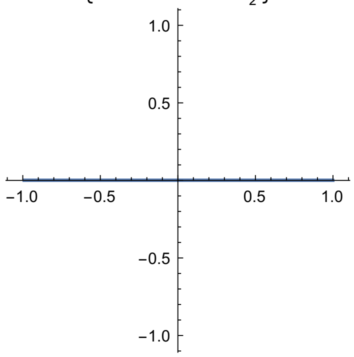
{North Pole, 0, θ= , 0}



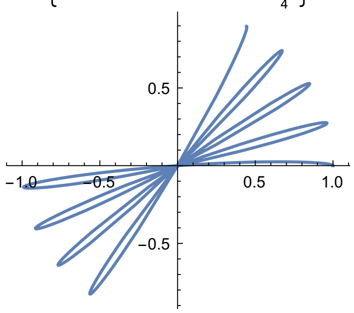
{Dallas, 1, θ= , $\frac{\pi}{4}$}



{Equator , 2, θ= , $\frac{\pi}{2}$}



{South America , 3, θ= , $\frac{3\pi}{4}$}



{South Pole , 4, θ= , π}