Physics 6321 Professor Olness
Homework 6 Friday February 22, 2024

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1) The density of a sphere of unit radius is proportional to $5\,z^2 + x\,y + 1$ with the center of the sphere at the origin. Determine the elements of the moment of inertia tensor in this system. Specify the coordinate system in which the moment of inertia tensor is diagonal and find the elements of the tensor in this system.

2) Three equal mass points are located at (a,0,0), (0, a, 2a) and (0, 2a, a). Find the principal moments of inertia about the origin and a set of principal axes.

3)

10.35 ⋆⋆ A rigid body consists of three masses fastened as follows: m at $(a, 0, 0)$, $2m$ at $(0, a, a)$, and $3m$ at $(0, a, -a)$. **(a)** Find the inertia tensor **I**. **(b)** Find the principal moments and a set of orthogonal principal axes.

4,5)

9.8 ⋆ What are the directions of the centrifugal and Coriolis forces on a person moving **(a)** south near the North Pole, **(b)** east on the equator, and **(c)** south across the equator?

9.9 ⋆ A bullet of mass m is fired with muzzle speed v_o horizontally and due north from a position at colatitude θ. Find the direction and magnitude of the Coriolis force in terms of m, v_o, θ, and the earth's angular velocity Ω. How does the Coriolis force compare with the bullet's weight if $v_o = 1000$ m/s and $\theta = 40$ deg?

6)

9.13 ⋆ Show that the angle α between a plumb line and the direction of the earth's center is well approximated by $\tan\alpha = (R_e\Omega^2 \sin 2\theta)/(2g)$, where g is the observed free-fall acceleration and we assume the earth is perfectly spherically symmetric. Estimate the maximum and minimum values of the magnitude of α.

7)

9.27 ⋆⋆ In Section 9.8, we discussed the path of an object that is dropped from a very tall stepladder above the equator. **(a)** Sketch this path as seen from a tower to the north of the drop and fixed to the earth. Explain why the object lands to the east of its point of release. **(b)** Sketch the same experiment as seen by an inertial observer floating in space to the north of the drop. Explain clearly (from this point of view) why the object lands to the east of its point of release. [*Hint:* The object's angular momentum about the earth's center is conserved. This means that the object's angular velocity $\dot{\phi}$ changes as it falls.]