Special Relativity

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Einstein Postulates:

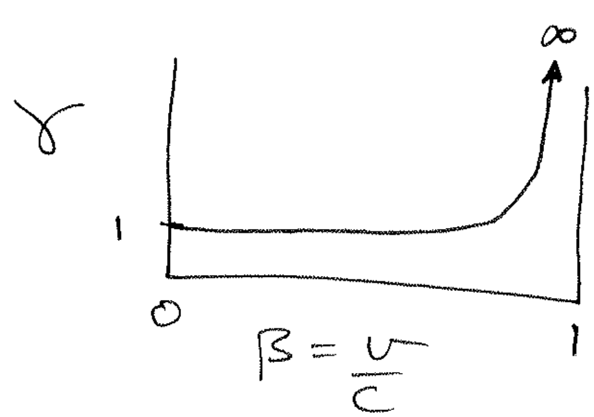
- Speed of light is constant c
- All Frames are equivalent

Consequences:

- Moving Clocks Run SLOW
- Moving Rulers are Short

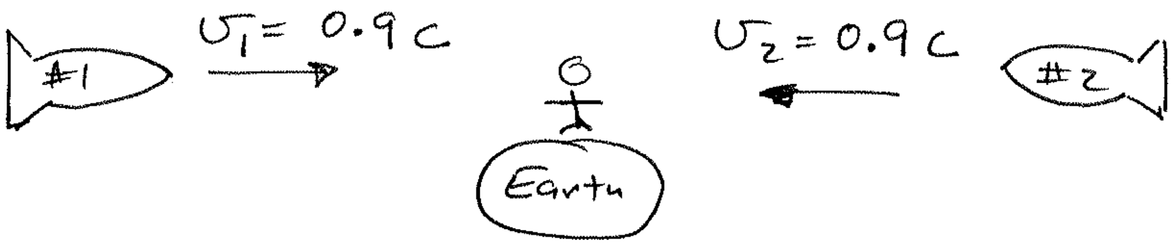
Both by Factor γ

$\beta = \frac{v}{c}$ $\quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$ $\quad \beta = \sqrt{1 - \frac{1}{\gamma^2}}$



Adding Velocities w/ Relativity

"... It's not just a good idea,
it's the law."



Old Fashioned Way: (Galilean Tx)

$$v_{TOT} = v_1 + v_2 = 0.9c + 0.9c$$

$$v_{TOT} = 1.8c$$

Ooooops!!!
Faster than Light

New, Improved Way: (Lorentz Tx)

$$v_{TOT} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{0.9c + 0.9c}{1 + (0.9)(0.9)} = \frac{1.80}{1.81} c$$

$$v_{TOT} = \frac{1.80}{1.81} c = 0.995c$$

OK!!!
Not faster than light!!!

Atmospheric Muons

Q: How can muons (μ) created in upper atmosphere reach surface of earth?

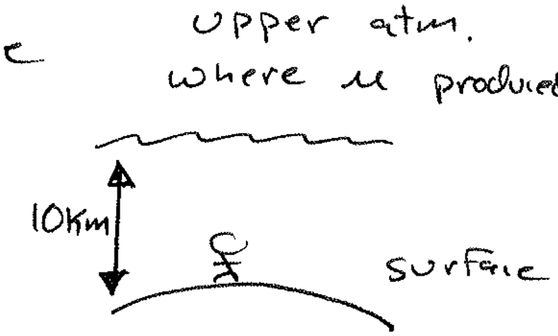
Lifetime $\tau = 2\mu s = 2\times10^{-6}\,s$

Speed $v \cong c = 3\times10^{8}\,m/s$

distance $x = vt \cong ct = (3\times10^{8}\,\frac{m}{s})(2\times10^{-6}\,s)$

$x \cong 600\,m = 0.6\,Km$

Suppose: height of atmosphere is about $\sim 10\,Km$ (I'm making up simple numbers.)



Then: muons won't reach surface without relativity.

Suppose $E \cong 2000\,MeV = 2\,GeV$

$mc^2 = 105\,MeV$

$$\gamma = \frac{E}{mc^2} \cong \frac{2000}{105} \cong 20$$

Q: Will the muon make it with relativity?

Part #1 In the Earth's Reference Frame:

The muon's clock runs slow by γ.

- with <u>NO</u> relativity, muon can travel 0.6 Km
- with relativity, muon can travel

$$\text{distance} = \gamma \cdot (0.6\,\text{Km}) = (20)(0.6\,\text{Km})$$

$$= 12\,\text{Km}$$

Conclusion: the muon <u>will</u> make it!!!

Part #2 In the muon's reference frame:

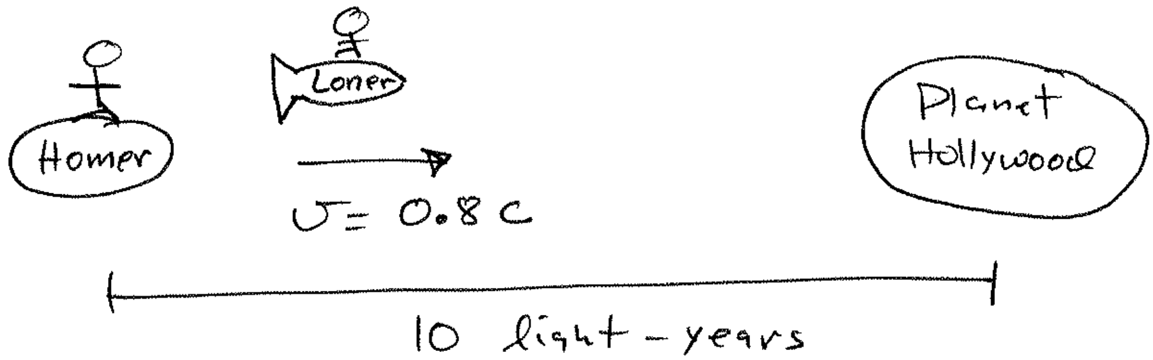
The thickness of the atmosphere is contracted by γ.

- with <u>NO</u> relativity, muon can only travel 0.6 Km through 10 Km of atmosphere.
- with relativity, thickness of atmosphere is not 10 Km but $\frac{10\,\text{Km}}{\gamma} = \frac{10\,\text{Km}}{20} = \frac{1}{2}\,\text{Km}$ $= 0.5\,\text{Km}$. Since muon can go 0.6 Km the muon <u>will</u> make it!!!

Punch-line For theory to be consistent, BOTH answers must agree!!!

The Twin Paradox:

Homer and Loner are twins.



$$\beta = \frac{v}{c} = 0.8 = \frac{4}{5}$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{5}{3}$$

Q: How long will it take Loner to make a round trip?

Part #1 Compute from Earth's reference frame.

$$x = vt \qquad t = \frac{x}{v} = \frac{x/c}{v/c} = \frac{20 \text{ light-years}}{4/5} = 25$$

$t = 25$ years for round trip

Part #2 Compute in Loner's frame.

Note: distance is short by γ

$$\therefore \quad \frac{20 \text{ light-years}}{\gamma} = \frac{20}{5/3} = 12 \text{ light-years}$$

$$x = vt \qquad t = \frac{x}{v} = \frac{x/c}{v/c} = \frac{12}{4/5} = 15 \text{ years}$$

Conclusion: Homer ages 25 years
Loner ages 15 years

Cross-Check In Earth frame, work out time that passes on Loner's clock.

Observe: Loner's clock runs slow by γ.

Loner's trip takes 25 years

But Loner's clock ticks off $\frac{25}{\gamma}$ years

$$\frac{25}{\gamma} = \frac{25}{5/3} = 15 \text{ years}$$

This is consistent !!!