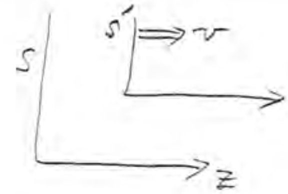
Ch 7

Lorentz Transformation Derived



Assume a linear relation:

$$z' = a_1 z + a_2 t$$
$$t' = a_3 t + a_4 z$$

Physical Laws Maintain their form in all inertial frames.
"Covariance"

The linear relation ensures Newton's First law is covariant.

$$\left.\begin{aligned} dz' &= a_1\,dz + a_2\,dt \\ dt' &= a_3\,dt + a_4\,dz \end{aligned}\right\} \quad \frac{dz'}{dt'} = \frac{a_1 \frac{dz}{dt} + a_2}{a_3 + a_4 \frac{dz}{dt}}$$

If $\frac{dz}{dt} = \text{const}$, then $\frac{dz'}{dt'} = \text{const}'$

Consider a point P fixed in S $\Rightarrow \Delta z = 0$

$$\Delta z' = a_2 \Delta t$$
$$\Delta t' = a_3 \Delta t$$

$$\frac{\Delta z'}{\Delta t'} = \frac{a_2}{a_3} = -v \qquad \boxed{a_2 = -v a_3}$$

Consider a point P' fixed in S' $\Rightarrow \Delta z' = 0$

$$a_1 \Delta z + a_2 \Delta t = 0$$

$$\frac{\Delta z}{\Delta t} = -\frac{a_2}{a_1} = +v \qquad \boxed{a_2 = -v a_1}$$

$$\Rightarrow a_1 = a_3$$

We want the velocity of light to be the same in all frames
(Michelson Morley)

$$\frac{\Delta z'}{\Delta t'} = c \Longleftrightarrow \frac{\Delta z}{\Delta t} = c$$

We want $a_1(v)$ to be an <u>even</u> function of v

$$a_1(+v) = a_1(-v)$$

$$\Rightarrow \left[a_1(v)\right]^2 = \frac{1}{1-\frac{v^2}{c^2}} \quad \Rightarrow \quad a_1(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \equiv \gamma$$

Why?

① We want to preserve time-reversal invariance:

$$\begin{aligned} z &\to +z \\ t &\to -t \\ v &\to -v \\ t' &\to -t' \end{aligned} \qquad a_1(v) \longrightarrow +a_1(-v)$$

② We want the <u>group</u> property:

$$S \xrightarrow{a_1(v)} S' \xrightarrow{a_1(-v)} \ ? \qquad S$$

$$\begin{tikzcd} S \arrow[r, "a_1(v_1)"'] \arrow[rr, bend right, "a_1(v_3)"'] & S' \arrow[r, "a_1(v_2)"'] & S'' \end{tikzcd}$$

$$\begin{tikzcd} S \arrow[r, "a_1(v_1)"'] & S' \arrow[r, "a_1(v_2)"'] & S'' \arrow[ll, bend left, "a_1(-v_3)"] \end{tikzcd}$$

Two Lorentz transformations in succession give another Lorentz transformation.

$$a_1\left(\frac{v_1+v_2}{1+\frac{v_1 v_2}{c^2}}\right) = a_1(v_1)\, a_1(v_2)\left[1+\frac{v_1 v_2}{c^2}\right]$$

$$\frac{d}{dv_2}\, a_1\left(\frac{v_1+v_2}{1+\frac{v_1 v_2}{c^2}}\right) = a_1(v_1)\left[\frac{d}{dv_2}\, a_1(v_2)\left[1+\frac{v_1 v_2}{c^2}\right] + a_1(v_2)\frac{v_1}{c^2}\right]$$

$$a_1'\left(\frac{v_1+v_2}{1+\frac{v_1 v_2}{c^2}}\right)\left[\frac{1}{1+\frac{v_1 v_2}{c^2}} - \frac{(v_1+v_2)\frac{v_1}{c^2}}{\left(1+\frac{v_1 v_2}{c^2}\right)^2}\right] = a_1(v_1)\left[a_1'(v_2)\left(1+\frac{v_1 v_2}{c^2}\right) - a_1(v_2)\frac{v_1}{c^2}\right.$$

$\underline{v_2 = 0}$

$$a_1'(v_1)\left[1-\frac{v_1^2}{c^2}\right] = a_1(v_1)\left[a_1'(0) + a_1(0)\frac{v_1}{c^2}\right]$$

$a_1(0) = 1 \Rightarrow z = z'$ if no relative velocity

$$a_1'(v_1)\left(1-\frac{v_1^2}{c^2}\right) - a_1(v_1)\left[a_1'(0) + \frac{v_1}{c^2}\right] = 0$$

differential equation. Solution:

If $a_1'(0) = 0$ then $a_1(v_1) = \dfrac{1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \gamma$

$$a_1(v+\delta v) = a_1(v)\, a_1(\delta v)\left[1+\frac{v\,\delta v}{c^2}\right]$$

$$\frac{da_1(v)}{dv} = \lim_{\delta v\to 0}\frac{a_1(v+\delta v)-a_1(v)}{\delta v} = \lim_{\delta v\to 0}\frac{a_1(v)\left\{a_1(\delta v)\left(1+\frac{v\,\delta v}{c^2}\right)-1\right\}}{\delta v}$$

$$= a_1(v)\frac{v}{c^2} \Rightarrow \left.\frac{da_1(v)}{dv}\right|_{v=0} = 0 = a_1'(0)$$

Verify:

Rotation leaves $L^2 = x^2+y^2+z^2$ inv.

$$\begin{pmatrix} c & s \\ -s & c \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cx+sy \\ -sx+cy \end{pmatrix} \qquad L \rightarrow x^2+y^2$$

Metric maps 2 v → scalar $\delta\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$

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time invariant $x^2+y^2+z^2 = c^2t^2$

or ⇒ $-x^2+c^2t^2 = 0$ inv.

Look for something like rotation

$$\begin{pmatrix} t \\ x \end{pmatrix} \quad \text{metric} \longrightarrow \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$\begin{pmatrix} c & s \\ s & c \end{pmatrix}\begin{pmatrix} t \\ x \end{pmatrix} \rightarrow \begin{pmatrix} ct+sx \\ st+cx \end{pmatrix} \xrightarrow{g} (ct+sx)^2 - (st+cx)^2$$

$$t^2(c^2-s^2) - x^2(c^2-s^2)$$

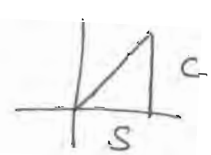
$$= t^2-x^2$$

Pure x: $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Tx: $\begin{pmatrix} c & s \\ s & c \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} s \\ c \end{pmatrix} =$

Do first: part at rest $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ disp. of part @ rest

$$\begin{pmatrix} c & s \\ s & c \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} c \\ s \end{pmatrix} \begin{matrix} t \\ x \end{matrix} \longrightarrow$$



$$\frac{v}{c} = \beta = \frac{\Delta x}{\Delta t} = \frac{s}{c} \Rightarrow s = \beta c$$

$s = \beta c$

$$c^2 - s^2 = 1$$

$$c^2 - c^2\beta^2 = 1$$

$$c^2 = \frac{1}{1-\beta^2} \qquad \cosh = \frac{1}{\sqrt{1-\beta^2}} = \gamma$$

$$\begin{pmatrix} t \\ x \end{pmatrix} \rightarrow \begin{matrix} \gamma t + \gamma\beta x \\ \gamma x + \gamma\beta t \end{matrix} \qquad \sinh = \beta\gamma$$

(2

$$\Lambda = L = \begin{pmatrix} c & s \\ s & c \end{pmatrix} \qquad L = L^{+}$$

$$\tilde{\Lambda} = L^{-1} = \begin{pmatrix} c & -s \\ -s & c \end{pmatrix}$$

$$ds = \vec{x}^2 = \vec{x} \circ \vec{x} = x^{\mu} g_{\mu\nu} x^{\nu} = \widetilde{\begin{pmatrix} x \\ t \end{pmatrix}} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$ds = x^2 = x'^2 = x' G x'$$

$$= \tilde{x} \tilde{\Lambda} G \Lambda x = \tilde{x} G \tilde{x}$$

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used to be $\underbrace{R^{+}}_{\text{orthog}} \mathbb{1} R = \mathbb{1} \Rightarrow R^{+} R$ orthog

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$$x \circ y = x^{\mu} y^{\nu} g_{\mu\nu}$$

$$x^{\mu} = (t, +\vec{x})$$

$$x_{\mu} = (t, -\vec{x})$$

Other 4-Vectors

$p^\mu = (E, \vec{P})$

$$\begin{pmatrix} c & s \\ s & c \end{pmatrix}\begin{pmatrix} E \\ P \end{pmatrix} = \begin{pmatrix} cE + sP \\ sE + cP \end{pmatrix} = \begin{pmatrix} \gamma E + \gamma\beta P \\ \gamma\beta E + \gamma P \end{pmatrix}$$

$A^\mu = (\phi, \vec{A})$

Lorentz gauge condition

$$\frac{\partial A^\mu}{\partial x^\mu} = \nabla_\mu \cdot A^\mu = \frac{\partial \phi}{\partial x_0} + \frac{\partial \vec{A}}{\partial \vec{x}} = \frac{\partial \phi}{\partial t} + \frac{\partial \vec{A}}{\partial \vec{x}} = 0$$

$\hookrightarrow \nabla \cdot A$

$$\Box^2 A = \frac{\partial^2}{\partial x\, \partial x} A = \left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) A^\mu = -\frac{4\pi}{c} j^\mu$$

where $j^\mu = (\rho, \vec{j})$

TX E and B ?

$F_{\mu\nu}$

$$P_1 \longrightarrow \quad \longleftarrow P_2$$

$$P_1 = (E_1\ 0\ 0\ p_1) \qquad \xrightarrow{CMS} \qquad (E_1\ 0\ 0\ p)$$

$$P_2 = (E_2\ 0\ 0\ p_1) \qquad \qquad \qquad (E_2\ 0\ 0\ {-p})$$

$$P_1^2 = m_1^2 = E_1^2 - p^2 \Rightarrow \quad E_1^2 = \sqrt{m_1^2+p^2}$$

$$E_2^2 = \sqrt{m_2^2+p^2}$$

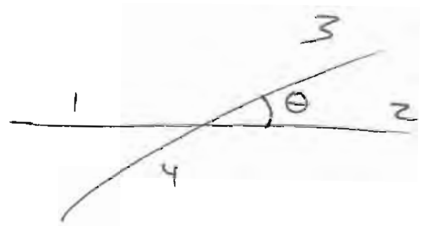
$$s = P_{TOT}^2 = (P_1+P_2)^2 = (E_1+E_2)^2 =$$

$$P_1 = \frac{1}{2\sqrt{s}}\left(s + m_1^2 - m_2^2,\ 0, 0,\ \underline{\Delta(s, m_1^2, m_2^2)}\right) = (E_1\ 0\,0\ \beta_1)$$

$$P_2 = \frac{1}{2\sqrt{s}}\left(s - m_1^2 + m_2^2,\ 0, 0, -\Delta(s, m_1^2\, m_2^2)\right) = (E_2\ 0\,0\ -\beta_1)$$

$$P_3 = \cancel{E_3\, 0\, 0\, \beta_2} \quad (E_3\ \ s_\theta \beta_2\ \ 0\ \ c_\theta \beta_2)$$

$$P_4 \quad (E_4\ -s_\theta\, \beta_2\ \ 0\ \ -c_\theta\, \beta_2)$$



$$P_3 = (E_3,\ s_\theta\, \beta,\ 0,\ c_\theta\, \beta)$$

Boost:
$$\begin{pmatrix} ch & sh \\ sh & ch \end{pmatrix}\begin{pmatrix} E_3 \\ s\beta \\ 0 \\ c\beta \end{pmatrix} = \begin{matrix} ch E_3 + sh\,(c\beta) \\ s\beta \\ 0 \\ sh\, E_3 + ch\,(c\beta) \end{matrix}$$

$$\operatorname{Tan}\theta' = \frac{s\,\beta}{sh\ E_3 + ch\,(\cos\ \beta)}$$

Boost to rest frame of P_1

$$\begin{pmatrix} ch & sh \\ sh & ch \end{pmatrix}\begin{pmatrix} E_1 \\ 0 \\ 0 \\ \Delta_1 \end{pmatrix} = \begin{pmatrix} ch\, E_1 + sh\,\Delta \\ \\ sh\, E_1 + ch\,\Delta \end{pmatrix} = \begin{pmatrix} m_1 \\ \\ 0 \end{pmatrix}$$

$$sh\ E_1 + ch\ \Delta = 0$$

$$\frac{sh}{ch} = -\frac{\Delta}{E_1}$$

rapidity :

$$y = \frac{1}{2} \ln\left(\frac{E + P_3}{E - P_3}\right)$$

$$P^\mu = (E,\ P_\perp,\ P_3) \qquad P^2 = E^2 - P_\perp^2 - P_3^2$$

$$\begin{pmatrix} c & s \\ s & c \end{pmatrix} \begin{pmatrix} E \\ P_\perp \\ P_3 \end{pmatrix} \longrightarrow \begin{pmatrix} cE + sP_3 \\ P_\perp \\ sE + cP_3 \end{pmatrix}$$

$$\downarrow$$

$$y = \ln\left(\frac{E + P_3}{E - P_3}\right) \qquad \ln\left(\frac{cE + sP_3 + sE + cP_3}{cE + sP_3 - sE - cP_3}\right)$$

$$= \ln\left(\frac{(E + P_3)(c + s)}{E(c-s) + P_3(s-c)}\right)$$

$$(E - P)(c - s)$$

$$y = y^{0} + \ln\left(\frac{c+s}{c-s}\right)$$

For Massless particles

$$P_1 = (P\ 0\ 0\ +P)$$
$$P_2 = (P\ 0\ 0\ -P)$$
$$P_1 + P_2 = P_3 + P_4$$
$$P_3 = P(1, S\ 0\ C)$$
$$P_4 = P(1, -S, 0, -C)$$

$$S = (P_1+P_2)^2 = (P_3+P_4)^2 = m_1^2 + m_2^2 + 2P_1 \cdot P_2$$
$$t = (P_1 - P_3)^2 = (P_4 - P_2)^2 = m_1^2 + m_3^2 - 2P_1 \cdot P_3$$
$$u = (P_1 - P_4)^2 = (P_3 - P_2)^2 = m_1^2 + m_4^2 - P_1 \cdot P_4$$

$m = 0$

$$2P_1 \cdot P_2 = 4P^2 = \sqrt{S = 4E^2}$$

$$-t = +2P_1 \cdot P_3 = \cancel{E(1-C)} \quad 2E^2(1-c)$$
$$-u = +2P_1 \cdot P_4 = 2E^2(1+c)$$

Verify $\quad s + t + u = \sum m_i^2$

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$$\eta = \frac{1}{2} \ln \left(\frac{P_{\parallel} + P_{\perp}}{P_{\parallel} - P_{\perp}} \right)$$

$$P_1 = (P\;0\;0\;P) \longrightarrow \left(\sqrt{2}P, \vec{0}, 0\right) \qquad x_\pm = \frac{x_0 \pm x_3}{\sqrt{2}}$$

$$P_2 = (P\;0\;0\;-P) \longrightarrow \left(0, \vec{0}, \sqrt{2}P\right) \qquad x_{0,3} = \frac{x_+ \pm x_-}{\sqrt{2}}$$

$$P_1 \cdot P_2 = 2P^2 \qquad (\sqrt{2}P)^2$$

$$= x_0^2 - \vec{x}^2 \qquad = x_+ y_- + x_- y_+ - x_T \cdot y_T$$

$$g = \begin{pmatrix} 0 & & & 1 \\ & -1 & & \\ & & -1 & \\ 1 & & & 0 \end{pmatrix}$$

$$P_1 = (P_+ \;\; \vec{0} \;\; 0)$$

$$P_2 = (0 \;\; \vec{0} \;\; P_-)$$

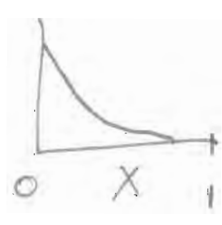
Parton model

Proton

$$= \begin{array}{lll} \circ & x_1 & = (x_1 P_+ \; \vec{0}, \vec{0}) = K_1 \\ \circ & x_2 & (x_2 P_+ \; \vec{0} \; 0) = K_2 \\ \circ & x_3 & \vdots \\ \circ & x_4 & \end{array}$$

$$K_1 + K_2 + \dots = P_1 \Rightarrow x_1 + x_2 + \dots = 1$$

$$F(x) = F$$



$$\int_0^1 dx \; x\, F(x) = 1$$

Masses $\quad P_1 = \left(P_+, 0, \frac{M^2}{2P_+}\right) \qquad P_1^2 = M^2$

Boost

$$(P_0\, 0\, 0\, P_3) \rightarrow (\mathrm{ch}\, P_0 + \mathrm{sh}\, P_3\ ,\ 0, 0,\ \mathrm{sh}\, P_0 + \mathrm{ch}\, P_3)$$

$$(P_+\, \vec{0}\, P_-) \rightarrow \left(\frac{(P_0+P_3)}{\sqrt{2}}(\mathrm{ch}+\mathrm{sh}),\ 0, 0,\ \frac{(P_0-P_3)}{\sqrt{2}}(\mathrm{ch}-\mathrm{sh})\right)$$

$$= \left(P_+ \underbrace{(\mathrm{ch}+\mathrm{sh})}_{e^{X}},\ 0, 0,\ P_- \underbrace{(\mathrm{ch}-\mathrm{sh})}_{e^{-X}}\right)$$

$$\begin{matrix}\mathrm{ch} \\ \mathrm{sh}\end{matrix} = \frac{e^{+X} \pm e^{-X}}{2}$$

Recall:

$$Y = \frac{1}{2}\ln\left(\frac{P_+}{P_-}\right) = \frac{1}{2}\ln\left(\frac{e^{X}}{e^{-X}}\right) = \frac{1}{2}\cdot 2X = X \Rightarrow X = Y$$

$\therefore$

$\therefore$

$$(P_+\, \vec{0}\, P_-) \rightarrow (P_+ e^{Y}, 0, P_- e^{-Y})$$

Lag. form of Rel Mechanics :

$L \neq T - V$ since $P \neq mv$

Still can use $\delta I = 0$

Trial + Error:

$$L = -mc^2\sqrt{1-\beta^2} - V(x) \neq T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial v} = \frac{mv}{\sqrt{1-\beta^2}} = mv\gamma = P$$

canonical mom. def still same

so $\dot{P} = -\frac{\partial V}{\partial x} = F_i$

$\Rightarrow$ cyclic coord $\Rightarrow \dot{P} = 0$

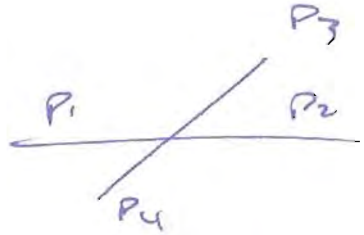
v dep pot $V(v)$

$$L = -mc^2\sqrt{1-\beta^2} - q\phi + \frac{q}{c}\vec{A}\cdot\vec{v}$$

$$\vec{P} = \frac{\partial L}{\partial v} = m\vec{v}\gamma + \frac{q}{c}\vec{A}$$

looks like
"minimal coupling

Kinematics



$$p_1 = (p\ 0\ 0\ p) \qquad p_3 = p(1\ s\ 0\ c)$$

$$p_2 = (p\ 0\ 0\ {-p}) \qquad p_4 = p(1\ {-s}\ 0\ {-c})$$

$$s = (p_1+p_2)^2 = 4p^2 \qquad p = \frac{\sqrt{s}}{2}$$

$$t = (p_1-p_3)^2 = -2p^2(1-c) \equiv -\frac{s}{2}(1-c)$$

$$u = (p_1-p_4)^2 = -2p^2(1+c) \equiv -\frac{s}{2}(1+c)$$

$$s+t+u = 0 = s - \frac{s}{2}(1+c) - \frac{s}{2}(1-c) = 0$$

HERA

$p \sim 820$ $\longrightarrow$ $\longleftarrow$ $e \sim 30$

$P = (P\ 0\ 0\ P)$ $\qquad e = (e\ 0\ 0\ {-e})$ $\qquad P_{TOT} = (P+e,\ 0\ 0\ P-e)$

$$S = (P+e)^2 = (P+e)^2 - (P-e)^2$$

$$= 2Pe + 2Pe = 4Pe$$

$$S = 4\,(820)(30) = 98400 \ \text{GeV}^2$$

$$\sqrt{s} = 314 \ \text{GeV}$$

$P_1 = (E\ 0\ 0\ p)$ $\qquad P_1^2 = E^2 - p^2 = m^2$

$E = 820$ GeV $\qquad E^2 - m^2 = p^2$

$m = 1$ GeV

$$\eta = -\ln\left[\mathrm{Tan}\left(\frac{\theta}{2}\right)\right] \qquad m = 0$$

$$Y = \frac{1}{2}\ln\left(\frac{E + P_z}{E - P_z}\right) \qquad m \neq 0, \text{ or } m = 0$$

For $m = 0$, $\eta = Y$

3A

$$B \quad v \quad = v' \qquad\qquad v'$$

$$\gamma \begin{pmatrix} 1 & \beta \\ \beta & 1 \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix} = \gamma \begin{pmatrix} E + \beta P \\ \beta E + P \end{pmatrix}$$

$$(E+P)(1+\beta)$$

$$E(1+\beta) + P(1+\beta)$$

$$Y = -\frac{1}{2} \ln \left(\frac{E+P}{E-P} \right) = \frac{1}{2} \ln \left(\frac{\cancel{\gamma}[E + \beta P + \beta E + P]}{\cancel{\gamma}[E + \beta P - \beta E - P]} \right)$$

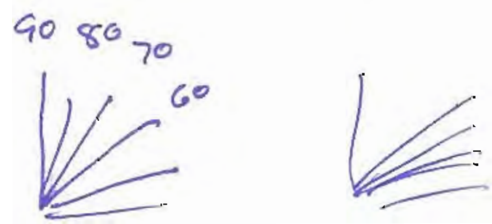
$$E(1-\beta) - P(1-\beta)$$

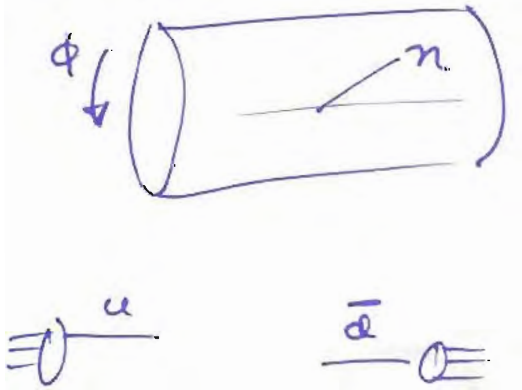
$$(E-P)(1-\beta)$$

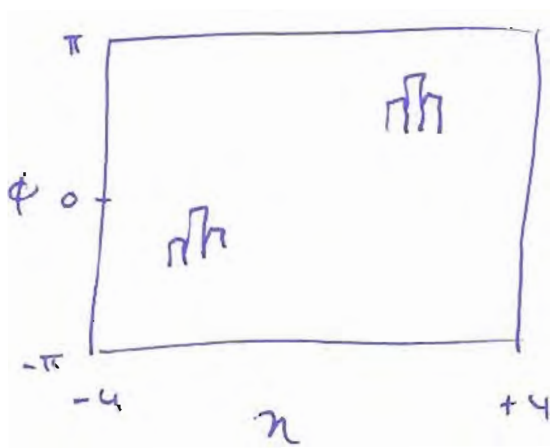
$$= \frac{1}{2} \ln \left[\frac{(E+P)(1+\beta)}{(E-P)(1-\beta)} \right]$$

$$= Y + \frac{1}{2} \ln \left[\frac{1+\beta}{1-\beta} \right]$$

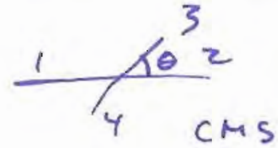
$$= Y + \text{cons}(\beta)$$

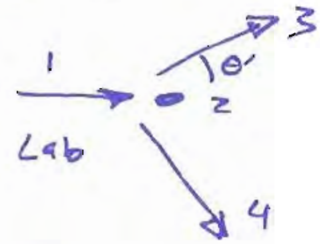






Boost CMS $\Longleftrightarrow$ Lab





$$\underset{\text{CMS}}{P_2} = (E_2\ 0\ 0\ -p)$$

$$P_2' = (m_2\ 0\ 0\ 0)$$

$$B\, P_2 = P_2'$$

$$\gamma \begin{pmatrix} 1 & \beta \\ \beta & 1 \end{pmatrix} \begin{pmatrix} E \\ -p \end{pmatrix} = \begin{pmatrix} m \\ 0 \end{pmatrix}$$

$$\gamma \begin{pmatrix} E - \beta p \\ \beta E - p \end{pmatrix} = \begin{pmatrix} m \\ 0 \end{pmatrix} \Longrightarrow \beta E = p$$

$$\beta = \frac{p}{E} = \frac{p}{E_2} = \frac{\dfrac{\Delta(s\ m_1^2\ m_2^2)}{2\sqrt{s}}}{\dfrac{s + m_2^2 - m_1^2}{2\sqrt{s}}}$$

$$\beta = \frac{\Delta(s, m_1^2, m_2^2)}{s + m_2^2 - m_1^2} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$P_3 \equiv (E_3,\ p\sin\theta,\ 0,\ p\cos\theta)$$

$$\tan\theta = \frac{P_{3x}}{P_{3z}}$$

$$B P_3 = P_3' \qquad \tan\theta' = \frac{P_{3x}'}{P_{3z}'}$$

$$\gamma \begin{pmatrix} 1 & \beta \\ \beta & 1 \end{pmatrix} \begin{pmatrix} E \\ P_z \end{pmatrix} = \gamma \begin{pmatrix} E + \beta P_z \\ \beta E + P_z \end{pmatrix} \Rightarrow P_3' = \begin{array}{cl} \begin{pmatrix} \gamma(E+\beta P_z) \\ p\sin\theta \\ 0 \\ \gamma(\beta E + P_z) \end{pmatrix} & \begin{array}{l} t \\ x \\ y \\ z \end{array} \end{array}$$

$$\tan\theta' = \frac{P_{3x}'}{P_{3z}'} = \frac{p\sin\theta}{\gamma(\beta E_3 + P)}$$

θ is CMS θ

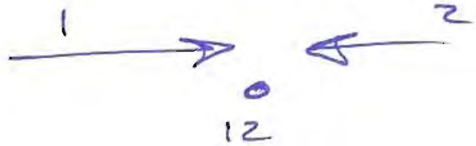
θ' is Lab θ

$$P = \frac{\Delta(s, m_3^2, m_4^2)}{2\sqrt{s}}$$

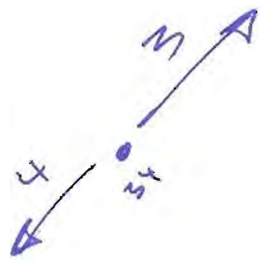
$$E_3 = \frac{s + m_3^2 - m_4^2}{2\sqrt{s}}$$

$$\beta = \frac{\Delta(s, m_1^2, m_2^2)}{s + m_1^2 - m_2^2}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$



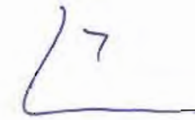
$$\Delta(s, m_1^2, m_2^2)$$

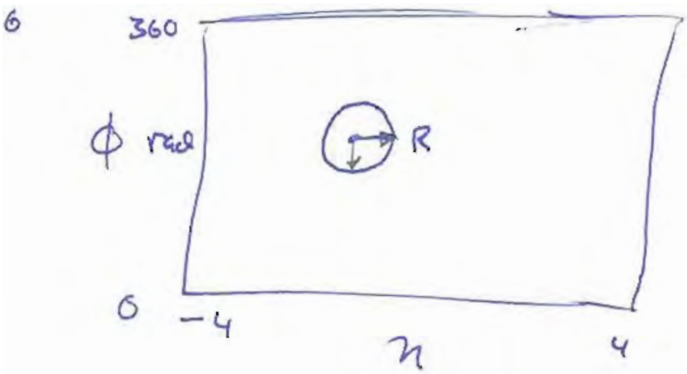


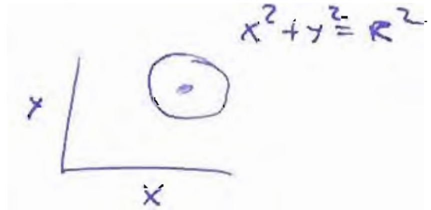
$$\Delta(s, m_3^2, m_4^2)$$

πr^2

1.5

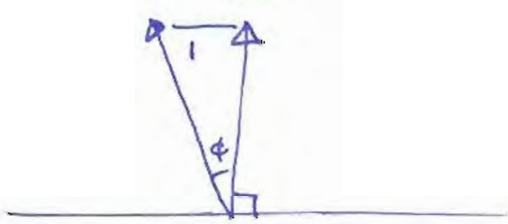






$$\phi^2 + n^2 = R^2$$

$n=0,\ \phi=0$



$\tan\phi = \frac{1}{100}$

$\phi \sim \frac{1}{100}$

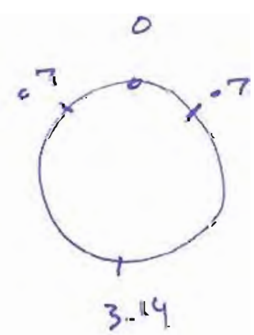
$P = E\ x\ y\ z$

$= (P\ P\ 0\ 0) = P(1\ 1\ 0\ 0)$

New $= \cancel{P(E\ 1\ \tfrac{1}{100}\ 1)}$

$= P(1\ 1\ 0\ \tfrac{1}{100})$

$$y = \frac{1}{2}\ln\left[\frac{E+P}{E-P}\right] = \frac{1}{2}\ln\left[\frac{1+\varepsilon}{1-\varepsilon}\right] = \varepsilon = \frac{1}{100}$$



$$\frac{0.7}{1.0} = R^2 = \phi^2 + n^2$$

0.4

2