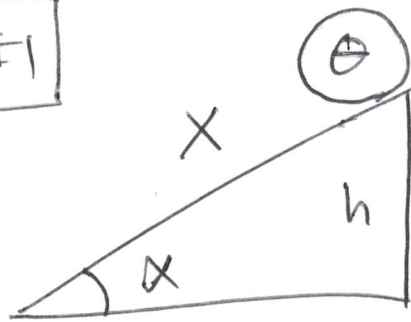


Method #1



Use $r\theta = x$

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \omega^2$$
$$\equiv \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \gamma m r^2 \dot{\theta}^2$$

$$L = T - V$$

$$V = mgh = mg \sin \alpha x$$

$$L = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \gamma m \dot{x}^2 - mg \sin \alpha x$$

$$\Rightarrow m \ddot{x} (1 + \gamma) - mg \sin \alpha = 0$$

$$\Rightarrow \ddot{x} = - \frac{g \sin \alpha}{(1 + \gamma)}$$

Lagrange Multiplier

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \lambda \frac{\partial F}{\partial q_i}$$

F is Constraint Equation ..

IF $F = x_1 - x_2 + L = 0$

$$\frac{\partial F}{\partial x_1} = (+1) \quad \frac{\partial F}{\partial x_2} = (-1)$$

Method #2

USE

$$r\dot{\theta} - \dot{x} = 0$$

$$L = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \gamma m r^2 \dot{\theta}^2 - mg \sin \alpha x$$

$$\frac{\partial L}{\partial \dot{x}} = m \dot{x}$$

$$\frac{\partial L}{\partial \dot{\theta}} = \gamma m r^2 \dot{\theta}$$

$$\frac{\partial L}{\partial x} = -mg \sin \alpha$$

USE: $(r) d\theta + (-1) dx = 0$

$$\frac{\partial L}{\partial r} = 0$$

$$\boxed{x} \quad m \ddot{x} + mg \sin \alpha = (-1) \lambda \quad (1)$$

$$\boxed{r} \quad \gamma m r^2 \ddot{\theta} + 0 = (r) \lambda \quad (2)$$

Also $r \ddot{\theta} = \ddot{x} \quad (3)$

$$(2) + (3) \Rightarrow r \lambda = \gamma m r^2 \ddot{\theta} = \gamma m r \ddot{x}$$

$$\Rightarrow \lambda = \gamma m \ddot{x}$$

$$(1) \Rightarrow m \ddot{x} + mg \sin \alpha = -\lambda \Rightarrow -\gamma m \ddot{x}$$

$$m(1+\gamma) \ddot{x} = -mg \sin \alpha$$

$$\ddot{x} = \frac{-g \sin \alpha}{(1+\gamma)}$$