Ch 14 Scattering

Example #1 p. 562

$$N_{sc} = N_{inc}\, n_{tar}\, \sigma$$

$$= (60)\left(\frac{50}{150\ ft^2}\right)\left(\frac{1}{2}\ ft^2\right) = 10$$

incident bullets — crows / Area — Cross section (area)

Example 14.2

$$N_{sc} = N_{inc}\, n_{tar}\, \sigma$$

$N_{inc} \to 10^4$, $\sigma \to 1.5 \times 10^{-28}\ m^2$

$V = At$

$n_{tar} = \frac{\#}{Area}$

$$\rho = \frac{M}{V} = \frac{M}{At}$$

m = one Al nucleus = 27 AMU $= 27 \times (1.66 \times 10^{-27}\ kg)$

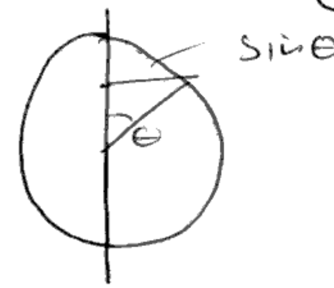
$$n_{tar} = \frac{\rho t}{m} = \frac{M}{m}\frac{t}{At} = \frac{M}{mA} = \frac{\#}{Area}$$

$$n_{tar} = \frac{(2.7 \times 10^{3}\ \frac{kg}{m^3})(10^{-4}\ m)}{[27 \times 1.66 \times 10^{-27}\ kg]} = 6 \times 10^{24}\ m^{2}$$

$\Longrightarrow N_{sc} = 9$

Solid Angle

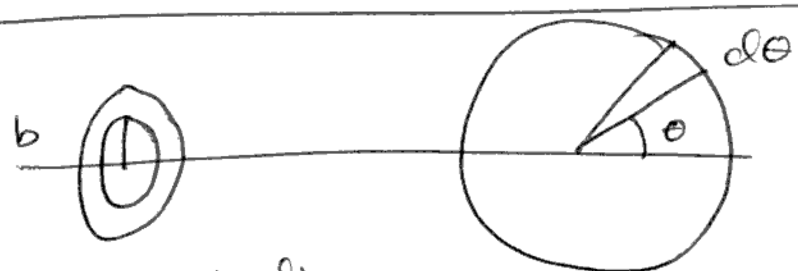
$$\Delta\Omega = \frac{A}{r^2} \longrightarrow \sin\theta \, d\theta \, d\phi$$



$$N_{sc}(\text{in } d\Omega) = N_{inc} \, n_{tar} \frac{d\sigma}{d\Omega} d\Omega$$

$$\int d\Omega = \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi = 2 \times 2\pi = 4\pi$$

14.5

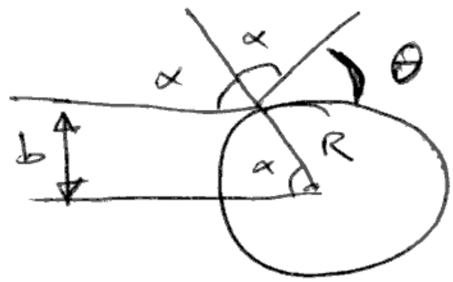


$$d\sigma = 2\pi b \, db$$

$$d\Omega = 2\pi \sin\theta \, d\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{2\pi b \, db}{2\pi \sin\theta \, d\theta} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$

Ex 14.5



$$2\alpha + \theta = \pi$$

$$\theta = \pi - 2\alpha$$

$$b = R\sin\alpha = R\sin\left(\frac{\pi-\theta}{2}\right) = R\cos(\theta/2)$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\frac{db}{d\theta} = \frac{R\cos(\theta/2)}{\sin\theta}\;\frac{R\sin(\theta/2)}{2} = \frac{R^2}{4}$$

w/ $\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) = \dfrac{\sin(\theta)}{2}$

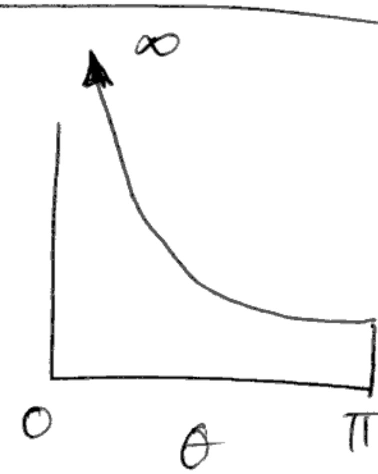
Note: $\dfrac{d\sigma}{d\Omega} = \text{const} = \text{isotropic}$

$$\sigma = \int \frac{d\sigma}{d\Omega}\,d\Omega = \int \frac{R^2}{4}\sin\theta\,d\theta\,d\phi$$

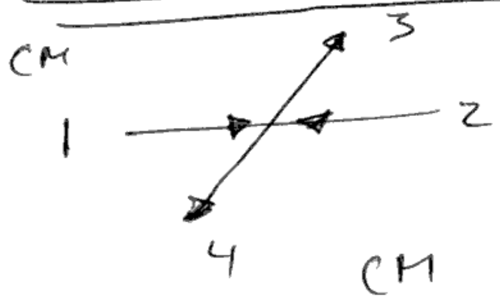
$$= \frac{R^2}{4}\cdot 4\pi = \pi R^2$$

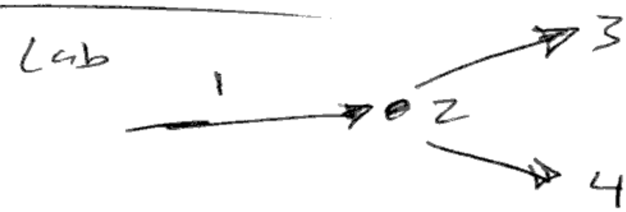
Rutherford Scattering

$$\frac{d\sigma}{d\Omega} = \left[\frac{Kq\,Q}{4E\,\sin^2(\theta/2)}\right]^2$$



Lab to CM frame

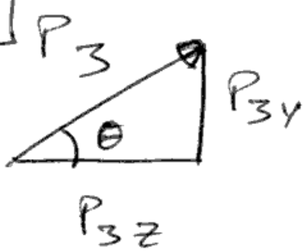




CM	LAB
$P_1 = P(0\,0\,1)$	$P_1' = P(0\,0\,2)$
$P_2 = P(0\,0\,-1)$	$P_2' = P(0\,0\,0)$
$P_3 = P(0\;S\;C)$	$P_3' = P(0\;S\;1+C)$
$P_4 = P(0\;-S\;-C)$	$P_4' = P(0\;-S\;1-C)$

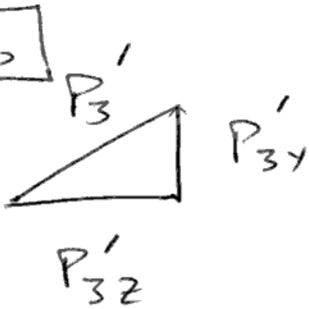
$\longrightarrow$ Add $P(0\,0\,1)$ to get

CMS



$$\tan\theta = \frac{P_{3y}}{P_{3z}} = \frac{\sin\theta}{\cos\theta} \equiv \tan\theta$$

Lab



$$\tan\theta' = \frac{P_{3y}'}{P_{3z}'} = \frac{\sin\theta}{1+\cos\theta} = \tan\left(\frac{\theta}{2}\right)$$

$$\boxed{\theta_{Lab} = \theta' = \frac{\theta_{cm}}{2}}$$

$$P_3' \cdot P_4' = P^2\left[-s^2 + (1+c)(1-c)\right] = P^2\left[1 - s^2 - c^2\right] = 0$$

P_3' and P_4' are <u>orthogonal</u> 90°

EXAMPLE 14.1 Shooting Crows in an Oak Tree

A hunter observes 50 crows settling randomly in an oak tree, where he can no longer see them. Each crow has a cross-sectional area $\sigma \approx \frac{1}{2}\ \text{ft}^2$, and the oak has a total area (as seen from the hunter's position) of 150 square feet. If the hunter fires 60 bullets at random into the tree, about how many crows would he expect to hit?

This situation closely parallels our simple scattering experiment. The target density is $n_{\text{tar}} = (\text{number of crows})/(\text{area of tree}) = 50/150 = 1/3\ \text{ft}^{-2}$. The number of incident projectiles is $N_{\text{inc}} = 60$, so, by the analog of (14.2), the expected number of hits is

$$N_{\text{hit}} = N_{\text{inc}}\, n_{\text{tar}}\, \sigma = 60 \times \left(\tfrac{1}{3}\ \text{ft}^{-2}\right) \times \left(\tfrac{1}{2}\ \text{ft}^{2}\right) = 10.$$

EXAMPLE 14.2 Scattering of Neutrons in an Aluminum Foil

If 10,000 neutrons are fired through an aluminum foil 0.1 mm thick and the cross section of the aluminum nucleus is about 1.5 barns,[4] how many neutrons will be scattered? (Specific gravity of aluminum = 2.7.)

The number of scatterings is given by (14.2), and we already know that $N_{inc} = 10^4$ and $\sigma = 1.5 \times 10^{-28}$ m^2. Thus all we need to find is the target density n_{tar}, the number of aluminum nuclei per area of the foil. (Of course, the foil contains lots of atomic electrons as well, but these do not contribute appreciably to the scattering of neutrons.) The density of aluminum (mass/volume) is $\varrho = 2.7 \times 10^3\,\text{kg/m}^3$. If we multiply this by the thickness of the foil ($t = 10^{-4}$ m), this will give the mass per area of the foil, and dividing this by the mass of an aluminum nucleus ($m = 27$ atomic mass units), we will have n_{tar}:

$$n_{tar} = \frac{\varrho t}{m} = \frac{(2.7 \times 10^3\,\text{kg/m}^3) \times (10^{-4}\,\text{m})}{27 \times 1.66 \times 10^{-27}\,\text{kg}} = 6.0 \times 10^{24}\,\text{m}^{-2}. \quad (14.3)$$

Substituting into (14.2) we find for the number of scatterings

$$N_{sc} = N_{inc}\, n_{tar}\, \sigma = (10^4) \times (6.0 \times 10^{24}\,\text{m}^{-2}) \times (1.5 \times 10^{-28}\,\text{m}^2) = 9.$$

Here, we used the given cross section σ to predict the number N_{sc} of scatterings we should observe. Alternatively, we could have used the observed value of N_{sc} to *find* the cross section σ.

EXAMPLE 14.5 Hard Sphere Scattering

As a first example of the use of (14.23), find the differential cross section for scattering of a point projectile off a fixed rigid sphere of radius R. Integrate your result over all solid angles to find the total cross section.

Our first task is to find the trajectory of a scattered projectile, as shown in Figure 14.10. The crucial observation is that when the projectile bounces off the hard sphere, its angles of incidence and reflection (both shown as α in the picture) are equal. (This "law of reflection" follows from conservation of energy and angular momentum — see Problem 14.13.) Inspection of the picture shows that the impact parameter is $b = R \sin \alpha$, and the scattering angle is $\theta = \pi - 2\alpha$. Combining these two equations we find that

$$b = R \sin \frac{\pi - \theta}{2} = R \cos(\theta/2), \tag{14.24}$$

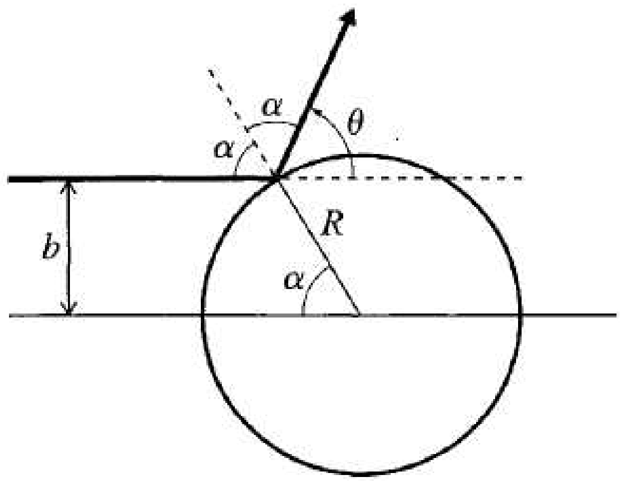


Figure 14.10 A point projectile bouncing off a fixed rigid sphere obeys the law of reflection, that the two adjacent angles labelled α are equal. The impact parameter is $b = R \sin \alpha$, and the scattering angle is $\theta = \pi - 2\alpha$.

and from (14.23), we find the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{R\cos(\theta/2)}{\sin\theta}\frac{R\sin(\theta/2)}{2} = \frac{R^2}{4}. \tag{14.25}$$

The most striking thing about this result is that the differential cross section is isotropic; that is, the number of particles scattered into a solid angle $d\Omega$ is the same in all directions. To find the total cross section, we have only to integrate this result over all solid angles:

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \int \frac{R^2}{4} d\Omega = \pi R^2,$$

which is, of course, the cross-sectional area of the target sphere.