Chpt 5 :

$$\vec{L} = m\, \vec{r} \times \vec{v} = m\, \vec{r} \times (\omega \times \vec{r}) = \overleftrightarrow{I} \cdot \vec{\omega}$$

$$\overleftrightarrow{I} = m\,(r^2 \overleftrightarrow{\mathbb{1}} - \vec{r}\,\vec{r})$$

$$T = \tfrac{1}{2} m v^2 = \tfrac{1}{2} I \omega^2 = \tfrac{1}{2}\, \vec{\omega} \cdot \overleftrightarrow{I} \cdot \vec{\omega}$$

$$N = \left(\frac{dL}{dt}\right)_s = \left(\frac{dL}{dt}\right)_b + \omega \times L = I \alpha + \omega \times (I \cdot \omega)$$

Take body axes along Princ axes.

For $\vec{N} = 0$

$$I_1 \dot{\omega}_1 = \omega_2 \omega_3 (I_2 - I_3) \qquad + \text{cyclic}$$

For $N = 0$

$$-\left(\frac{dL_i}{dt}\right)_b = \omega \times L = \varepsilon_{ijk}\, \omega_j L_k$$

$$-\left(\frac{dL_x}{dt}\right)_b = \omega_y L_z - \omega_z L_y$$

Let $\vec{\omega} = \omega\, \vec{n}$ where $\vec{n} = \alpha \hat{i} + \beta \hat{j} + \gamma \hat{k}$

$$T = \frac{1}{2}\, \omega \cdot I \cdot \omega = \frac{1}{2}\, \omega^2 \left(\vec{n} \cdot \overleftrightarrow{I} \cdot \vec{n}\right)$$

$( ) = \ldots\ldots$ Eq 5-38

For Princ Axes: I is diag:

$$I = I_{xx}\, \alpha^2 + I_{yy}\, \beta^2 + I_{zz}\, \gamma^2$$

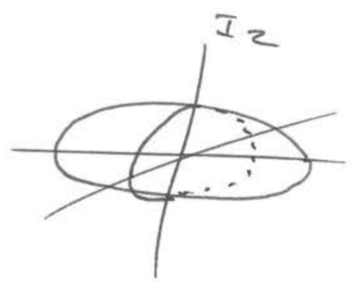
Equ for ellipse of rad. $\sqrt{I}$

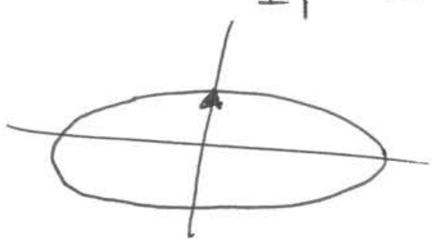
Define $\vec{\rho} = \dfrac{\vec{n}}{\sqrt{I}} = \dfrac{\vec{\omega}}{|\omega|} \dfrac{1}{\sqrt{I}}$

$$1 = I_x\, \rho_x^2 + I_y\, \rho_y^2 + I_z\, \rho_z^2$$

$\therefore$ $T = \frac{1}{2}\, \omega^2 I \Rightarrow \vec{\omega} = \omega \sqrt{I}\, \vec{\rho}$

moves on surface of ellipsoid $= \sqrt{2T}\, \vec{\rho}$





$$T = \frac{1}{2}\, \vec{\omega} \cdot \overleftrightarrow{I} \cdot \vec{\omega} = \frac{1}{2}\, \omega^2 I \, (\vec{\rho} \cdot \overleftrightarrow{I} \cdot \vec{\rho})$$

$$\Rightarrow \quad 1 = \vec{\rho} \cdot \overleftrightarrow{I} \cdot \vec{\rho}$$

For $\vec{P} = P_x \hat{x} \quad \hookrightarrow \quad P_x^2 I_x$

$$P_x^2 I_x = P_y^2 I_y$$

Larger I, smaller P

Energy conservation

Consider $F(\rho) = \rho \cdot I \cdot \rho = \rho_i^2 I_i$

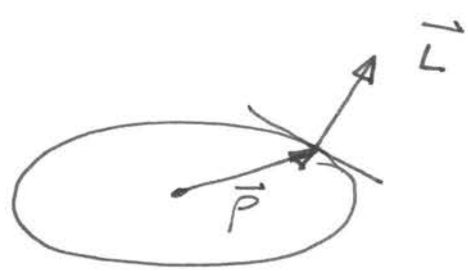
$$\nabla_\rho F(\rho) = 2 \overleftrightarrow{I} \cdot \vec{\rho} = \frac{2 I \cdot \omega}{\sqrt{2T}} = \sqrt{\frac{2}{T}}\, \vec{L}$$

F defines surface

$\nabla_\rho F$ defines normal $\parallel \vec{L}$

Force free motion: $\frac{d\vec{L}}{dt} = \vec{L} \times \vec{\omega}$

$\vec{\omega} \parallel \vec{\rho}$



$\vec{L} \times \vec{\omega} = \otimes$

pwan so

$\boxed{\text{Poinsot's Construction}}$

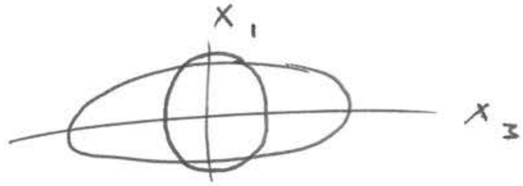
Poisson

Binet Const:

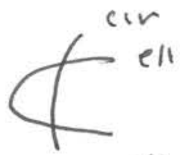
$$T = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{L}{I} = \sum_i \frac{L_i^2}{2 I_i} \qquad \text{Ellipse}$$

$$\text{Cons } L \Rightarrow L^2 = \sum L_i^2 = \text{const} \qquad \text{sphere}$$

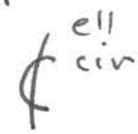
Binet ellipse
+ circle



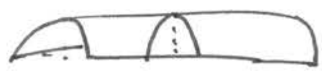
case 1



case 2)



case 3



unstable

For Steady state motion: $\vec{\alpha} = \dot{\vec{\omega}} = 0$

$$\Rightarrow \omega \times L = 0 = \omega \times (I \cdot \omega)$$

$$= \omega_1 \omega_2 (I_1 - I_2) \pm \text{cyclic}$$

$$\Rightarrow \vec{\omega} = (\omega, 0, 0) \quad \text{one direction}$$

Force Free motion

Take $I_1 = I_2 < I_3$

$$I\vec{\alpha} = -\omega \times L$$

$$I_1 \dot{\omega}_1 = (I_1 - I_3)\,\omega_2 \omega_3$$

$$I_1 \dot{\omega}_2 = -(I_1 - I_3)\,\omega_3 \omega_1$$

$$I_3 \dot{\omega}_3 = 0 \Rightarrow \omega_3 = \text{const}$$

Take #1 : $\frac{\partial}{\partial t} \Rightarrow$

$$I_1 \ddot{\omega}_1 = (I_1 - I_3)\,\omega_3 \dot{\omega}_2$$

$$\ddot{\omega}_1 = \left[-\frac{(I_1 - I_3)^2 \omega_3^2}{I_1^2} \right] \omega_1 \equiv -\Omega^2 \omega_1$$

$$\omega_1 = A \cos(t\Omega) \qquad \Omega_1 = \left(\frac{I_3 - I_1}{I_1}\right)\omega_3$$

$$\omega_2 = A \sin(t\Omega)$$

since $I_3 > I_1$

Note: $T = \frac{1}{2} I \omega^2 = \frac{1}{2}\left[I_1 A^2 + I_3 \omega_3^2 \right] = \text{const}$

$$L^2 = I^2 \omega^2 = I_1^2 A^2 + I_3^2 \omega_3^2 = \text{const}$$