$\boxed{\text{Lagrangian}}$ $L(q, \dot{q})$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

$$\frac{\partial L}{\partial q} = \dot{P}$$

$$\frac{\partial L}{\partial \dot{q}} = P$$

$\boxed{\text{Hamiltonian}}$ $H(q, P)$

$$H(q\,P) = \dot{q}P - L(q, \dot{q})$$

$$\frac{\partial H}{\partial P} = +\dot{q}$$

$$\frac{\partial H}{\partial q} = -\dot{P}$$

$\boxed{\text{SHO Lagrangian}}$ $L = T - V = \frac{m\dot{q}^2}{2} - \frac{K}{2}q^2$

Eqs $\Rightarrow$ $m\ddot{q} + Kq = 0$ $\quad q = e^{i\omega t} \Rightarrow \omega = \sqrt{\frac{K}{m}}$

One 2nd-order D.E.Q.

$\boxed{\text{SHO Hamiltonian}}$ $H = T + V = \frac{P^2}{2m} + \frac{Kq^2}{2}$

$$\frac{\partial H}{\partial P} = \frac{P}{m} = \dot{q} \qquad \frac{\partial H}{\partial q} = Kq = -\dot{P}$$

$$P = m\dot{q} \qquad -Kq = \dot{P}$$

$$P = mv \qquad -Kx = F \quad \text{Hooke's Law}$$

Goldstein Ch 9.3 Canonical TX

$$H = \frac{p^2}{2m} + \frac{Kq^2}{2} \equiv \frac{1}{2m}\left(p^2 + m^2\omega^2 q^2\right)$$

IF

$$p = f(P)\cos Q$$
$$q = f(P)\sin Q$$

$$K = H = \frac{f^2(P)}{2m}$$

Cyclic in Q

Generating Function $F_1(q\,Q) = \frac{m\omega q^2}{2}\cot Q$

For $F_1(q\,Q)$

$$p = \frac{\partial F_1}{\partial q} = m\omega q \cot Q \quad (1)$$

$$P = -\frac{\partial F_1}{\partial Q} = \frac{m\omega q^2}{2\sin^2 Q} \quad (2)$$

$$K = H + \frac{\partial F_1}{\partial t} \equiv H$$

(2) $\Rightarrow$ $q^2 = \frac{2P}{m\omega}\sin^2 Q$

(1) $\Rightarrow$ $p^2 = m^2\omega^2 \frac{2P}{m\omega}\sin^2 Q \frac{\cos^2 Q}{\sin^2 Q} = 2m\omega P\cos^2 Q$

$$q = \sqrt{\frac{2P}{m\omega}}\sin Q$$

$$p = \sqrt{2P m\omega}\cos Q$$

Goldstein Typo

$$H = K = \omega P$$

$$f(P) = \sqrt{2Pm\omega}$$

$$K(Q\ P) = \omega P \equiv H(Q\ P)$$

$$\frac{\partial H}{\partial P} = \omega = +\dot{Q} \implies Q = \omega t + \alpha$$

$$\frac{\partial H}{\partial Q} = 0 = -\dot{P} \implies P = \text{const} = \frac{E}{\omega}$$

Having Solved $\{Q\ P\} \implies \{q\ p\}$

$$q = \sqrt{\frac{2P}{m\omega}}\ \sin Q = \sqrt{\frac{2E}{m\omega^2}}\ \sin(\omega t + \alpha)$$

out of phase

$$p = \sqrt{2m\omega P} \times \cos Q = \sqrt{2mE}\ \cos(\omega t + \alpha)$$

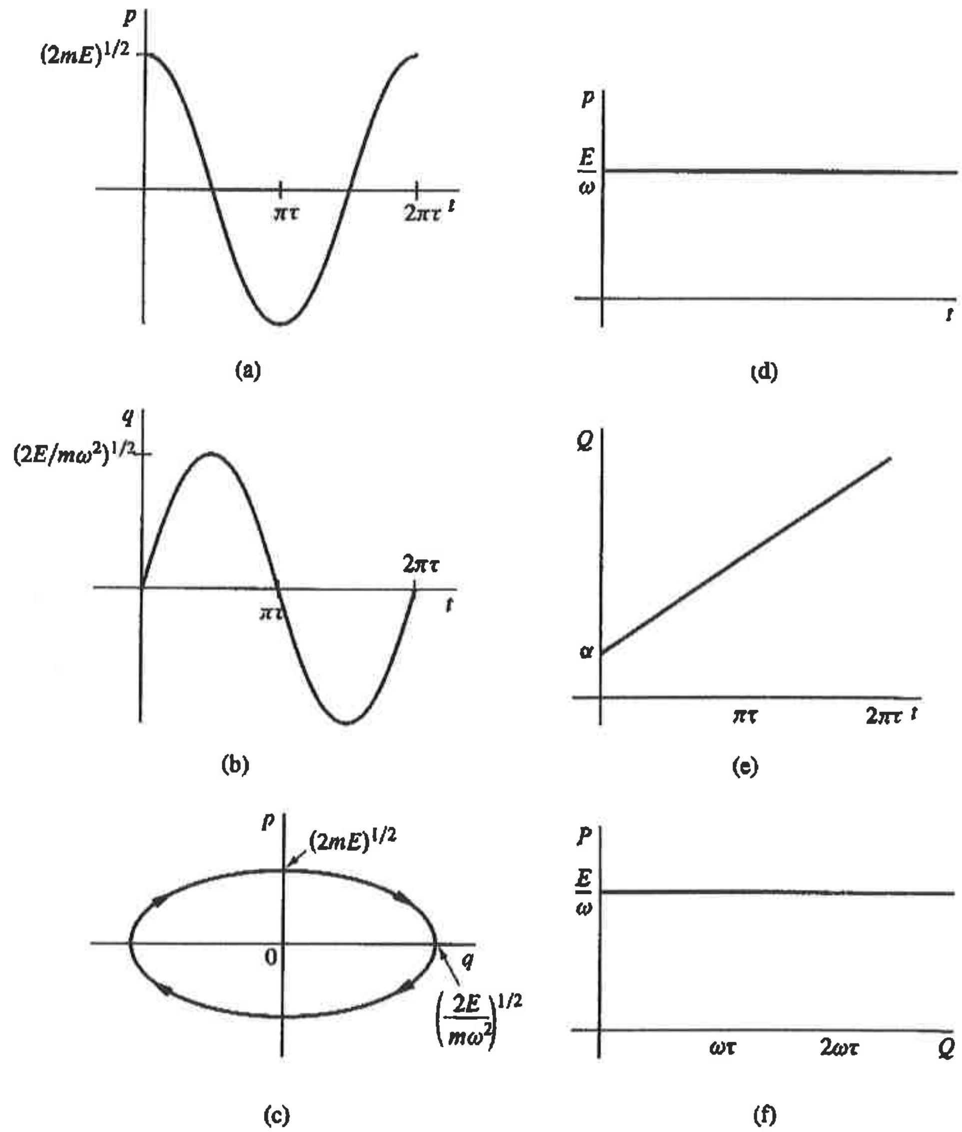

FIGURE 9.1 The harmonic oscillator in two canonical coordinate systems. Drawings (a)–(c) show the q, p system and (d)–(f) show the P, Q system.

When we invoke quantum mechanics, we write $E = \hbar\omega$, where $\hbar = h/2\pi$, and h is Planck's constant. The coordinate and momentum q and p can be normalized as

$$q' = \sqrt{\frac{m\omega^2}{2E}}q \qquad \text{and} \qquad p' = \frac{p}{\sqrt{2mE}}.$$

to make the phase space plot of p' versus q' a circle of area π. This normalized form will be useful in Section 11.1 on chaos.