Classical Preliminary Exam

Profs. T.E. Coan and F.I. Olness
Spring 2007

Printed Name ____

BOX YOUR ANSWERS

BOX YOUR ANSWERS

We never make mistakes

Two legs are better than four

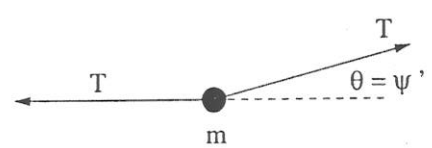
B, B, B, BOX YOUR ANSWERS

Q1. Required. A small uniform sphere of mass m and radius ρ starts at rest from the top of a fixed hemisphere of radius R. The small sphere *rolls without slipping* along the hemisphere's surface. Find the force F_h that the hemisphere exerts on the sphere and determine the height H at which the sphere leaves the hemisphere. The hemisphere's "equator" is parallel to the ground, similar to an orange cut in half and placed face down.

Q2. Required. Consider the transmission and reflection of waves from a mass m at $x = 0$ on a string of linear mass density ρ and tension T. The string stretches from $x = -\infty$ to $x = +\infty$. so that we can assume that the wave $\psi(x,t)$b has the form

$$\begin{aligned} \psi(x,t) &= Ae^{ikx}e^{-i\omega t} + RAe^{-ikx}e^{-i\omega t} & x < 0 \\ &= \tau Ae^{ikx}e^{-i\omega t} & x \geq 0, \end{aligned}$$

where ω is the angular frequency of the wave and $k = 2\pi/\lambda$, with λ the wavelength of the wave. Using the abbreviation $\epsilon = (m\omega^2)/(kT)$, determine τ and R. Assume that the mass moves only vertically and that the tension in the string is equal on each side of the mass. This problem requires no special knowledge of waves and is less hard than it might appear. The figure below might help.



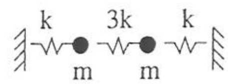
NQ1. Required. *Estimate* the mass of Earth's atmosphere M_a. Make the simple assumptions that the atmosphere's density is independent of altitude and that the acceleration due to gravity is also independent of height above Earth's surface for the heights relevant for this problem. If you do not have a calculator, then approximate (intelligently).

NQ2. Do 1 of 2 optional. *Estimate* the number of atoms N_a in your body.

NQ3. Do 1 of 2 optional. You are an astronaut at rest with respect to Earth at some distance from Earth. You see red neon lights from Las Vegas. *Estimate* how fast you must travel radially away from Earth so that your eye can no longer detect red light? Express your answer β^* as a fraction of the speed of light.

Q3. Required. Pions can be produced in proton-proton collisions via the strong interaction. A beam of protons (each of mass m_p) strikes a stationary target of protons in the reaction $p + p \to p + p + \pi^+\pi^-\pi^0$. Assuming $m_{\pi^\pm} = m_{\pi^0} = m_\pi$, determine the minimum total energy E_p the incident proton must have for the reaction to occur. Express your answer in terms of $\{m_p, m_\pi\}$.

Q4. Required. Two masses m are connected by springs of spring constants $\{k, 3k, k\}$ as shown in the beautiful drawing below. The equilibrium spring length is a. Consider longitudinal motion only and determine the normal modes (also called eigenmodes) of oscillation. Also determine the oscillation frequency for each normal mode.



Farro
Firan
Gibbs
Kasmi

Classical Exam

Prof. T.E. Coan (x8-2497)
Prof. J. Ye
Aug. Jan 2006

PRINTED Name ____________________________________

BRAIN ONLY EXAM:
NO REFERENCES ALLOWED.
CALCULATORS OK.

† DO FOUR PROBLEMS.

† BOX your final answers. We will NOT hunt for your answer amid a sea of algebra.

† Scratch out irrelevant calculations so that we can follow your reasoning.

THIS EXAM IS CLOSED BOOK

(1) 25 pts A uniform rod of length a is freely pivoted at one end. It is initially held horizontally and then released from rest.

a) 10 pts Find the angular velocity at the instant when the rod is vertical. When the rod is vertical it breaks at its midpoint. Assume that no impulsive forces are generated when the rod breaks.

b)10 pts Find the largest angle from the vertical reached by the upper part of the rod in its subsequent motion.

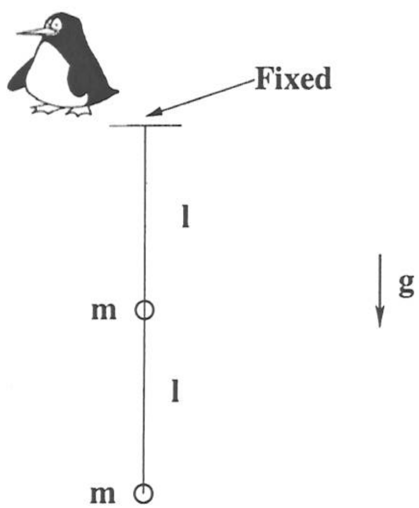
c) 5 pts Describe the motion of the lower part of the rod. Be clear if you use no math.

(2) 25 pts This problem requires no special knowledge of particle physics. The proposed NOνA experiment at Fermilab seeks to determine, among several things, the mass ordering of the neutrino mass eigenstates. The experimental technique relies on generating a beam of energetic muon neutrinos ν_μ. This is accomplished by first colliding a beam of protons into a carbon target, and then letting the subsequently produced charged pions decay in flight. The charged pions have a typical energy of 10 GeV and decay isotropically in their *rest* frame into a muon and a muon neutrino ($\pi^+ \to \mu^+ + \nu_\mu$). Useful information includes $m_\pi = 140\,\mathrm{MeV}$ and $m_\mu = 106\,\mathrm{MeV}$. Assume $m_\nu < 1\,\mathrm{eV}$.

10 pts a) What is the neutrino energy E_ν and the muon energy E_μ in the pion *rest* frame? Express your compact answers as functions of m_π, m_ν and m_μ.

10 pts b) Show that for neutrinos produced in the *lab* frame and parallel to the flight direction of the pions, the neutrino energy $E_\nu \simeq 0.43 E_\pi$, where E_π is the energy of the pion in the lab frame.

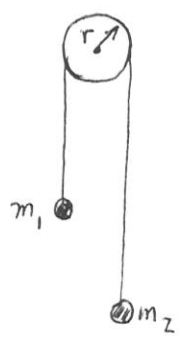
(3) **25 pts** The AMANDA and ICECUBE Antarctic neutrino experiments rely for signal detection on photomultiplier tubes (PMTs) strung along vertical cables frozen in the ice. Holes are first drilled in the ice, a PMT string is lowered and the ice is then allowed to refreeze, fixing the positions of the PMTs. For simplicity, assume each such string has only 2 PMTs, each of identical mass m, and let the distance between the 2 PMTs and the distance between the top most PMT and the surface be l. (See the figure.)



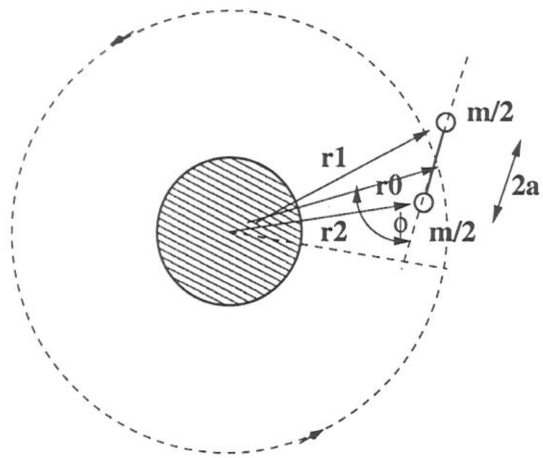
a) **15 pts** If one end of the PMT string is always fixed at the surface, what are the frequencies of the normal modes of oscillation of the PMTs before the ice refreezes? Assume the hole diameter is large enough so that it does not interfere w/ a swinging PMT and ignore the effects of the ice water in the bore hole.

b) **5 pts** Sketch each normal modes and match it with its respective eigenfrequency.

(4) (25 points) Atwood's machine consists of two weights of mass m_1 and m_2 connected by a massless, non-stretchable string of length l. The string passes over a pulley of radius r and moment of inertia I. The pulley is fixed in space but can rotate without any friction. The string does not slide on the pulley. Choose a proper coordinate and find the equation of motion for this system from the Lagrange equations, assuming the system is stationary at the beginning.



5) 25 **pts** We are interested in the stability of the orientation or "attitude" of a dumbbell-shaped satellite in its circular orbit about earth. The satellite is composed of two small spherical masses of identical mass $m/2$ joined by a thin massless cylinder of length $2a$. See the figure. Let the polar coordinates r and θ describe the position of the center-of-mass of the satellite and let the angle ϕ describe the attitude or angular orientation of the satellite axis relative to the radius vector $\vec{r}_0$.



a) 5 pts) To start, consider the motion of a point mass m moving in 1 dimension and in a conservative force field whose potential energy function is $V(q)$, where q denotes the particle's displacement from the origin. Also assume that $q = 0$ is a point of equilibrium. After Taylor series expanding $V(q)$ around $q = 0$ and keeping only the lowest non-vanishing term, write the lagrangian L for this system. Ignore arbitrary constants in the series expansion. (By arbitrary, I mean those terms that can be set to zero without loss of generality.)

b) 5pts What is the equation of motion for this simple system? (Be as explicit as you can. The symbol L should not appear in this equation.)

c) 5pts For motion near $q = 0$, what is the condition on $(d^2 V/dq^2)|_q = 0$ for oscillatory motion about $q = 0$ and what is the angular frequency of oscillation ω in terms of m and $(d^2 V/dq^2)|_{q=0}$?

d) 5pts Back to the problem at hand. Write the potential energy of the satellite in terms of r_1, r_2, m, M_E and G. The symbols have an obvious meaning.

e) 5pts Examining the figure and from the law of cosines, we can write

$$r_{1,2} = (r_0^2 \pm 2r_0 a \cos\phi)^{1/2} = (r_0^2 + a^2)^{1/2}(1 \pm \epsilon \cos\phi)^{1/2},$$

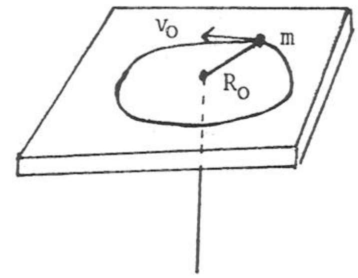
where $\epsilon = 2r_0 a/(r_0^2 + a^2)$. Since $a \ll r_0$, ϵ is small and we can then use the binomial series expansion $(1+x)^{-1/2} = 1 - x/2 + 3x^2/8 + \cdots$, where $x = \pm\epsilon\cos\phi$. With these approximations, simplify your answer for V in part d) and rewrite it as

$$V(\phi) = -\frac{GM_E m}{r_0}\{\cdots\},$$

where *you* need to fill in the $\{\cdots\}$. Just keep the lowest order terms.

f) 5 pts Determine the value of ϕ for *stable* oscillations. Sketch the attitude of the satellite in its stable configuration.

4. A mass m moves on a smooth horizontal plane with velocity v_o at a radius R_o. The mass is attached to a string which passes through a smooth hole in the plane, as shown. ("Smooth" means frictionless.)



(a) What is the tension in the string?

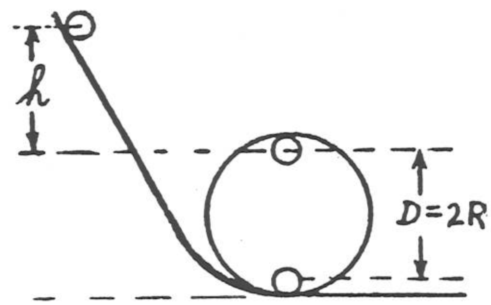
(b) What is the angular momentum of m?

(c) What is the kinetic energy of m?

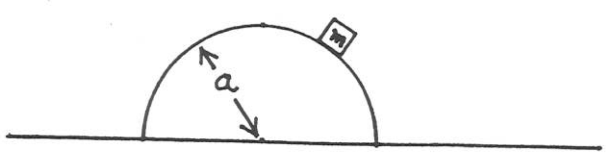
(d) The tension in the string is increased gradually and finally m moves in a circle of radius $R_o/2$. What is the final value of the kinetic energy?

(e) Why is it important that the string be pulled gradually?

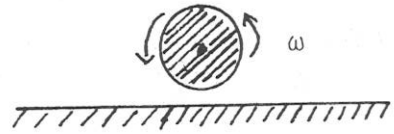
9. Find the minimum height h (above the top position in the loop) that will permit a spherical ball of radius r (which rolls without slipping) to maintain constant contact with the rail of the loop.
(The moment of inertia of a sphere about the center is $2/5\ m\ r^2$.)



3. **A mass m slides without friction on a hemispherical hill of radius a. If it starts at rest at the top of the hill (unstable equilibrium), at what height h does it leave (separate from) the hill?**



12. A coin spinning about its axis of symmetry with angular frequency ω is set down on a horizontal surface. After it stops slipping, with what velocity does it roll away?



Classical Exam

Prof. T.E. Coan (x8-2497)
May 2005

PRINTED Name ______________________________

BRAIN ONLY EXAM:
NO REFERENCES ALLOWED
NO CALCULATORS ALLOWED.
OK, YOU CAN DRINK BEER.

† DO ALL PROBLEMS.

† BOX your final answers. I will NOT hunt for your answer amid a sea of algebra.

† Scratch out irrelevant calculations so that I can follow your reasoning.

THIS EXAM IS CLOSED BOOK

(1) 10 pts Here are some easy questions to start with. Answer **only five** of them. All answers should have at least 1 significant figure precision and use proper "conventional" units such as meters, kilograms, seconds, centimeters, grams, etc. I do **NOT** want to see energy units for mass.

1) What is the density ρ_{Fe} of iron?

2) What is the mass $M_{\oplus}$ of the earth?

3) What is the value of Avogadro's number N_0?

4) What is the mass m_p of a proton?

5) What is the mass m_e of an electron?

6) What is the wavelength λ of visible light?

7) What is the mass $M_{\odot}$ of the sun?

8)What is the mean radius R_{ES} of earth's orbit about the sun?

9) What is the value of atmospheric pressure P at sea level? (Do NOT write "1 atmosphere.")

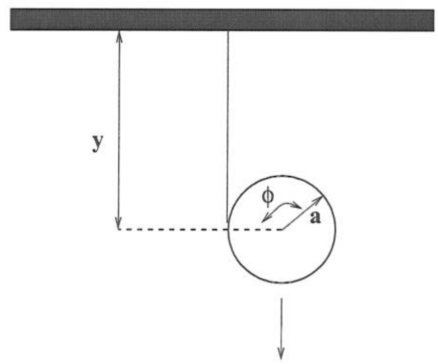
10) What is the value of the air density ρ in this room?

(2) 20 pts This problem requires no special knowledge of particle physics. A charged pion (π^+) decays into a muon μ^+ and a muon neutrino ν_μ. Let the mass of these particles be denoted in an obvious notation by m_π, m_μ and m_ν.

10 pts a) What is the neutrino three-momentum p_ν in the pion *rest* frame? Express your answer in terms of the three particle masses and a well-known physical constant.

10 pts b) What must be the pion total energy E_π as measured in the *lab* frame if the neutrino is produced at rest in the *lab* frame? Now you see why producing neutrinos at rest is very difficult. Perhaps this could be your thesis topic.

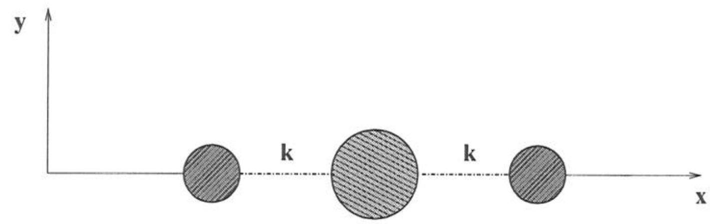
(3) 20 pts Consider the behavior of a yo-yo. (This does not refer to the famous cellist Yo-Yo Ma.) A yo-yo is a toy in the form of a disk with a string wrapped around it. Suppose our yo-yo falls under the influence of gravity only and that the trajectory of the disk's center-of-mass is completely vertical. Our yo-yo has the end of its string that is not attached to the disk, attached to a fixed support. The figure summarizes the situation. Let the yo-yo have a mass m, radius a and start falling from rest. Ignore the mass of the string. To keep me sane while I try to grade these, use the variable names as shown in the figure.



10 pts a) Find the equations of motion for the falling disk. You do NOT need to integrate them: you can just give them to me as uncoupled differential equations.

10 pts b) Find the forces of constraint and tell me their *physical* significance.

(4) 20 pts There is strong evidence that earth's atmosphere is heating up. One of the molecules involved in this process is CO_2. Let's see if we can understand something about this molecule's vibrational modes. CO_2 is a symmetric molecule with all 3 atoms in a line, as shown in the figure. Let the mass of the carbon and oxygen atoms be m_C and m_O, respectively. Let k be the effective spring constant between the carbon and oxygen atoms.



10 pts a) What are the three eigenfrequencies and eigenmodes ("normal modes") for the vibrational motion of CO_2?

8 pts b) For the mode(s) with non-zero eigenfrequency, what is the algebraic relation between the non-zero amplitudes of motion of the relevant atoms? Sketch the normal mode associated with each eigenfrequency, including any possible zero-frequency eigenmode.

2 pts c) What is the *numerical* value of the ratio(s) of the non-zero eigenfrequencies?

For Pavel, Sophia

Classical Exam

Prof. T.E. Coan (x8-2497)
Prof. Y Gao
Spring 2003

Name ______________________________

BOX your final answers

Do ALL problems

BOX your final answers

Do ALL problems

Do What with your answers?

Classical 1. Most of the surface area of earth is liquid water and the core of the earth is presumably liquid iron. Consider earth as an isothermal, homogeneous liquid sphere with the same mean mass density and radius as the real earth. Ignore rotational and surface tension effects.

i) Calculate the radius R_p at which the hydrostatic pressure inside this fully liquid earth is a *maximum*.

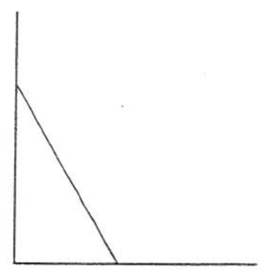
ii) What is this maximum pressure P_{max}? I need both a formula and a number. If you do not have a calculator, estimate the calculation. No units, no credit.

Classical 2. Consider two identical pendula joined as in the figure. Treat the rods as inflexible and massless. The rods each have a length l and the bobs each have mass m. Consider motion only in the plane of the pendula.

i) Find Lagrange's equations of motion for the system. Do not assume small angles.

ii) For the case of small angles ($\sin\theta \sim \theta$), what are the eigenfrequencies?

Classical 3. A ladder is leaning against a wall at some angle θ as shown in the figure. The ladder now starts to fall down in such a way that its ends are always in contact with the two surfaces. For simplicity, assume that there is no friction between the ladder ends and the wall and floor surfaces.



i) Calculate the equation(s) of motion for the system.

ii) Determine the forces that the wall and floor exert on the ladder.

Classical 4. Refer to the figure below. A star, located at O is at rest with respect to you, located at P. The star explodes, producing a spherically expanding shell of gas moving with speed v. You observe the light from this expanding shell of gas. Assume that there is vacuum between you and the star and let the distance $\overline{OP} = R$. You can assume $R \gg r$, where r is the radius of the shell. Show that the apparent ('observed') speed of then expanding shell can exceed the speed of light in vacuum. Specifically,

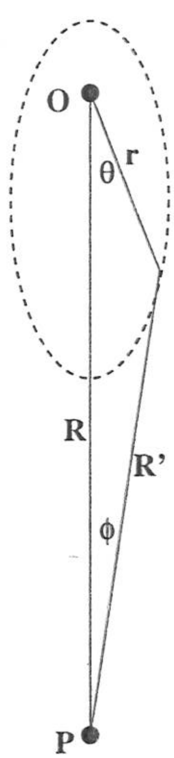
i) Show that the *observed* speed $u_\perp$ of the shell at right angles to the line of sight is given by

$$u_\perp = \frac{v \sin\theta}{1 - (v/c)\cos\theta}.$$

Hints: Star and you are at rest relative to each other. Define 3 times,

t'_e = emission time of light;
t'_0 = explosion time of star (start time of expanding shell);
t'_{obs} = time when shell light reaches P.

Construct relations between times and the geometry of the problem.



10 pts Here are some easy questions to start with. Answer **only five** of them. All answers should have at least 1 significant figure precision and use proper "conventional" units such as meters, kilograms, seconds, centimeters, grams, etc. I do **NOT** want to see energy units for mass.

1) What is the radius R_E of the earth?

2) What is the mass M_E of the earth?

3) What is the value of Avogadro's number N_0?

4) What is the mass of a proton?

5) What is the mass of an electron?

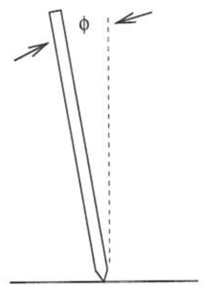
6) What is the value of Newton's gravitational constant G?

7) What is the mass M_S of the sun?

8)What is the mean radius R_{ES} of earth's orbit about the sun?

9) What is the value of the gravitational acceleration g in vacuum of an object dropped near the surface of the earth?

10 pts A homogeneous pencil of total length L and total mass M is placed nearly vertically on a table and allowed to fall over. See the figure. Just *after* the pencil is released, what is the force F_T that the table exerts on the pencil?



Classical Exam

Prof. T.E. Coan (x8-2497)
Fall 2002

Name ____________________________________

CLOSED BOOK: NO REFERENCES OR CALCULATORS ALLOWED.

† DO ALL PROBLEMS.

† BOX your final answers. I will NOT hunt for your answer amid a sea of algebra.

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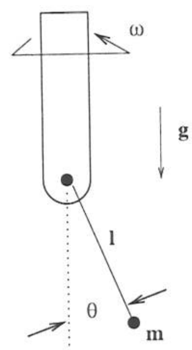
THIS EXAM IS CLOSED BOOK

20 pts The bearing of a rigid pendulum is forced to rotate uniformly about a vertical axis with angular speed ω. The pendulum has a rod length l and a bob mass m. Let θ be the angle between the pendulum and the vertical. See the figure. Neglect the inertia of the bearing as well as the rod connecting it to the mass. Neglect friction but do include the effects of gravity.



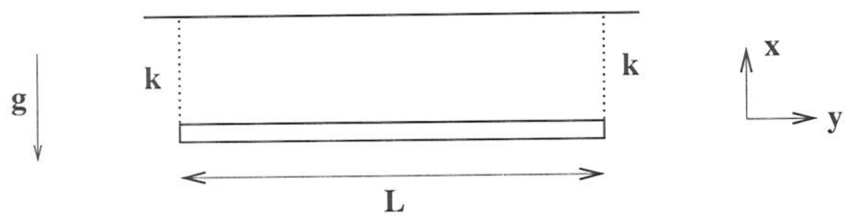
a) Determine the differential equation for θ.

b) At what angular speed ω_c does the stationary point $\theta = 0$ become unstable?

c) For $\omega > \omega_c$, what is the stable equilibrium value of θ?

d) What is the frequency Ω of small oscillation about this point?

20 pts total A rigid homogeneous bar of total length L and mass M is suspended in equilibrium in a horizontal position by identical massless springs of spring constant k at each of its two ends. See the figure. Assume that the bar's center-of-mass can move *only* parallel to the x-direction and that the bar is further constrained to move *only* in the x-y plane.



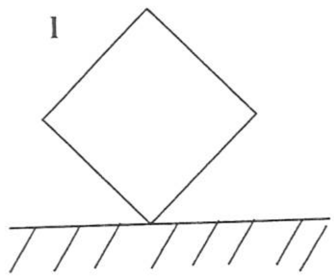
15 pts a) What are the eigenfrequencies of oscillation?

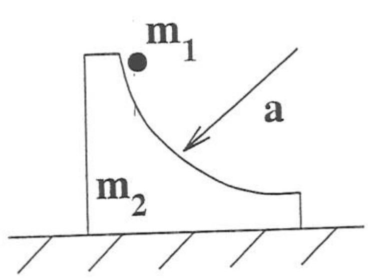
5 pts b) Sketch clearly the normal modes of oscillation and identify the normal mode with its correct eigenfrequency.

20 pts A proton with $\gamma = 1/\sqrt{(1 - v^2/c^2)}$ collides elastically with a proton at rest. If the two protons rebound with equal energies, what is the angle between them?

for Eliana Viahello
by Tom Coan

A homogeneous cube, of edge length l, is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then given a small displacement and allowed to fall. Find the angular velocity when one face strikes the plane if sliding can occur without friction.





Refer to the figure above. A particle of mass m_1 slides down a smooth circular surface of radius of curvature a of a wedge of mass m_2 that is free to move horizontally along the smooth horizontal surface on which it rests.

a. Find the equations of motion for each mass.

b. Find the force of constraint exerted by the wedge on the particle.

8/21/01

Classical Exam

Prof. T.E. Coan (x8-2497)
Fall 2001

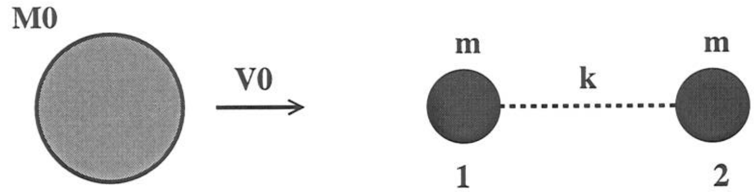
Name ________________________________

BOX YOUR FINAL ANSWERS

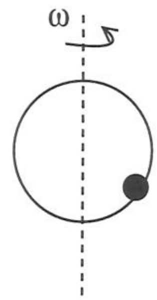
BOX YOUR FINAL ANSWERS

Classical 1. Estimate the mass of earth's atmosphere. Do not write down just a number, but show your reasoning based on familiar physical quantities. If you do not know the numerical value of some quantity, indicate clearly what your assumption is.

What fraction of earth's mass is this?



Classical 2. A ball of mass M_0 moves with velocity V_0 on a horizontal frictionless surface and strikes the first of two stationary target balls of mass m each. The balls are joined together by a massless spring of spring constant k. See the figure. You can consider the collision as head-on, elastic and instantaneous. Describes the motion of target ball 2 as a function of M_0, m, V_0, k and the time after collision t. Assume that there is only 1 collision between M_0 and the target balls.



Classical 3. A particle of mass m is constrained to move without friction on a circular wire of radius a that rotates with constant angular velocity ω about a vertical diameter. See the figure. Recall that Lagrange's equations are:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_j}) - \frac{\partial L}{\partial q_j} = 0, \quad j = 1, 2, \ldots$$

A Find the equilibrium position of the particle.

B Calculate the frequency of small oscillations about the equilibrium position.

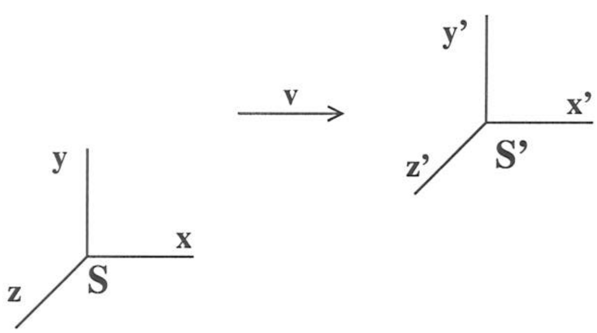
Classical 4. It is estimated that earth's mass accumulation rate due to meteorites is 10^9 grams/yr. Assuming that this mass accumulation rate has been constant since earth's formation and that the mass is accumulated uniformly over earth's surface, calculate the total change in the length of the day due to this effect. Indicate whether the day has increased or decreased. If you do not know the value of some physical quantity, clearly indicate what your assumption is.

Classical 5. An astronaut accelerates radially away from earth at a uniform rate (as measured from earth) of $g = 9.8\,\mathrm{m/sec^2}$.

A Calculate the astronaut's speed at which the red neon lights of Las Vegas become invisible to the human eye. If you do not know the value of a physical parameter, guess it and clearly label your assumption.

B Calculate how long T it takes, as seen from earth, for the astronaut to reach this speed. **Hint:** You can transform continuously to the astronaut's reference frame so that for each transformation the astronaut's proper speed is zero. Useful transformation formulae in an obvious notation are below and refer to the figure. The table contains a useful integral.

$u_x = \text{x velocity in frame S}$	$u_y = \text{y velocity in frame S}$	$\int \frac{d\beta}{1-\beta^2} = \frac{1}{2}\ln\frac{1+\beta}{1-\beta}$
$u'_x = \text{x velocity in frame S}'$	$u'_y = \text{y velocity in frame S}'$	
$u_x = \frac{u'_x+v}{1+vu'_x/c^2}$	$u_y = \frac{u'_y/\gamma}{1+vu'_y/c^2}$	$t = \gamma(t' + vx'/c^2)$
$a_x = \frac{a'_x}{\gamma^3(1+vu'x/c^2)^3}$	$a_y = \frac{a'_y}{\gamma^2(1+vu'_x/c^2)} - \frac{(vu'_y/c^2)a'_x}{\gamma^2(1+vu'_x/c^2)^3}$	



Du, Magar, Wang

Classical Exam

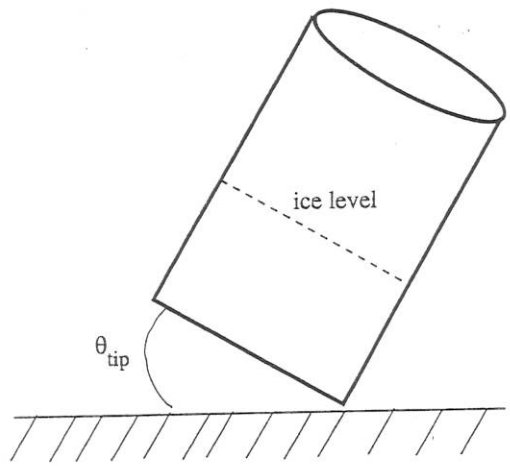
Prof. T.E. Coan
Spring 2000

Name ____________________

Both problems are required.

BOX YOUR FINAL ANSWERS

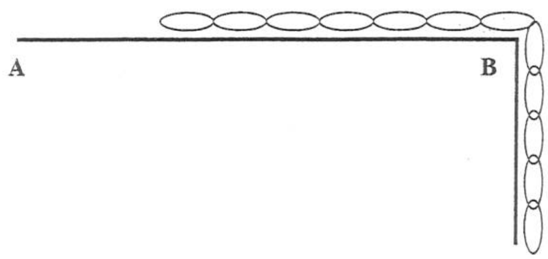
BOX YOUR FINAL ANSWERS



Classical 1. Refer to the figure above. The Texas heat has melted your brain and you have taken to eating ice. A cylindrical glass full of ice has a mass four times that of the empty glass. At what intermediate level of filling is the glass *least* likely to tip over? (The tip angle θ_{tip} is defined in the figure.) I am looking for the height h of the ice level above the bottom of the glass. For simplicity, ignore the mass at the bottom of the glass. Let m be the empty glass mass, ρ be the ice density, r be the glass radius, h_0 be the glass length, and h be the ice level. Assume that the top surface of the ice is always parallel to the bottom of the glass as shown in the figure.

(a) First, convince yourself (and me!) that the maximum tip angle corresponds to the *minimum* center-of-mass (CM) height. Do this by deriving a simple relationship between Z_{CM}, the center of mass of the ice+glass system measured from the glass bottom and along the symmetry axis of the glass, θ_{tip} and the glass radius r. You may draw figures to aid your mathematical explanation.

(b) Determine h. Hint: the actual value of r is totally irrelevant and the mass of the ice in the glass is proportional to h.



Classical 2. A uniform chain of total length a has a portion $0 < b < a$ hanging over the edge of a smooth table AB. (Refer to the figure above.) Show that the time t for the chain to slide off the table if it starts frorm rest is $t = \sqrt{a/g}\ln[\frac{(a+\sqrt{a^2-b^2})}{b}]$. You may find the integral $\int \frac{dx}{\sqrt{x^2-c^2}} = \ln[2\sqrt{x^2-c^2}+2x]$, where c =constant, useful.

Daeschler, Trout - Aug 08
by Coan/Fao

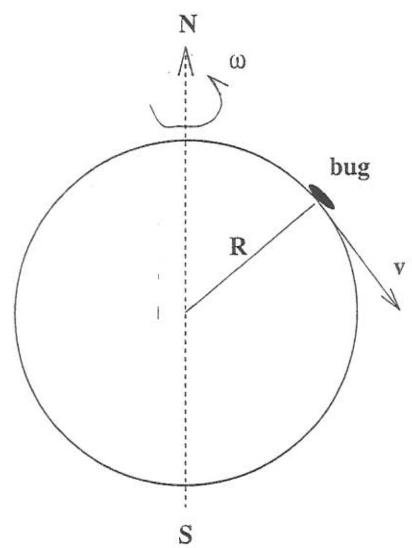
A chain of mass M and length l is suspended vertically above the horizontal surface of a scale so that its lower end just touches the scale. The chain is released and falls onto the scale. What is the scale reading when a length of chain x has fallen? Neglect the size of the chain links.

A globe rotates freely without friction with an intial angular velocity ω_0. A small bug of mass m starts at the North pole and travels to the South pole along a meridian with constant velocity v. The axis of rotation of the globe is fixed. See the figure below. Let M be the mass of the globe and R be the globe radius. You can assume that the globe is solid with uniform mass density so that its moment of inertia $I_0 = \frac{2}{5}MR^2$. Finally, suppose that the time of the bug's trip from pole-to-pole is T. Show that, during the time the bug is travelling, the globe rotates through the angle

$$\Delta\theta = \frac{\pi\omega_0 R}{v}\sqrt{2M/(2M+5m)}.$$

A useful integral is

$$\int_0^{2\pi} \frac{dx}{a+b\cos x} = \frac{2\pi}{\sqrt{a^2-b^2}}, \qquad (a^2 > b^2).$$

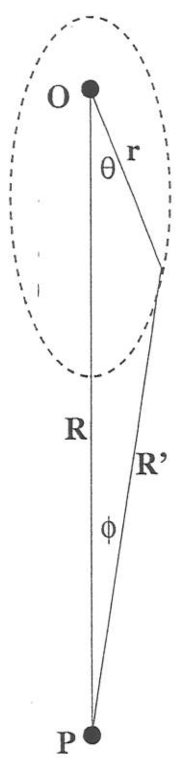


Refer to the figure below. A star, located at O, is at rest with respect to you, located at P. The star explodes, producing a spherically expanding shell of gas moving with speed v. You observe the light from the expanding shell of gas. Assume that there is vacuum between you and the star and let the distance $\overline{OP} = R$. You can assume also that $R \gg r$, where r is the radius of the shell. Show that the *apparent* ('observed') speed of the expanding shell can exceed the speed of light in vacuum c. Specifically,

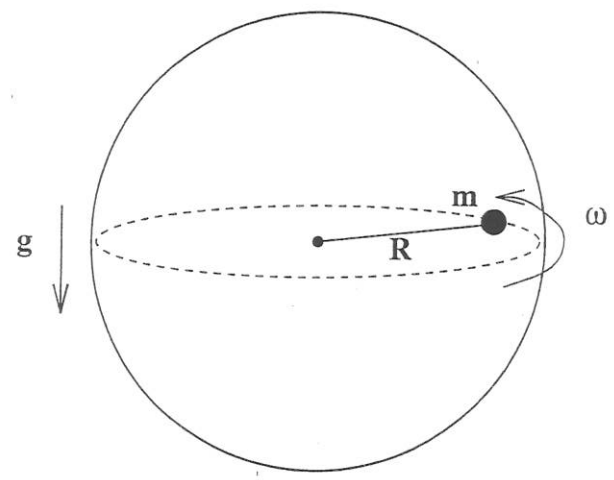
a) **20 pts** Show that the *observed* speed $u_\perp$ of the shell at right angles to the line of sight is given by

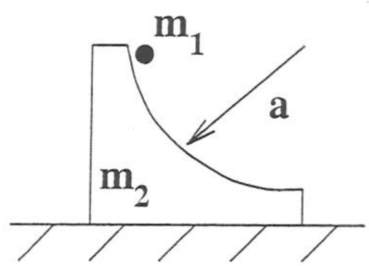
$$u_\perp = \frac{v \sin\theta}{1 - (v/c)\cos\theta}.$$

b) **5 pts** Determine the maximum value $u_{\perp,\max}$ of $u_\perp$ for fixed v, and the angle θ for which this occurs.



A particle constrained to move on the surface of a hollow spherical shell of radius R is projected horizontally from a point at the level of the sphere's center so that its angular velocity about the vertical symmetry axis of the sphere is ω. See the figure below. (You can imagine that the particle is shot from the outside through a small hole in the shell which is then magically sealed.) If $\omega^2 R \gg g$, where g is the acceleration due to gravity, show that the particle's maximum depth below the level of the center is approximately $z \simeq \frac{2g}{\omega^2}\sin^2(\frac{\omega t}{2})$.





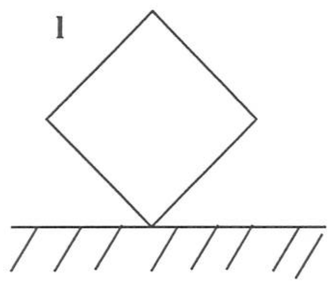
Refer to the figure above. A particle of mass m_1 slides down a smooth circular surface of radius of curvature a of a wedge of mass m_2 that is free to move horizontally along the smooth horizontal surface on which it rests.

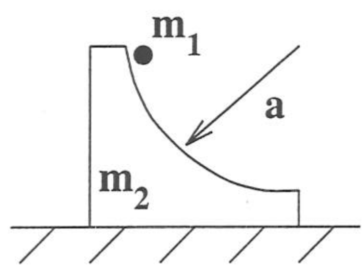
a. Find the equations of motion for each mass.

b. Find the force of constraint exerted by the wedge on the particle.

MAY 99 for Eliana Vianello by Tom Caan

A homogeneous cube, of edge length l, is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then given a small displacement and allowed to fall. Find the angular velocity when one face strikes the plane if sliding can occur without friction.





Refer to the figure above. A particle of mass m_1 slides down a smooth circular surface of radius of curvature a of a wedge of mass m_2 that is free to move horizontally along the smooth horizontal surface on which it rests.

a. Find the equations of motion for each mass.

b. Find the force of constraint exerted by the wedge on the particle.

Classical Exam

Prof. T.E. Coan (x8-2497)
Prof F.I. Olness (x8-2500)
Winter 2011

Printed Name ______________________________

DIRECTIONS:

0. If we can't read it, we can't grade it.

1. **BOX YOUR FINAL ANSWERS**

2. **BOX YOUR FINAL ANSWERS**

3. Paginate all pages. Label the problem number clearly.
4. Staple your pages together, in order.
5. Good luck.

Q1 5 pts. Estimate the number N of atoms in the Sun. Make whatever approximations seem reasonable.

Q2 5 pts. Estimate the number density n of solar photons near earth's surface. The unit conversion 1 electron-Volt $= 1.6 \times 10^{-19}$ Joules may be useful. Make whatever approximations seem reasonable.

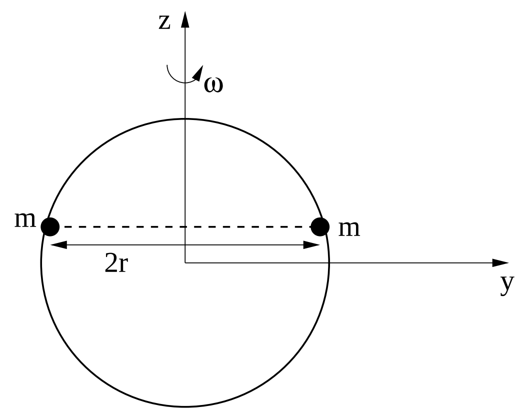
Q3 10 pts. Humanity consumes energy at an average rate of approximately 10^{13} Watts. Suppose you develop a technique to furnish this energy by slowing the spinning of earth on its axis so that its period of rotation about its axis is increased by 1 minute per day. How long could T could your technique supply the required energy, assuming humanity's power consumption P doesn't change? This is an approximate calculation so a precise number is *not* required nor is a calculator. Make whatever approximations seem reasonable to you.

Q4 10 pts. A particle is acted on by a central force $f = f(r)$ such that its motion is a spiral orbit

$$r(\theta) = Ae^{\beta\theta},$$

where A and β are constants. Find the time dependent trajectory $(r(t), \theta(t))$ for the motion. **Hint:** Think about angular momentum for central forces and let L be the particle's angular momentum at the start of its motion.

Q5 20 pts. Consider a system of two particles (beads), a spring and a circular wire (hoop). The beads are connected by a spring and they slide without friction on the wire. See the figure below. A Cartesian coordinate system is used with the center of the hoop at the center of the coordinate system. The $y-z$ plane of the coordinate system is horizontal and the spring is initially parallel to the y-axis (so that the beads always have the same z-coordinate). Each bead has mass m, the force constant of the sprig is k, and the radius of the hoop is R. The equilibrium length of the spring $2r_0$ is less than than the hoop diameter, i.e., $r_0 < R$. Finally, the hoop rotates about the z-axis of the inertial frame $Oxyz$ with constant angular frequency ω.



(a) Express the lagrangian of this system in terms of cylindrical coordinates (r, ϕ, z) and show that it can be written in the one-dimensional form

$$L = \frac{1}{2}\mu\dot{z}^2 - V_e(z)$$

where

$$\mu = 2m(1 - z^2/R^2)^{-1}$$

is a position dependent mass and

$$V_e(z) = 2k(\sqrt{R^2 - z^2} - r_0)^2 - m\omega^2(R^2 - z^2)$$

is a one-dimensional effective potential.

(b) Determine the equilibrium points z_ω of the particles. You may find the variable

$$\xi(\omega) = 2kr_0/(2k - m\omega^2)$$

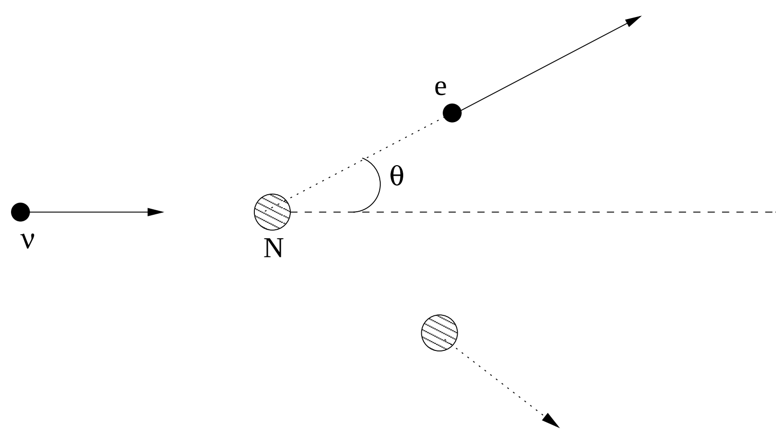
useful.

(c) Determine the *stability* of the equilibrium points and show that there is a critical angular frequency

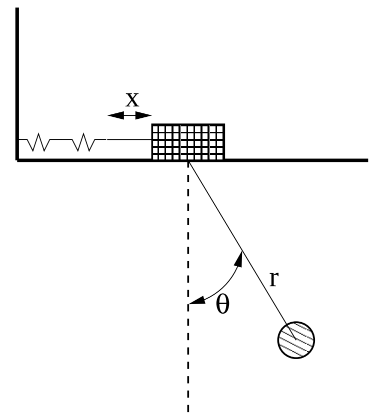
$$\omega_c = \sqrt{\frac{2k}{m}(1 - \frac{r_0}{R})}.$$

Q6 20 pts. This problem requires NO special knowledge of particle physics. In an elastic collision (i.e., a collision where kinetic energy is conserved) between an incident electron-type neutrino and a nucleon initially at rest in the laboratory, the incident energy of the neutrino E_ν can be determined from the total energy E_e and the scattering angle θ of the electron produced in the interaction. That is, the elastic collision between the incident neutrino and the at-rest nucleon produces a final state electron and a moving nucleon. The neutrino disappears in the interaction. See the figure below. Let θ be the angle between the initial direction of the neutrino and the direction of the final state electron, all measured in the laboratory frame. Furthermore, let E_N be the total energy of the final state nucleon. You can assume that the neutrino is massless and that the electron's mass is much less than its energy, $m_e c^2 \ll E_e$. However, do not assume that the nucleon mass is much less than its total energy. Show that:

$$E_\nu = \frac{E_e}{1 - \frac{E_e}{m_N c^2}(1 - \cos\theta)}$$



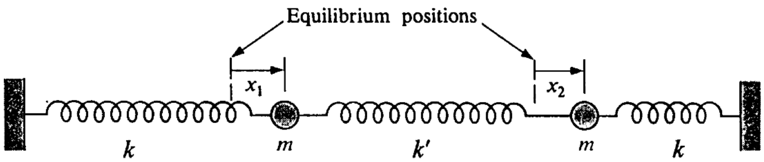
Q7 20 pts. A pendulum of mass m and length r is attached to a support, also of mass m that can move along a frictionless, horizontal track. A spring, of force constant k, is attached to the support and to a nearby wall. See the figure below. The values of the mass m, spring constant k, and the pendulum length r are such that $2mg = kr$, that is, if the spring were to support the weight of the two masses, it would be stretched a distance equal to the length of the pendulum. Find the normal mode frequencies of the system.



Closed-book. The pass is set at 60/100.

Name: ____________________

1. (20 points) Two masses each with mass m are connected by 3 springs and move along a straight line. Assume x_1 and x_2 are displacements from equilibrium positions. The two springs that connect to a mass to the wall have the same spring constant k. The spring that connects the two masses has a spring constant k'. Find out the normal modes.



2. (15 points) Obtain Hamilton’s equations of motion for a one-dimensional harmonic oscillator.

3. (15 points) A uniform chain of length $l = \pi R$ and mass M is placed on the upper half of a uniform thin disc of radius R and mass M. The disc is placed vertically and can rotated freely about its center that is fixed in space. With a small disturbance the chain starts to fail. Find out the angular velocity of the disc when the chain leaves it.

4. (20 points) A bug of mass m crawls radially outwards with a constant speed v' on a disc that rotates with a constant angular velocity ω about a vertical axis. The speed v' is relative to the center of the disc. Assume a coefficient of static friction μ_s, find out where on the disc the bug starts to slip.

5. (15 points) A particle is placed on top of a smooth sphere of radius R. If the particle is slightly disturbed, at what point will it leave the sphere?

6. (15 points) A particle of mass m moves in two dimensions under the following potential energy function:

$$V(\vec{r}) = \frac{1}{2}k\left(x^2 + 4y^2\right)$$

Find the resulting motion, given the initial condition at $t = 0 : x = a, y = 0, \dot{x} = 0, \dot{y} = v_0$

Closed-book. The pass is set at 60/100. Each problem has 20 points. Name: ____________________

1. A bullet is fired straight up with an initial speed v_0. Assume quadratic air resistance with the drag constant c_2, show that when the bullet turns back and hits the ground, its speed is $\dfrac{v_0 v_t}{\sqrt{v_0^2 + v_t^2}}$, with $v_t = \sqrt{mg/\ c_2}$, the terminal speed

2. A mass m moves along the x-axis subject to an attractive force given by $17\beta^2 mx/2$ and a retarding force given by $3\beta m\dot{x}$, where x is its distance from the origin and β is a constant. A driving force given by $mA \cos \omega t$, where A is a constant, is applied to the particle along the x-axis.

(a) What value of ω results in steady-state oscillations about the origin with maximum amplitude?

(b) What is the maximum amplitude?

3. A smooth rod of length l rotates in a horizontal plane with a constant angular velocity ω about an axis fixed at one end of the rod and is perpendicular to the plane of rotation. A bead of mass m is released at the midpoint of the rod. Calculate (a) the position of the bead on the rod as a function of time; (b) the time and velocity (relative to the rod) when the bead leaves the end of the rod.

4. A satellite is placed into a low-lying orbit by launching it with a two-stage rocket from Cape Canaveral with speed v_0 inclined from the vertical by an elevation angle θ_0. On reaching apogee of the initial orbit, the second stage is ignited, generating a velocity boost Δv_1 that places the payload into a circular orbit

(**a**) Calculate the additional speed boost Δv_1 required of the second stage to make the final orbit circular.

(**b**) Calculate the altitude h of the final orbit. Ignore air resistance and the rotational motion of the Earth. The mass and radius of the Earth are $M_E = 5.98 \times 10^{24}$ kg and $R_E = 6.4 \times 10^3$ km, respectively. Let $v_0 = 6$ km/s and $\theta_0 = 30°$.

5. A uniform chain lies in a heap on a table. If one end is raised vertically with uniform velocity v, show that the upward force that must be exerted on the end of the chain is equal to the weight of a length $z + (v^2/g)$ of the chain, where z is the length that has been uncoiled at any instant.

SMU Department of Physics
QUALIFYING EXAMINATION
Saturday , August 20, 2016
9:00AM to 12:00PM

Classical Mechanics

Three hours are permitted for the completion of this section of the examination. There are 5 problems included in this section, each is worth 20 points. Apportion your time carefully.

Please write only on ONE SIDE of the paper, and DO NOT staple your sheets; they will be scanned in the auto-feeder. Clearly mark your initials on each page, and number each of your pages.

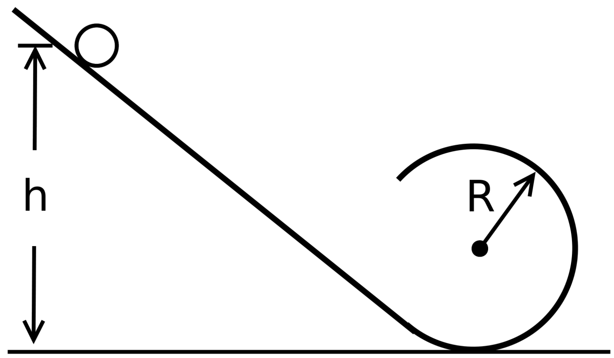
No reference materials or books are permitted. (If you believe there is a key piece of information or formula missing you may ask the proctor to check; but we've checked this carefully.)

Simple calculators are permitted; cell phone calculators can NOT be used.

Questions should be directed to the proctor.

Good Luck!

1. As shown in the figure, a solid brass ball of mass 0.271 g will roll smoothly along a loop-the-loop track when released from rest along a straight section. The circular loop has radius $R = 0.05$ m, and the ball has radius $r \ll R$. What is h if the ball is on the verge of leaving the track when it reaches the top of the loop?



2

A spider is hanging by a silk thread from a tree in Dallas. Find the orientation and the value of the equilibrium angle that the thread makes with the vertical (i.e. with the direction of gravity), taking into account the rotation of the Earth. Assume that the latitude of Dallas is $\theta \approx 33$ and the radius of the Earth is $R \approx 6,400\text{km}$. *[Important; note the spider is stationary. This should simplify the problem. Think.]*

3) Note $s = |p_1 + p_2|^2$ for 4-vectors p_1 and p_2.
(I need to see work; no credit if you happen to have memorized any of these answers.)

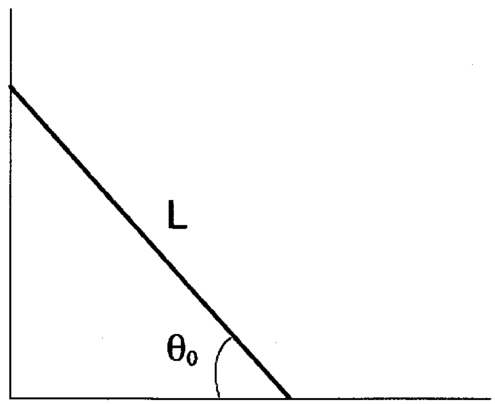
a) Compute the invariant energy $\sqrt{s}$ for a fixed target system for a proton beam of mass m and a proton target of mass m. Express the result in terms of the beam energy E and the proton mass. (Eliminate the proton 3-momentum.) [Please DO NOT neglect m^2 terms.]

b) Compute the invariant energy $\sqrt{s}$ for a collider system for two proton beams of mass m and energy E. Express the result in terms of the beam energy E and the proton mass. [Please DO NOT neglect m^2 terms.]

c) For a fixed target system compute the minimum beam energy E that you need to create anti-protons via the reaction $pp \to ppp\bar{p}$. Express the result in terms of the proton mass.

d) For a collider system compute the minimum beam energy E (each beam has energy E) that you need to create anti-protons via the reaction $pp \to ppp\bar{p}$. Express the result in terms of the proton mass.

4. A uniform ladder of mass M and length L is placed with one end against a frictionless wall and the other end on a frictionless floor. The ladder initially makes an angle θ_0 with the floor, as shown below.



The ladder is released, and slides under the influence of gravity.

(a) Write the Lagrangian for the sliding ladder as a function of θ (the angle of the ladder with respect to the floor).

(b) At what angle θ does the ladder lose contact with the wall?

(Note: The moment of inertia of a uniform rod of mass M and length L rotating about an axis through its center of mass is $I = \frac{1}{12}ML^2$)

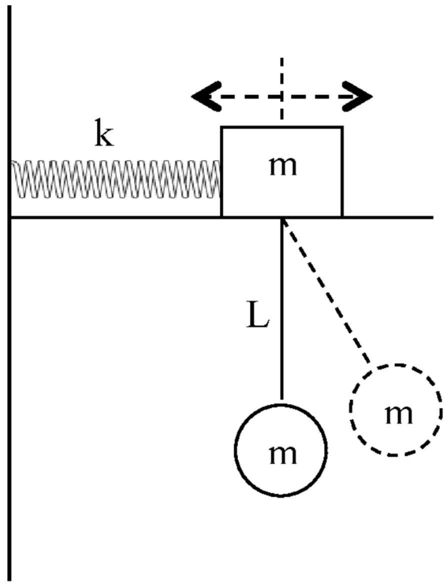
5. A pendulum of length L and mass m is connected to a block also of mass m that is free to move horizontally on a frictionless surface. The block is connected to a wall with a spring of spring constant k. For the special case where

$$\sqrt{\frac{k}{m}} = \sqrt{\frac{g}{L}} = \omega_0 \tag{1}$$

determine:

(a) The frequencies of the normal modes of this system for small oscillations around the equilibrium positions.

(b) The motion of each of the normal modes.



SMU Department of Physics
QUALIFYING EXAMINATION
Saturday , August 19, 2017
9:00AM to 11:00PM

Classical Mechanics

Two hours are permitted for the completion of this section of the examination. There are 5 problems included in this section, each is worth 20 points. Apportion your time carefully.

Please write only on ONE SIDE of the paper, and DO NOT staple your sheets; they will be scanned in the auto-feeder. Clearly mark your initials on each page, and number each of your pages.

No reference materials or books are permitted. (If you believe there is a key piece of information or formula missing you may ask the proctor to check.)

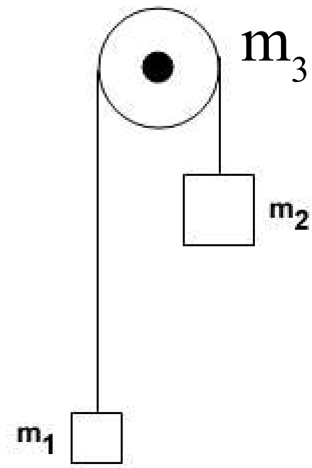
Simple calculators are permitted; <u>cell phone calculators can NOT be used</u>.

Questions should be directed to the proctor.

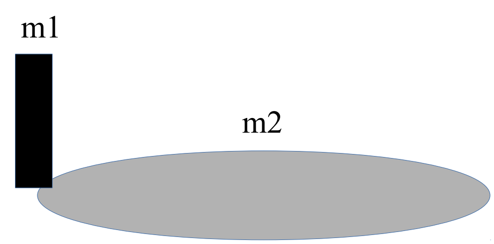
Good Luck!

Problem: 1

a) [10 points] An Atwood machine has masses m_1, m_2 and a frictionless <u>massive</u> pulley of mass m_3 in the shape of a disk ($I=m_3r^2/2$).
Write the equations of motion for the system, and compute the acceleration of the system in terms of the masses.



b) [10 points] A physics student (mass m_1=100kg) is on the edge of a merry-go-round of mass m_2=50Kg, radius r=10m, and $I=m_2r^2/2$. Initally, the merry-go-round is spinnning at w=10 RPM (revolutions per minute). The student then moves from the edge (r=10m) to r=5m. Find the final angular speed of the merry-go-round in both RPM (revolutions per minute) and rad/sec.



Problem: 2

[20 points]

a) [10 points] A pion π^+ at rest decays to a muon μ^+ and a neutrino ν, ($\pi^+ \rightarrow \mu^+ \nu$). In terms of the pion and muon masses (take the neutrino mass to be zero) $\{m_{\pi+}, m_{\mu}, m_{\nu}=0\}$, find the energy, momentum, and β=v/c of the muon.

b) [10 points]The pions are produced in the fixed-target interaction **pp→ πππpp** where a beam of protons hits a stationary target of protons, and produces a final state of 3 pions and 2 protons. Find the minimum energy of the proton beam for this reaction to occur.
[Assume pions of charge (+,-,0) are all the same mass (m_π).]
Express your answer in terms of the proton (m_p) and pion (m_π) masses.

Problem: 3

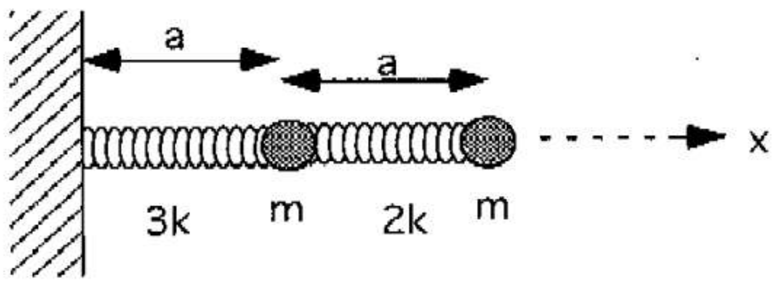
[20 points]

A bullet is fired straight up with an initial speed v_0. Assume quadratic air resistance with the drag constant c_2, show that when the bullet turns back and hits the ground, its speed is

$$\frac{v_0 v_t}{\sqrt{v_0^2 + v_t^2}} \qquad \text{with} \qquad v_t = \sqrt{mg/c_2} \text{ being the terminal speed.}$$

Assuming an air resistance force given by $\mathbf{-c_2\, v^2}$, where c_2 is the drag coefficient.

Problem: 4

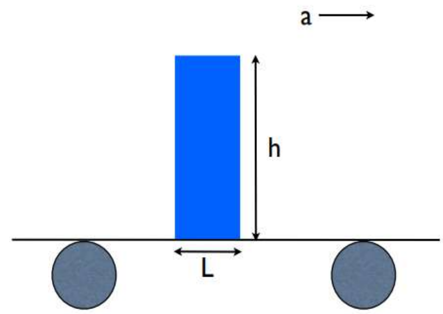


Two objects of mass m are attached to each other by a spring, and the left mass is also attached by a spring to a fixed wall. The springs are of equilibrium length a. The masses are on a frictionless surface and can only move along the x-axis. The left spring has spring constant 3k, and the right has 2k.

a) [10 points] Find the Lagrangian for this system.
b) [10 points] Find the normal modes and their frequencies.

Problem: 5

A truck carries a rectangular block of uniform mass density with height h, a square base with width L, and total mass M. The truck accelerates with constant acceleration "a". Assume the box does not slide.



(a) [6 points] In the frame of reference of the truck, draw all forces and pseudo forces acting on the block (and where they act) when the block just starts tipping over.

(b) [8 points] Calculate for what value of "a" the block starts to tip over.

(c) [4 points] If there is an additional small mass m glued to the center of the top of the block, for what value of "a" does the block start to tip over now?

(d) [2 point] Verify that your answer to (c) is smaller than what you found in part (b).