We already studies

Now we consider

**Drell-Yan Process**

\[ e^+ e^- \]

Important for Tevatron and LHC

Fundamental process for hadron-hadron and lepton-lepton collisions, e.g.,

\[ \text{Drell-Yan} \quad e^+ e^- \rightarrow 2 \text{ jets} \]

A Drell-Yan Example: Discovery of J/ψ

The Process: \( p + Be \rightarrow e^+ e^- X \)

at BNL AGS

very narrow width \( \Rightarrow \) long lifetime

Drell-Yan and \( e^+ e^- \) have an interesting historical relation
We'll look at Drell-Yan
Specifically W/Z production

Schematically:
\[ d\sigma(q\bar{q} \rightarrow l^+ l^-) = d\sigma(q\bar{q} \rightarrow \gamma^*) \times d\sigma(\gamma^* \rightarrow l^+ l^-) \]

For example:
\[ \frac{d\sigma}{dQ^2 dy}(q\bar{q} \rightarrow l^+ l^-) = \frac{d\sigma}{dt}(q\bar{q} \rightarrow \gamma^*) \times \frac{\alpha}{3\pi Q^2} \]
Partonic CMS has longitudinal momentum w.r.t. the hadron frame.

\[ p_{12} = (p_1 + p_2) = (E_{12}, 0, 0, p_L) \]
\[ E_{12} = \frac{\sqrt{s}}{2} (x_1 + x_2) \]
\[ p_L = \frac{\sqrt{s}}{2} (x_1 - x_2) \equiv \frac{\sqrt{s}}{2} x_F \]

The rapidity is defined as:

\[ y = \frac{1}{2} \ln \left( \frac{E_{12} + p_L}{E_{12} - p_L} \right) \]

Partonic CMS has longitudinal momentum w.r.t. the hadron frame.

\[ k_1 = x_1 P_1 \quad k_2 = x_2 P_2 \]

\[ p_1 = x_1 P_1 \quad p_2 = x_2 P_2 \]

\[ x_F = \frac{1}{2} \ln \left( \frac{x_1}{x_2} \right) \]

\[ \tau = x_1 x_2 = \frac{s}{\sqrt{s}} \equiv \frac{Q^2}{s} \]

\[ y = \frac{1}{2} \ln \left( \frac{x_1}{x_2} \right) \]

\[ \tau = x_1 x_2 \]

\[ x_{1,2} = \sqrt{\tau} e^{\pm y} \]
Kinematics for W production at Tevatron & LHC

How do we measure the W-boson mass?

Can't measure W directly
Can't measure ν directly
Can't measure longitudinal momentum

We can measure the $P_T$ of the lepton

The Jacobian Peak

Suppose lepton distribution is uniform in θ

The dependence is actually $(1+\cos\theta)^2$, but we'll worry about that later

What is the distribution in $P_T$?

We find a peak at $P_T^{\max} \approx M_W/2$

Drell-Yan Cross Section and the Scaling Form

Using: $\hat{\sigma}_0 = \frac{4\pi\alpha^2}{9\hat{s}} Q_i^2$ and $\delta(Q^2-\hat{s}) = \frac{1}{sx_1} \delta(x_2-\frac{\tau}{x_1})$

we can write the cross section in the scaling form:

$Q^4 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9} \sum_{q,q'} Q_i^2 \int_0^{1/sx_1} \frac{dx_1}{x_1} \tau \left| q(x_1)\bar{q}(\tau/x_1) + q(x_1)\bar{q}(\tau/x_1) \right|$
\[ e^+ e^- \quad \text{R ratio} \]

\[ R = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)} \]

\[ e^+ e^- \quad \text{NLO corrections} \]

Define the energy fractions \( E_i \):

\[ x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i \cdot q}{s} \]

Energy Conservation:

\[ \sum_i x_i = 2 \]

Range of \( x \):

\[ x_i \subset [0, 1] \]

Exercise: show 3-body phase space is flat in \( dx_1 dx_2 \).
3-Particle Phase Space

\[ x_i = \frac{E_i}{\sqrt{s}/2} \]
\[ x_i \in [0, 1] \]
\[ x_1 + x_2 + x_3 = 2 \]

\[ d\Gamma \sim dx_1 \, dx_2 \]

3-Particle Configurations

\[ \sigma_0 = \frac{4\pi\alpha^2}{s} \sum c_i^2 \quad C_F = 4/3 \]

After symmetrization

\[ \frac{1}{\sigma_0} \frac{d\sigma}{dx_1 \, dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2 + x_3^2}{(1 - x_1)(1 - x_2)(1 - x_3)} \]

Singularities cancel between 2-particle and 3-particle graphs

\[ \sigma_2^{(\epsilon)} = \sigma_0 \, C_F \, \frac{\alpha_s}{\pi} (...) \left[ +\frac{1}{\epsilon^2} + \frac{-3}{2\epsilon} + \frac{\pi^2}{2} - 4 \right] \]

\[ \sigma_3^{(\epsilon)} = \sigma_0 \, C_F \, \frac{\alpha_s}{\pi} (...) \left[ +\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{-\pi^2}{2} - \frac{19}{4} \right] \]

\[ \sigma_2^{(\epsilon)} + \sigma_3^{(\epsilon)} = \sigma_0 \, C_F \, \frac{\alpha_s}{\pi} (...) \left[ 0 + 0 + 0 + \frac{-35}{4} \right] \]

Same result with gluon mass regularization

\[ e^+ e^- \]

Differential Cross Sections
Differential Cross Section

What do we do about soft and collinear singularities????

Introduce the concept of “Infrared Safe Observable”

The soft and collinear singularities will cancel **ONLY**
if the physical observables are appropriately defined.

Infrared Safe Observables

Observables must satisfy the following requirements:

**Soft**

$$\mathcal{O}_{n+1}(p_1, \ldots, p_n, p_s) \rightarrow \mathcal{O}_n(p_1, \ldots, p_n)$$

if $$p_s \rightarrow 0$$

**Collinear**

$$\mathcal{O}_{n+1}(p_1, \ldots, p_n, p_a, p_b) \rightarrow \mathcal{O}_n(p_1, \ldots, p_n + p_b, \ldots, p_n)$$

if $$p_a \parallel p_b$$

Examples: Infrared Safe Observables

- Event shape distributions
- Jet Cross sections

Un-Safe Infrared Observables:

- Momentum of the hardest particle
  (affected by collinear splitting)
- 100% isolated particles
  (affected by soft emissions)
- Particle multiplicity
  (affected by both soft & collinear emissions)
Infrared Safe Observables: Define Jets

**Soft**

if \( p_s \rightarrow 0 \)

\[ \mathcal{O}_{n+1}(p_1, \ldots, p_n, p_s) \rightarrow \mathcal{O}_n(p_1, \ldots, p_n) \]

**Collinear**

if \( p_a \parallel p_b \)

\[ \mathcal{O}_{n+1}(p_1, \ldots, p_n, p_b, \ldots, p_n) \rightarrow \mathcal{O}_n(p_1, \ldots, p_n + p_b, \ldots, p_n) \]

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Jet Cone

\[ R^2 = (\Delta \eta)^2 + (\Delta \phi)^2 \]

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**Pseudo-Rapidity vs. Angle**

\[ \eta = -\ln(\tan(\theta/2)) \]

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Jet Cone

\[ R^2 = (\Delta \eta)^2 + (\Delta \phi)^2 \]
**HOMEWORK: Jet Cone Definition**

**PROBLEM #2:** In a Tevatron detector, consider two particles traveling in the transverse direction:

\[
p_1^\perp = \{ E, 100, 0, 1 \}
\]

\[
p_2^\perp = \{ E, 100, 1, 0 \}
\]

where the components are expressed in GeV units. \( E \) is defined such that the particles are massless.

a) Compute \( E \).

b) For each particle, compute the pseudorapidity \( \eta \) and azimuthal angle \( \phi \).

c) Explain how the above exercise justifies the correct jet radius definition to be:

\[
R = \sqrt{\eta^2 + \phi^2}
\]

In particular, why is the above correct and \( R = \sqrt{\eta^2 + 2\phi^2} \), for example, incorrect.
\[ P_{\mu} = \{P_t, P_x, P_y, P_z\} \]
\[ P_{\mu} = \{P_+, \mathbf{P}_\perp, P_-\} \]
\[ \mathbf{P}_\perp = \{P_x, P_y\} \]
\[ P_{\pm} = \frac{1}{\sqrt{2}} (P_t \pm P_z) \]

1) Compute the metric \( g_{\mu\nu} \) in the light-cone frame, and compute \( \mathbf{P}_1 \cdot \mathbf{P}_2 \) in terms of the light-cone components.

2) Compute the boost matrix \( B \) for a boost along the \( z \)-axis, and show the light-cone vector transforms in a particularly simple manner.

3) Show that a boost along the \( z \)-axis uniformly shifts the rapidity of a vector by a constant amount.

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**HOMEWORK: Rapidity vs. Pseudo-Rapidity**

**PROBLEM #1**: Consider the rapidity \( y \) and the pseudo-rapidity \( \eta \):

\[ y = \frac{1}{2} \ln \left( \frac{E + P_z}{E - P_z} \right) \]
\[ \eta = -\ln \left[ \tan \left( \frac{\theta}{2} \right) \right] \]

a) Make a parametric plot of \( \{y, \eta\}\) as a function of the particle.

b) Show that in the limit \( m \to 0 \) that \( y \to \eta \).

c) Make a table of \( \eta \) for \( \theta = [0^\circ, 180^\circ] \) in steps of 5

d) Make a table of \( \theta \) for \( \eta = [0, 10] \)

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**Infrared Safe Observables: Define Jets**

**Problem:**
The cone definition is simple, **BUT** it is too simple

\[ R^2 = (\Delta\eta)^2 + (\Delta\phi)^2 \]

 Such configurations can be mis-identified as a 3-jet event

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See talk by 
Dave Soper & 
Andrew Larkoski
End of lecture 4: Recap

Drell-Yan: Tremendous discovery potential

Need to compute 2 initial hadrons

e\text{e}^+\text{e}^- processes:

Total Cross Section: singularity

Infrared Safe Observables
Stable under soft and collinear emissions

Jet definition
Cone definition is simple:
... it is TOO simple

Final Thoughts

Scaling, Dimensional Analysis, Factorization, Regularization & Renormalization, Infrared Safety...

Can you find the Nobel Prize???

Hi ET
Jet Excess

CDF Collaboration, PRL 77, 438 (1996)
H1 Collaboration, ZPC74, 191 (1997)
ZEUS Collaboration, ZPC74, 207 (1997)

Hi Q Excess

Can you find the Nobel Prize???

Mmm
GeV
cross section
p + N \rightarrow m^+ m^- + X

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and the many web pages where I borrowed my figures...
Keep an open mind!!!

END OF LECTURE

4