Homework #2

```
Off[General::spell];
Off[General::spell1];
Off[Solve::ifun];
```

■ Problem 1: The right way:

Note: We will use Hobbiton as our vertex, and compute 3 angles. The angle from Erebor to Dagorlad, Dagorlad to Corsairs, and Erebor to Corsairs. If the sum of the first two angles adds up to the third, the space is flat.

Import["/Net/thy2/olness/3368/ttt.bmp"];

Show[%];

```
rule = {
  Thread[{a, b, c} → {813, 960, 735}],
  Thread[{a, b, c} → {960, 1112, 780}],
  Thread[{a, b, c} → {813, 1112, 1498}]
 }
```

{{$a \to 813$, $b \to 960$, $c \to 735$}, {$a \to 960$, $b \to 1112$, $c \to 780$}, {$a \to 813$, $b \to 1112$, $c \to 1498$}}

```
eq1 = {c^2 == a^2 + b^2 - 2 a b Cos[γ]}
```

$\{c^2 == a^2 - 2\, b \cos(\gamma)\, a + b^2\}$

```
sol1 = Solve[eq1, γ][[2]]
```

$\left\{\gamma \to \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2\, a\, b}\right)\right\}$

```
result = γ / Degree /. sol1 /. rule // N
```

{48.1057, 43.4594, 101.048}

```
result[[1]] + result[[2]]
```

91.5651

■ Problem 1: The wrong way:

```
rule = {Thread[{a, b, c} → {813, 735, 960}], Thread[{a, b, c} -> RotateLeft[{813, 735, 960}]],
  Thread[{a, b, c} -> RotateRight[{813, 735, 960}]]}
```

{{$a \to 813$, $b \to 735$, $c \to 960$}, {$a \to 735$, $b \to 960$, $c \to 813$}, {$a \to 960$, $b \to 813$, $c \to 735$}}

```
eq1 = {c^2 == a^2 + b^2 - 2 a b Cos[γ]}
```

$\{c^2 == a^2 - 2\, b \cos(\gamma)\, a + b^2\}$

```
sol1 = Solve[eq1, γ][[2]]
```

$\left\{\gamma \to \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2\, a\, b}\right)\right\}$

```
result = γ / Degree /. sol1 /. rule // N
```

{76.4704, 55.4239, 48.1057}

```
Plus @@ result
```

180.

■ Problem 3:

```
<< Miscellaneous`PhysicalConstants`
```

```
<< Miscellaneous`Units`
```

```
? Miscellaneous`PhysicalConstants`*
```

```
? PlanckConstantReduced
```

PlanckConstantReduced is PlanckConstant/(2 Pi), a universal constant.

```
PlanckConstantReduced
```

1.05457×10^{-34} Joule Second

```
ProtonMass
```

1.67262×10^{-27} Kilogram

```
proton = PlanckConstantReduced / 2 / (2 ProtonMass) // Simplify
```

$$\frac{1.57623 \times 10^{-8} \text{ Joule Second}}{\text{Kilogram}}$$

```
MKS[proton / SpeedOfLight^2] //. {Joule → Newton Meter, Newton → Kilogram Meter / Second^2}
```

1.75379×10^{-25} Second

```
% SpeedOfLight
```

5.25772×10^{-17} Meter

```
electron = PlanckConstantReduced / 2 / (2 ElectronMass) // Simplify
```

$$\frac{0.0000289419 \text{ Joule Second}}{\text{Kilogram}}$$

```
MKS[electron / SpeedOfLight^2] //. {Joule → Newton Meter, Newton → Kilogram Meter / Second^2}
```

3.22022×10^{-22} Second

```
% SpeedOfLight
```

9.65398×10^{-14} Meter

■ Problem 4:

```
result = A / (4 π r^2) * 4 π + π
```

$$\frac{A}{r^2} + \pi$$

```
result /. {A → 0}
```

$$\pi$$

```
result /. {A -> 4 π r^2}
```

$$5\,\pi$$

■ Problem 5:

```
2^16
```

65536

■ Problem 6

```
inside[n_] = 2 π^(n/2) r^n / (n Gamma[n / 2])
```

$$\frac{2\,\pi^{n/2}\,r^n}{n\,\Gamma\left(\frac{n}{2}\right)}$$

```
outside[n_] = D[inside[n], r]
```

$$\frac{2\,\pi^{n/2}\,r^{n-1}}{\Gamma\left(\frac{n}{2}\right)}$$

```
Table[{n, inside[n], outside[n]}, {n, 1, 10}]
```

$$\begin{pmatrix} 1 & 2\,r & 2 \\ 2 & \pi\,r^2 & 2\,\pi\,r \\ 3 & \frac{4\,\pi\,r^3}{3} & 4\,\pi\,r^2 \\ 4 & \frac{\pi^2\,r^4}{2} & 2\,\pi^2\,r^3 \\ 5 & \frac{8\,\pi^2\,r^5}{15} & \frac{8\,\pi^2\,r^4}{3} \\ 6 & \frac{\pi^3\,r^6}{6} & \pi^3\,r^5 \\ 7 & \frac{16\,\pi^3\,r^7}{105} & \frac{16\,\pi^3\,r^6}{15} \\ 8 & \frac{\pi^4\,r^8}{24} & \frac{\pi^4\,r^7}{3} \\ 9 & \frac{32\,\pi^4\,r^9}{945} & \frac{32\,\pi^4\,r^8}{105} \\ 10 & \frac{\pi^5\,r^{10}}{120} & \frac{\pi^5\,r^9}{12} \end{pmatrix}$$

```
Integrate[1, {r, -r, r}]
```

$$2\,r$$

```
D[%, r]
```

$$2$$

```
Integrate[ r, {r, 0, r}, {ϕ, 0, 2 π}]
```

$$\pi r^2$$

```
D[%, r]
```

$$2 \pi r$$

```
Integrate[ r^2 Sin[θ], {r, 0, r}, {ϕ, 0, 2 π}, {θ, 0, π}]
```

$$\frac{4 \pi r^3}{3}$$

```
D[%, r]
```

$$4 \pi r^2$$

```
Integrate[ r^3 Sin[θ]^2 Sin[ρ], {r, 0, r}, {ϕ, 0, 2 π}, {θ, 0, π}, {ρ, 0, π}]
```

$$\frac{\pi^2 r^4}{2}$$

```
D[%, r]
```

$$2 \pi^2 r^3$$

```
Integrate[ r^4 Sin[θ]^3 Sin[ρ]^2 Sin[ω], {r, 0, r}, {ϕ, 0, 2 π}, {θ, 0, π}, {ρ, 0, π}, {ω, 0, π}]
```

$$\frac{8 \pi^2 r^5}{15}$$

```
D[%, r]
```

$$\frac{8 \pi^2 r^4}{3}$$