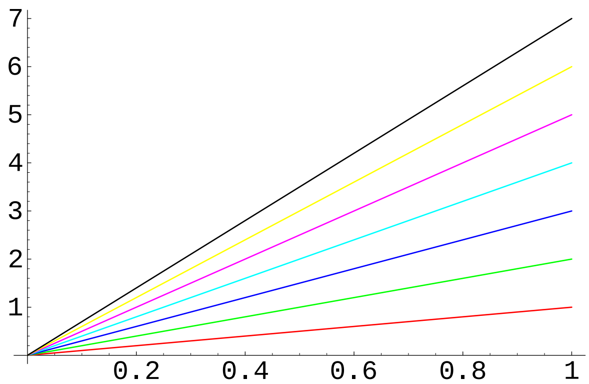
Homework #3:

```
Off[Clear::wrsym]
Off[General::spell]
Off[General::spell1]
```

■ Setup:

```
colors = {red, green, blue, cyan, magenta, yellow, black} =
   {RGBColor[1, 0, 0]
    , RGBColor[0, 1, 0]
    , RGBColor[0, 0, 1]
    , CMYKColor[1, 0, 0, 0]
    , CMYKColor[0, 1, 0, 0]
    , CMYKColor[0, 0, 1, 0]
    , CMYKColor[0, 0, 0, 1]
   };
Plot[Table[i x, {i, 1, 7}] // Evaluate, {x, 0, 1}, PlotStyle → colors];
```




```
Protect[{colors, red, green, blue, cyan, magenta, yellow, black}];
```

■ Problem #1:

■ Right−HandedCoordinate system:

```
{x, y, z} = DiagonalMatrix[{1, 1, 1}]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
s = Cross[x, y]
```

{0, 0, 1}

```
s.z
```

1

Left−HandedCoordinate system: Invert all axes.

```
{x, y, z} = - DiagonalMatrix[{1, 1, 1}]
```

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
s = Cross[x, y]
```

{0, 0, 1}

```
s.z
```

−1

Problem #2:(TEXT #2)

```
Clear["Global`*"]
```

```
<< Miscellaneous`PhysicalConstants`
<< Miscellaneous`Units`
```

```
? Miscellaneous`PhysicalConstants`*
```

```
AccelerationDueToGravity    ElectronCharge              IcePoint
PlanckConstant              SolarLuminosity
AgeOfUniverse               ElectronComptonWavelength   MagneticFluxQuantum
PlanckConstantReduced       SolarRadius
AvogadroConstant            ElectronGFactor             MolarGasConstant            PlanckMass
SolarSchwarzschildRadius
BohrRadius                  ElectronMagneticMoment      MolarVolume
ProtonComptonWavelength     SpeedOfLight
BoltzmannConstant           ElectronMass                MuonGFactor
ProtonMagneticMoment        SpeedOfSound
ClassicalElectronRadius     FaradayConstant             MuonMagneticMoment          ProtonMass
StefanConstant
CosmicBackgroundTemperature FineStructureConstant       MuonMass
QuantizedHallConductance    ThomsonCrossSection
DeuteronMagneticMoment      GalacticUnit                NeutronComptonWavelength
RydbergConstant             VacuumPermeability
DeuteronMass                GravitationalConstant       NeutronMagneticMoment
SackurTetrodeConstant       VacuumPermittivity
EarthMass                   HubbleConstant              NeutronMass
SolarConstant               WeakMixingAngle
EarthRadius
```

The formula for resolving power is: λ /a≃ Sin[θ]≃θ, where a is the apperature. From the text, Fig.1, the parallax angle is: θ = d/D where d = 1a.u., and D is the distance to the star.

```
eq1 = λ / a == 1.22 θ
eq2 = θ == d / D
```

$\frac{\lambda}{a} == 1.22\,\theta$

$\theta == \frac{d}{D}$

```
sol1 = Solve[{eq1, eq2}, {D, θ}][[1]]
```

$\left\{D \to \frac{1.22\,a\,d}{\lambda}, \theta \to \frac{0.819672\,\lambda}{a}\right\}$

```
EarthSunDistance = 500. Second * SpeedOfLight
```

1.49896×10^{11} Meter

```
distance = D /. sol1 /. {a → 200 Inch, d → EarthSunDistance  , λ → 5000 Angstrom}
```

$\frac{7.31494 \times 10^{9}\ \text{Inch Meter}}{\text{Angstrom}}$

```
distance = MKS[distance]
```

1.85799×10^{18} Meter

```
Convert[distance, LightYear]
```

196.395 LightYear

■ Problem #3:(TEXT #3)

```
Clear["Global`*"]
```

```
<< Graphics`Graphics`
```

```
eq1 = ℓ / ℓret == 100.^((mref - M) / 5)
```

$\frac{3.96825 \times 10^{7}\ \text{Meter}^2\ \ell}{\text{Watt}} == 100.^{\frac{1}{5}(m_{\text{ref}} - M)}$

```
ℓret = 2.52 * 10^-8 Watt / Meter^2
```

$\frac{2.52 \times 10^{-8}\ \text{Watt}}{\text{Meter}^2}$

```
sol1 = Solve[{eq1}, {ℓ}][[1]]
```

$\left\{\ell \to \frac{2.52 \times 10^{-8}\ 100.^{\frac{1}{5}(m_{\text{ref}} - M)}\ \text{Watt}}{\text{Meter}^2}\right\}$

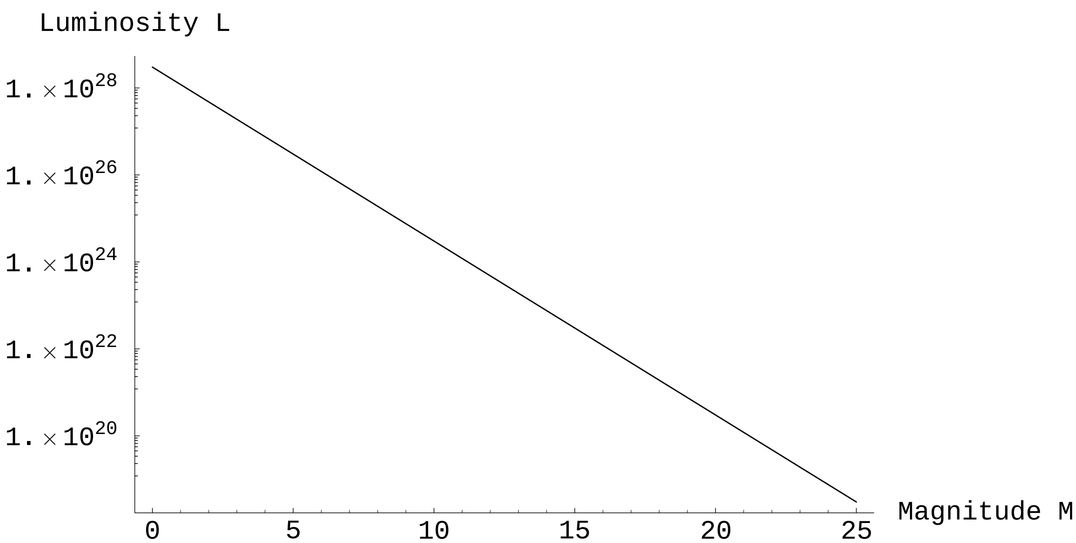
```
L = ℓ 4 π (10 Parsec)^2
```

$400\ \text{Parsec}^2\ \pi\,\ell$

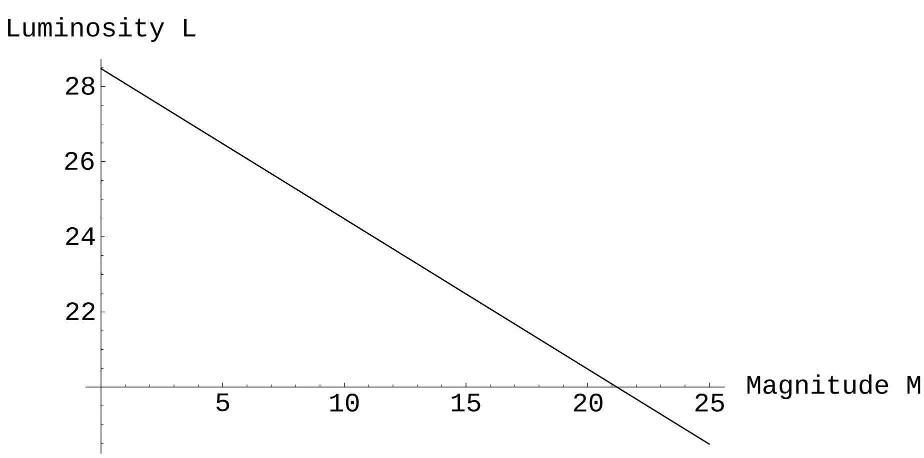
```
L = MKS[L /. sol1 /. {m_ref → 0}]
```

$3.01521 \times 10^{28}\ 100.^{-M/5}$ Watt

```
LinearLogPlot[L / Watt // Evaluate,
  {M, 0, 25}, AxesLabel → {"Magnitude M", "Luminosity L"}];
```




```
Plot[Log[10., L / Watt ] // Evaluate,
  {M, 0, 25}, AxesLabel → {"Magnitude M", "Luminosity L"}];
```



■ Problem #4:(TEXT #4)

```
eq1 = m - M == 5 Log[10., D / 10 / Parsec] (* eq .2 .2 .8  *)
```

$$m - M == 2.17147 \log\left(\frac{D}{10\,\text{Parsec}}\right)$$

```
sol1 = Solve[eq1, m][[1]]
```

$$\left\{m \to 1.\,M + 2.17147 \log\left(\frac{0.1\,D}{\text{Parsec}}\right)\right\}$$

```
m /. sol1 /. {M → 4.72, D → 3 * 10^4 Parsec}
```

22.1056

Yes, m is just above 22.7. If we increase D by 10^3

```
m /. sol1 /. {M → 4.72, D → 10^3 * 3 * 10^4 Parsec}
```

37.1056

■ Problem #5:(TEXT #5)

```
Clear["Global`*"]
```

■ Outside:

```
eq1 = m v^2 / r == G M m / r^2
```

$$\frac{m\,v^2}{r} == \frac{G\,m\,M}{r^2}$$

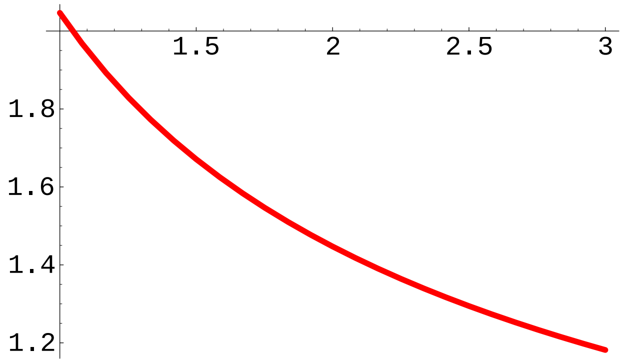
```
sol1 = Solve[eq1, v][[2]]
```

$$\left\{v \to \frac{\sqrt{G}\,\sqrt{M}}{\sqrt{r}}\right\}$$

```
rules = {G → 1, ρ → 1, M → 4 / 3 π}
```

$$\left\{G \to 1,\ \rho \to 1,\ M \to \frac{4\,\pi}{3}\right\}$$

```
p1 = Plot[v /. sol1 //. rules // Evaluate, {r, 1, 3}, PlotStyle → {Thickness[0.010], red}];
```



■ Inside:

```
eq1 = m v^2 / r == G M m / r^2 //. {M → ρ Volume, Volume → (4 / 3) π r^3}
```

$$\frac{m v^2}{r} == \frac{4}{3} G m \pi r \rho$$

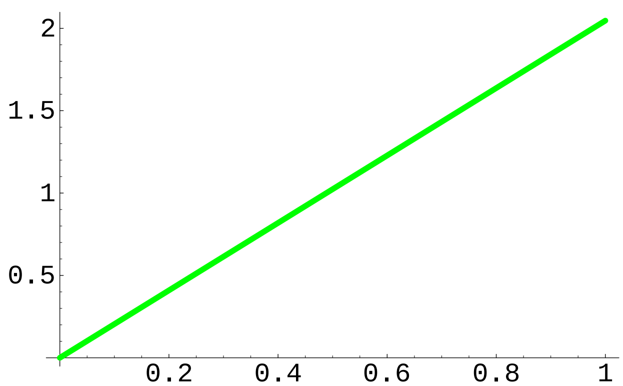
```
sol1 = Solve[eq1, v][[2]]
```

$$\left\{v \to 2 \sqrt{G} \sqrt{\frac{\pi}{3}} r \sqrt{\rho}\right\}$$

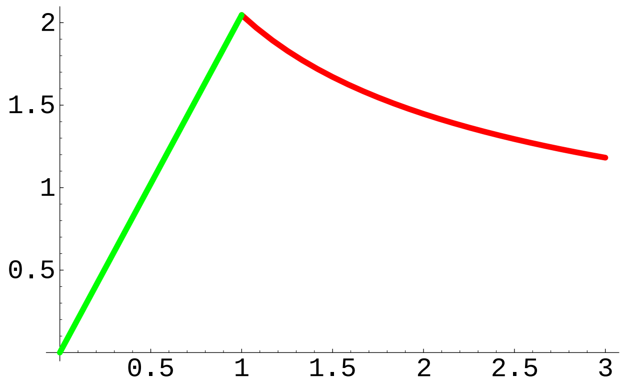
```
v /. sol1 //. rules
```

$$2 \sqrt{\frac{\pi}{3}} r$$

```
p2 = Plot[v /. sol1 //. rules // Evaluate,
   {r, 0, 1}, PlotStyle → {Thickness[0.010], green}];
```



```
Show[p1, p2];
```



■ Problem #6:(TEXT #8)

```
Clear["Global`*"]

<< Miscellaneous`PhysicalConstants`

ϵ0 = VacuumPermittivity;
(* k= 1/(4 π ϵ0) *)

forceE = k q1 q2 / r^2
forceG = G m1 m2 / r^2
```

$\frac{k\, q_1\, q_2}{r^2}$

$\frac{G\, m_1\, m_2}{r^2}$

```
moonMass = 7.36 * 10^22 Kilogram;
earthMoonDistance = 3.84 * 10^8 Meter;

lhs = forceG /.
  {G → GravitationalConstant, m1 → EarthMass, m2 → moonMass, r → earthMoonDistance}
```

1.98956×10^{20} Newton

```
rhs = forceE //. {q1 → κ m1 / ProtonMass * ElectronCharge,
   q2 → κ m2 / ProtonMass * ElectronCharge, m1 → EarthMass,
   m2 → moonMass,
   k → 1 / (4 π ϵ0),
   r → earthMoonDistance,
   Ampere → Coulomb / Second,
   Coulomb → Joule / Volt,
   Joule → Newton Meter
  }
```

2.45881×10^{56} Newton κ^2

```
Solve[lhs == rhs, κ][[2]]
```

$\{\kappa \to 8.99531\times10^{-19}\}$

■ Problem #7:

```
Clear["Global`*"]
```

```
<< Miscellaneous`PhysicalConstants`
```

```
eq1 = 1 / 2 m v^2 - G M m / r == 0
```

$$\frac{m v^2}{2} - \frac{G m M}{r} == 0$$

```
eq2 = v == H r
```

$v == H r$

```
eq3 = ρ == M / volume
```

$$\rho == \frac{M}{\text{volume}}$$

```
volume = (4 / 3) π r^3
```

$$\frac{4 \pi r^3}{3}$$

```
sol1 = Solve[{eq1, eq2, eq3}, {ρ, M, v}][[1]]
```

$$\left\{M \to \frac{H^2 r^3}{2 G}, \rho \to \frac{3 H^2}{8 G \pi}, v \to H r\right\}$$

```
{GravitationalConstant, HubbleConstant}
```

$$\left\{\frac{6.67266 \times 10^{-11} \text{ Meter}^2 \text{ Newton}}{\text{Kilogram}^2}, \frac{3.2 \times 10^{-18}}{\text{Second}}\right\}$$

```
ρCRITICAL = ρ /. sol1 /. {G → GravitationalConstant, H -> HubbleConstant} /.
  {Newton → Kilogram Meter / Second^2}
```

$$\frac{1.83182 \times 10^{-26} \text{ Kilogram}}{\text{Meter}^3}$$

```
ρGAL = 3. 10^-28 Kilogram / Meter^3
```

$$\frac{3. \times 10^{-28} \text{ Kilogram}}{\text{Meter}^3}$$

```
ratio = ρGAL / ρCRITICAL
```

0.0163772

```
ratio * 100 (* Convert to Percent *)
```

1.63772