Homework #4:

```
Off[Clear::wrsym]
Off[General::spell]
Off[General::spell1]
```

■ Setup:

■ Problem #1: Parallax for Pluto

```
Clear["Global`*"]
```

```
<< Miscellaneous`PhysicalConstants`
<< Miscellaneous`Units`
```

```
?Miscellaneous`PhysicalConstants`*
```

The formula for resolving power is: λ /a$\simeq$ 1.22 Sin[θ]$\simeq$ 1.22 θ, where a is the apperature. From the text, Fig.1, the parallax angle is: θ = d/D where d = 5.91 * 10^{12} m, and D is the distance to the star.

```
eq1 = λ / a == 1.22 θ
eq2 = θ == d / D
```

$$\frac{\lambda}{a} == 1.22\,\theta$$

$$\theta == \frac{d}{D}$$

```
sol1 = Solve[{eq1, eq2}, {D, θ}][[1]]
```

$$\left\{D \to \frac{1.22\,a\,d}{\lambda},\ \theta \to \frac{0.819672\,\lambda}{a}\right\}$$

■ For Earth:

```
distance = D /. sol1 /. {a → 200 Inch, d → 1.5 * 10^11 Meter , λ → 5000 Angstrom}
```

$$\frac{7.32\times 10^{9}\ \text{Inch Meter}}{\text{Angstrom}}$$

```
distance = MKS[distance]
```

1.85928×10^{18} Meter

```
Convert[distance, LightYear]
```

196.531 LightYear

```
Convert[distance, Parsec]
```

60.2547 Parsec

For Pluto:

```
distance = D /. sol1 /. {a → 200 Inch, d → 5.91 * 10^12 Meter , λ → 5000 Angstrom}
```

$$\frac{2.88408\times 10^{11}\ \text{Inch Meter}}{\text{Angstrom}}$$

```
distance = MKS[distance]
```

7.32556×10^{19} Meter

```
Convert[distance, LightYear]
```

7743.32 LightYear

```
Convert[distance, Parsec]
```

2374.04 Parsec

Ratio:

```
{5.91 * 10^12 Meter / (1.5 * 10^11 Meter), 1595 Parsec / (40.48 Parsec)}
```

{39.4, 39.4022}

Problem #2:
The sun at 4 light−years

```
Clear["Global`*"]

eqs =
 {ℓ == L / (4 π d^2),
  m == -2.5 Log[10., ℓ / ℓ0],
  M == m - 5 Log[10., d / d0]}
```

$$\left\{\ell == \frac{L}{4 d^2 \pi}, m == -1.08574 \log\left(\frac{\ell}{\ell 0}\right), M == m - 2.17147 \log\left(\frac{d}{\text{d0}}\right)\right\}$$

```
sol1 = Solve[eqs, {L, m, ℓ}][[1]] // Simplify
```

$$\left\{L \to 12.5664\, \text{d0}^2\, e^{-0.921034 M}\, \ell 0, m \to 1.\, M + 2.17147 \log\left(\frac{d}{\text{d0}}\right), \ell \to \frac{1.\, e^{-0.921034 M}\, \ell 0}{\left(\frac{d}{\text{d0}}\right)^{2.}}\right\}$$

```
array = {d / parsec, L, ℓ, M, m} /. sol1
```

$$\left\{\frac{d}{\text{parsec}}, 12.5664\, \text{d0}^2\, e^{-0.921034 M}\, \ell 0, \frac{1.\, e^{-0.921034 M}\, \ell 0}{\left(\frac{d}{\text{d0}}\right)^{2.}}, M, 1.\, M + 2.17147 \log\left(\frac{d}{\text{d0}}\right)\right\}$$

```
parsec = 3.09 * 10^16;
lightYear = parsec / 3.26;
d0 = 10 parsec;
ℓ0 = 2.52 * 10^-8;
earthSunDistance = 1.5 * 10^11;
```

```
array /. {M → 4.72, d → 4 lightYear}
```

$\{1.22699, 3.91316\times10^{26}, 2.16629\times10^{-8}, 4.72, 0.164212\}$

■ Problem #3: Table of a star at different distances

```
Clear["Global`*"]
```

```
parsec = Convert[1.0 Parsec, Meter]
```

3.0857×10^{16} Meter

```
lightYear = Convert[1.0 LightYear, Meter]
```

9.4605×10^{15} Meter

```
d0 = 10 parsec;
ℓ0 = 2.52 * 10^-8 Watt / Meter^2;
```

```
solL = Solve[ℓ == L / (4 π d^2), L][[1]]
```

$\{L \to 4\, d^2\, \pi\, \ell\}$

```
L0 = L /. solL /. {d → d0, ℓ → ℓ0}
```

3.01521×10^{28} Watt

```
ℓ = L / (4 π d^2)
m = -2.5 Log[10., ℓ / ℓ0]
M = m - 5 Log[10., d / d0]
```

$$\frac{L}{4\, d^2\, \pi}$$

$$-1.08574 \log\left(\frac{3.15784\times10^{6}\, L\, \text{Meter}^2}{d^2\ \text{Watt}}\right)$$

$$-2.17147 \log\left(\frac{3.24076\times10^{-18}\, d}{\text{Meter}}\right) - 1.08574 \log\left(\frac{3.15784\times10^{6}\, L\, \text{Meter}^2}{d^2\ \text{Watt}}\right)$$

```
array[d_] = {d / parsec, L, ℓ, M, m} /. {L → L0}
```

$$\left\{\frac{3.24076\times10^{-17}\, d}{\text{Meter}},\ 3.01521\times10^{28}\ \text{Watt},\ \frac{2.39943\times10^{27}\ \text{Watt}}{d^2},\right.$$
$$\left.-2.17147 \log\left(\frac{3.24076\times10^{-18}\, d}{\text{Meter}}\right) - 1.08574 \log\left(\frac{9.52154\times10^{34}\ \text{Meter}^2}{d^2}\right),\ -1.08574 \log\left(\frac{9.52154\times10^{34}\ \text{Meter}^2}{d^2}\right)\right\}$$

```
Table[ array[ 10^n parsec], {n, -1, 3, 1}] //
  TableForm[#, TableHeadings → {None, {"d", "L", "ℓ", "M", "m"}}] & // Chop[#, 10^-14] &
```

d	L	ℓ	M	m
0.1	3.01521×10^{28} Watt	$\frac{0.000252\ \text{Watt}}{\text{Meter}^2}$	0	−10.
1.	3.01521×10^{28} Watt	$\frac{2.52\times10^{-6}\ \text{Watt}}{\text{Meter}^2}$	0	−5.
10.	3.01521×10^{28} Watt	$\frac{2.52\times10^{-8}\ \text{Watt}}{\text{Meter}^2}$	0	0
100.	3.01521×10^{28} Watt	$\frac{2.52\times10^{-10}\ \text{Watt}}{\text{Meter}^2}$	0	5.
1000.	3.01521×10^{28} Watt	$\frac{2.52\times10^{-12}\ \text{Watt}}{\text{Meter}^2}$	0	10.

■ Problem #4
Berry Text, #6: Gravitational Red Shift

```
Clear["Global`*"]
```

```
<< Miscellaneous`PhysicalConstants`
<< Miscellaneous`Units`
```

```
energyLoss = G M m / r
```

$$\frac{G\,m\,M}{r}$$

```
(*  Find effective mass for photon of energy: h ν *)
sol1 = Solve[h ν == m c^2, m][[1]]
```

$$\left\{m \to \frac{h\,\nu}{c^2}\right\}$$

```
z = energyLoss / (h ν) /. sol1
```

$$\frac{G\,M}{c^2\,r}$$

```
z = z /. {G → GravitationalConstant, c → SpeedOfLight,
   r → 1000 Parsec, M → 10^9 SolarMass, Newton → Kilogram Meter / Second^2}
```

$$\frac{7.42433\times10^{-22}\ \text{Newton Second}^2\ \text{SolarMass}}{\text{Kilogram}^2\ \text{Parsec}}$$

```
MKS[z]
```

$$\frac{4.8095\times10^{-8}\ \text{Newton Second}^2}{\text{Kilogram Meter}}$$