Homework #4 Due: Thursday Oct. 3, 2000
Physics 3368 Professor Olness

NOTE: Homework problem #1 is due at the beginning of class Thursday, October 3, 2000. The rest of the homework problems are due Friday, but I will go over #1 in class Thursday.

#1) (30 Points) The Twin Paradox.

We are going to work through this in detail.
Homer stays on earth, and his twin sister Loner leaves in a space ship for planet X, a distance of 8 light−years away. Loner travels at a speed of 80% the speed of light.

a) Compute β, γ, and $\beta\gamma$.
b) The Boost matrix will convert from one frame to the other. B = {{γ, $-\beta\gamma$},{$-\beta\gamma$, γ}}. The inverse matrix is given by boosting with the negative velocity: B^{-1} = {{γ, $+\beta\gamma$},{$+\beta\gamma$, γ}}.
Compute the Boost matrix, and its inverse, and verify that: $B \bullet g \bullet B^{-1} = g$,
where g is the metric tensor (matrix) g={{1,0},{0,–1}}.

Let's start by working in Homer's reference frame.

c) Homer stays on earth, (x=0), and travels 10 years into the future; therefore, his coordinates are: v_1={t,x} = {10,0}. Sketch this on a {t,x} graph, label the end point for Homer, and compute the invariant length, s, of v_1 using the formula: $\Delta s^2 = \Delta t^2 - \Delta x^2$.

d) Loner's space–time path in Homer's reference frame is as follows. Loner also starts at {t,x}={0,0}.
After 10 years, he is on planet X, 8 light–years away.
Therefore, his final coordinate is: v_2={t,x}={10, 8}.
Sketch this on a {t,x} graph, label the end point for Loner, and compute the invariant length, s, of v_2 using the formula: $\Delta s^2 = \Delta t^2 - \Delta x^2$.

e) The slope of Loner's trajectory is (in some manner) related to his velocity. (Compare with Homer's slope.) Compute the slope of Loner's trajectory = $\Delta t/\Delta x$, and relate to β, γ, and $\beta\gamma$.

Let's now work in Loner's reference frame.

f) Using the boost matrix B, let's transform Homer's vector v_1={t,x} = {10,0}, into Loner's reference frame. Sketch this on a {t,x} graph, label the end point for Homer, and compute the invariant length, s, of v_1 using the formula: $\Delta s^2 = \Delta t^2 - \Delta x^2$. Compare Δs^2 with the previous result, and comment.

g) Using the boost matrix B, let's transform Loner's vector v_2={t,x} = {10,8}, into Loner's reference frame. Sketch this on a {t,x} graph, label the end point for Loner, and compute the invariant length, s, of v_1 using the formula: $\Delta s^2 = \Delta t^2 - \Delta x^2$. Compare Δs^2 with the previous result, and comment.

h) On the {t,x} graph, sketch contours of constant Δs^2. Comment on how Lorentz transformations move the vectors around the {t,x} graph using Homer and Loner as specific examples.

The following are due Friday, October 4.

#2) (10 Points) Muons are similar to electrons, but heavier. They have a lifetime of approximately 2micro–seconds, (2×10^{-6}s). Traveling at the speed of light, they would only be able to cover 600m were it not for the time–dilation factor, γ. Assume the muon is moving at 99.9% the speed of light.

a) Compute γ.

Let first work in the earth reference frame.

b) Working in the earth reference frame, explain how the muons produced in the upper atmosphere can survive to reach the surface of the earth 10km below. I.e., if the muons decay after 600m, how can they travel more than 10km?

Now let's work in the muon reference frame.

c) Working in the muon reference frame, explain how the muons produced in the upper atmosphere can survive to reach the surface of the earth 10km below. I.e., if the muons decay after 600m, how can they travel more than 10km?

#3) (10 Points) Berry Text, #14.

#4) (10 Points) Berry Text, #16.

#5) (10 Points) Challenge Problem:

The x–y plane with Cartesian coordinates has a metric of g={{1,0},{0,1}}. This metric is very physical; if you take a vector v={x,y}, the combination v.g.v gives the length–squared of the vector. A physical model for this system would be: you are walking on a flat surface with no obstacles, and this is the shortest distance you would have to walk to get to that point.

On an x–y plane with Cartesian coordinates, if we use a different metric, namely g={{1,1},{1,1}}, this will lead to a different physical quantity. Specifically, the combination v.g.v NO LONGER gives the length–squared of the vector, it now gives another quantity. Find a physical model where the distance–squared that you have to walk is given by this new quantity: v.g.v.

Hint: For my answer, I have in mind an example that you encounter everyday. However, that need not be the only answer.