

Homework #2: Phys 3320: Prof. Olness Fall 2004

Due Oct. 8

Consider the equation:

$$x'' + 2 \gamma x' + \omega_0^2 x = Q_0 \cos(\omega_D t)$$

The term $2 \gamma x'$ is the friction (dissipative) term.

The term $Q_0 \cos(\omega_D t)$ is the driving term.

- 1) Case: $x'' + 0 + \omega_0^2 x = 0$.
- 2) Case: $x'' + 2 \gamma x' + \omega_0^2 x = 0$
- 3) Case: $x'' + 0 + \omega_0^2 x = Q_0 \cos(\omega_D t)$
- 4) Case: $x'' + 2 \gamma x' + \omega_0^2 x = Q_0 \cos(\omega_D t)$

Note, you may find it convenient to define: $\omega_1^2 = \omega_0^2 - \gamma^2$.

Note, also find it easier to replace $Q_0 \cos(\omega_D t)$ by $Q_0 \text{Exp}(i \omega_D t)$.

Boundary Conditions: $x[t]=x_0, x'[t]=v_0$.

For each of the 4 cases above, use Mathematica to obtain the solution including boundary conditions. Simplify the solution to roughly match the form you derived on paper. Plot the solution for a selection of variables that display the key features of the solution.

Here are a few observations I expect you to make. This is an example—I expect you to come up with more.

- For case 2, display a) underdamped, b) critically damped, and c) overdamped, and plot these curves on the same plot for a selection of BC. Comment.
- For case 3, vary γ and ω_D , and comments. What happens when γ is small? When ω_D is close to ω_0 ?