Pythagorean - $1 + 2 + 3 + 4 = 10$

All tones found by pure 5th

\[
\begin{align*}
C & : 9/8 \\
D & : \frac{256}{243} \\
E & : \frac{9}{8} \\
F & : \frac{9}{8} \\
G & : \frac{9}{8} \\
A & : \frac{256}{243} \\
B & : \frac{9}{8} \\
C' & : \frac{9}{8}
\end{align*}
\]

Pythagorean comma = difference between B# arrived at by 5ths (702¢) and C arrived at by 7 octaves (1200¢) - 12 5ths, 7 8va

\[
\text{Comma} = 24\frac{4}{3}
\]

\[
\frac{\frac{3}{2}}{\frac{3}{2}} = \frac{9}{4} \times \frac{1}{2} = \frac{9}{8} \times \frac{3}{2} = \frac{27}{16} \times \frac{3}{2} = \frac{81}{32} \times \frac{1}{2} = \frac{81}{64}
\]

Just - based on pure 5th plus pure 3rd (54) $\frac{5}{4} = \frac{80}{64}$

\[
\frac{8\frac{1}{3}}{2\frac{1}{4}}
\]

\[
\frac{5\frac{1}{2}}{3\frac{1}{2}}
\]

\[
\frac{4\frac{1}{3}}{1\frac{1}{9}}
\]

\[
C, D, E, F, G, A, B, C
\]

\[
\begin{align*}
C & : \frac{9}{8} \\
D & : \frac{10}{9} \\
E & : \frac{16}{15} \\
F & : \frac{9}{8} \\
G & : \frac{9}{8} \\
A & : \frac{16}{15}
\end{align*}
\]

\[
\begin{align*}
C & : \frac{5}{3} \\
D & : \frac{5\frac{1}{2}}{3} \\
E & : \frac{5\frac{1}{3}}{3} \\
F & : \frac{4\frac{1}{3}}{3}
\end{align*}
\]

\[
\frac{4\frac{1}{3}}{3} \times \frac{3}{5} = \frac{16}{15}
\]
TEMPERAMENT (cont.)

MEAN TONE - BASED ON ALTERING SUCCESSIVE 5ths USED IN ARRIVING AT 3rd IN ORDER TO GET PURE 3rd INSTEAD OF PYTH. 3rd. THIS MEANS SPREADING \( \frac{1}{4} \) DIATONIC COMMA (81/80 OR 22 CENTS) OVER EACH OF THE 4 5ths (702 CENTS) USED. \( \frac{1}{4} \) OF THE 22 CENTS IS 5 1/2 CENTS, SO EACH 5th IS 696 1/2 CENTS INSTEAD OF 702.

\[
\begin{align*}
C & \rightarrow G & \rightarrow D & \rightarrow A & \rightarrow E \\
696\frac{1}{2} & 696\frac{1}{2} & 696\frac{1}{2} & 696\frac{1}{2} & 702 - \text{MINUS } 2\frac{1}{2}(240) = 386 \\
& & & & \text{PURE} \\
& & & & \text{vs. 409 PYTH.}
\end{align*}
\]

So - 8vo = 1200 CENTS

5th = 696 1/2 CENTS

3rd = 386 CENTS

2nd = 386/2 (MEAN TONE) = 193 CENTS

TO TUNE

FROM C ADD 5ths THROUGH B

FROM C SUBTRACT 5ths FOR F, B, E

Then

\[
\begin{align*}
F^# &= D + 386 \\
C^# &= A + 386 \\
G^# &= E + 386
\end{align*}
\]

SO ACCIDENTALS ARE - B, E\textsuperscript{6}/F, C\textsuperscript{#} G\textsuperscript{#}

WOLF TONE

WOLF TONE IS DIFF. BET A\textsuperscript{#} G\textsuperscript{#} ARRIVED AT BY COMPARING TUNING METHODS ABOVE. EASIER WAY IS - G\textsuperscript{#} IS TWO 386 UP FROM C - 772
Comparison in Cents Among Various Tuning/Temperament systems

Physics 1320
Professors Olness and Tunks

<table>
<thead>
<tr>
<th></th>
<th>Pythagorean</th>
<th>Just</th>
<th>Mean Tone*</th>
<th>Equal</th>
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<tbody>
<tr>
<td>C</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>C#</td>
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<tr>
<td>D</td>
<td>204</td>
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<td>Eb</td>
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<td>F#</td>
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<td>G#</td>
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<td>A</td>
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<td>890</td>
<td>900</td>
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<td>Bb</td>
<td>996</td>
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<td>1007</td>
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<td>B</td>
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<tr>
<td>C</td>
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*1/4 comma (Aron, 1523)
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<th>C</th>
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<th>E</th>
<th>F</th>
<th>G</th>
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<td>+5.5</td>
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<td>Equal Temperament</td>
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<td>-2</td>
<td>+16</td>
<td>+12</td>
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Comparison of scales with the just diatonic scale as base.
THE "IN TUNE" CHORDS OF JUST INTONATION

In the following chords the notes of equal temperament are considered to be "0" (zero) pitch. All of the IN TUNE chords are based on the root "C" which is equal tempered "0" pitch. Cents +X, or -X indicates the cents difference necessary to be IN TUNE from the equal tempered "0" pitch.

Regardless of the frequency of the starting point the structure of the chord remains constant. Therefore, the pitches of the notes shown here would occur on each of the twelve equal tempered roots...only the frequencies would be different.

Thanks to Mr. Ward Widener and his fabulous AccuTone Tuner for calculating the notes of these IN TUNE chords of just intonation.

JACK HOLLAND PRODUCTIONS 2722 BRENTWOOD BLVD. ST. LOUIS, MO 63144 (314) 962-2005