Homework #2: Phys 3320: Prof. Olness Fall 2008

Due Sept 26

Consider the equation:

$$x'' + 2\gamma\ x' + \omega_0^2\ x = Q_0 \operatorname{Cos}(\omega_D\ t)$$

The term $2\gamma\ x'$ is the friction (dissipative) term.
The term $Q_0 \operatorname{Cos}(\omega_D\ t)$ is the driving term.

1) Case: $x'' + 0 + \omega_0^2\ x = 0$.
2) Case: $x'' + 2\gamma\ x' + \omega_0^2\ x = 0$
3) Case: $x'' + 0 + \omega_0^2\ x = Q_0 \operatorname{Cos}(\omega_D\ t)$
4) Case: $x'' + 2\gamma\ x' + \omega_0^2\ x = Q_0 \operatorname{Cos}(\omega_D\ t)$

Note, you may find it convenient to define: $\omega_1^2 = \omega_0^2 - \gamma^2$.
Note, also find it easier to replace $Q_0 \operatorname{Cos}(\omega_D\ t)$ by $Q_0 \operatorname{Exp}(i\ \omega_D\ t)$.

Boundary Conditions: x[t]==x0, x'[t]==v0.

For each of the 4 cases above, use Mathematica to obtain the solution including boundary conditions. Simplify the solution to roughly match the form you derived on paper. Plot the solution for a selection of variables that display the key features of the solution.

Here are a few observations I expect you to make. This is an example—I expect you to come up with more.

- For case 2, display a) under-damped, b) critically damped, and c) over-damped, and plot these curves on the same plot for a selection of BC. Comment.
- For case 3, vary γ and ω_D, and comments. What happens when γ is small? When ω_D,is close to ω_0?