Wave Eq

$$c^2 \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial t^2}$$

$$c^2 \partial_x^2 = \partial_t^2$$

$$\boxed{\psi = e^{\pm i(Kx \pm \omega t)}}$$

IF $\omega^2 = c^2 K^2$

$$c = \frac{\omega}{K}$$

Diffusion / Heat Eq

$$\partial_t = \partial_x^2 = \nabla_x^2$$

Schrodinger Eq

$$i\,\partial_t = \nabla_x^2$$

$$C = v = f\lambda = (2\pi f)\left(\frac{\lambda}{2\pi}\right)$$

$$= \omega \quad 1/K$$

angular Frequency

wave number

$$\boxed{K = \frac{2\pi}{\lambda}}$$

wave number

Trig Solution: $\psi(x,t) = \sin(Kx)\cos(\omega t)$

$\sin(Kx)$ $K = 1, 2, 3 \ldots$

$x \in [0, \pi]$

$$c^2 \partial_x^2 \psi = \partial_t^2 \psi$$

$$c^2(-K^2)\psi = (-\omega^2)\psi$$

$$c^2K^2 = \omega^2$$

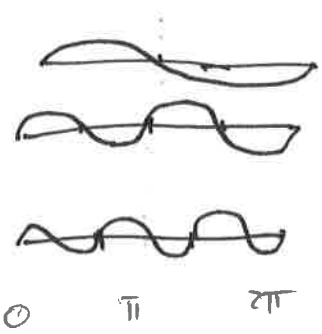
$$K^2 = \omega^2$$

$$K = \pm\omega$$

$\boxed{c = 1}$ natural units

$\sin(Kx) = 0$ if $K = 1, 2, 3 \ldots$ at $x = \pi, 0$

$$\boxed{K = \pm\omega}$$



sin(1x) K=1 ω=1

sin(2x) K=2 ω=2

sin(3x) K=3 ω=3

Wave Eq Notes

$\partial_x^2 = 1/c^2\,\partial_t^2$ 1) Check units $c = v = \frac{x}{t}$

Guess $\psi = e^{\pm i(kx \pm \omega t)}$

$$c = v = f\lambda = (2\pi f)\left(\frac{\lambda}{2\pi}\right) = \frac{\omega}{k}$$

so $\omega = 2\pi f$ $k = \frac{2\pi}{\lambda}$ wave number

Solve: $k^2 = 1/c^2\,\omega^2 \Rightarrow k = \omega$

BC $\psi = 0$ at $x = 0, \pi \Rightarrow$

$\sin(kx) \Rightarrow k = 1, 2, 3 \ldots$

so $\omega = k = 1, 2, 3 \ldots$

or $\omega_n = n\,\omega_1$ Harmonic Series

Animate

$$\psi = \sin(\underset{\substack{\downarrow \\ n}}{k}x)\cos(\underset{\substack{\downarrow \\ n}}{\omega}t)$$

same value

Maxwell's Eq ⟹ Wave Eq

$\nabla \cdot E = \rho$ $\qquad$ $\nabla \times E = \frac{\partial B}{\partial t} + 0$

$\nabla \cdot B = 0$

$\nabla \times B = \frac{\partial E}{\partial t} + \mu_0 I$

> Homework for students:
> Repeat with all μ_0 ε_0 to find c in terms of $\mu_0 \varepsilon_0$

In Vacuum: $\rho \to 0$ $\quad$ $I \to 0$

$$\nabla \times \left(\nabla \times E = \frac{\partial B}{\partial t} \right) \Rightarrow \nabla^2 E = \frac{\partial}{\partial t} (\nabla \times B)$$

$$= \frac{\partial}{\partial t} \left(\frac{\partial E}{\partial t} \right) = \frac{\partial^2 E}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 E = \partial_t^2 E}$$

Likewise $\nabla^2 B = \partial_t^2 B$

214 444-9068

Diff Form

$$\nabla \cdot E = \rho \qquad \nabla \times E = 0 + \frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0$$

$$\nabla \times B = \mu_0 I + \frac{\partial E}{\partial t}$$

Integral Form

$$\int E \cdot dA = \frac{Q}{\varepsilon_0} \qquad \oint E \cdot d\ell = \frac{\Delta \Phi_B}{dt}$$

$$\oint B \cdot d\ell = \mu_0 I + \frac{\partial \Phi_E}{\partial t}$$

$$\int B \cdot dA = 0$$