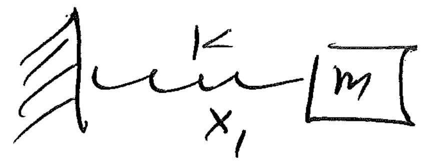
SHO 1-D



$$T = \frac{1}{2} m v^2 \qquad V = \frac{K}{2} x^2 \qquad L = T - V$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = m\ddot{x} \qquad \frac{\partial L}{\partial x} = -Kx$$

Eq: $\quad m\ddot{x} + Kx = 0$

$$x = e^{i\omega t} \qquad \ddot{x} = -\omega^2 x$$

$\therefore$ Eq $\longrightarrow \quad (K - m\omega^2)\, x = 0$

$$\Rightarrow \quad \omega^2 = \frac{K}{m} \qquad x = x_0\, e^{i\omega t}$$

SHO Coupled Modes

$$T = \frac{1}{2}\left(m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2\right)$$

$$V = \frac{k}{2}(x_1 - x_2)^2 = \frac{k}{2}\left(x_1^2 + x_2^2 - 2x_1 x_2\right)$$

$$L = T - V$$

$$\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \qquad \frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2$$

$$\frac{\partial L}{\partial x_1} = -k\, x_1 + k x_2 \qquad \frac{\partial L}{\partial x_2} = -k x_2 + k x_1$$

Eqs
$$\begin{cases} m_1 \ddot{x}_1 + k(x_1 - x_2) = 0 \\ m_2 \ddot{x}_2 + k(x_2 - x_1) = 0 \end{cases} \qquad \begin{aligned} x_{1,2} &= e^{i\omega t} \\ \ddot{x}_{12} &= -\omega^2 x_{1,2} \end{aligned}$$

$$k(x_1 - x_2) - \omega^2 m_1 = 0$$

$$k(x_2 - x_1) - \omega^2 m_2 = 0$$

$$\begin{pmatrix} k & -k \\ -k & k \end{pmatrix} - \omega^2 \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = 0 \qquad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Eigen values:

$$\omega^2 = 0 \qquad v_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \omega_\varphi^2 = \frac{2k}{m} \qquad v_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} +1 \\ -1 \end{pmatrix}$$