SHO

In[1]:=
```
Clear["Global`*"]
```

In[2]:=
```
T = (1 / 2) m1 v1^2 + (1 / 2) m2 v2^2;
T // Expand // Factor
```

Out[3]= $\frac{1}{2}\left(m1\ v1^2 + m2\ v2^2\right)$

In[4]:=
```
V = (1 / 2) k (x1 - x2)^2;
V // Expand
```

Out[5]= $\frac{k\ x1^2}{2} - k\ x1\ x2 + \frac{k\ x2^2}{2}$

In[6]:=
```
D[T, v1]
```

Out[6]= $m1\ v1$

In[7]:=
```
D[T, v2]
```

Out[7]= $m2\ v2$

In[8]:=
```
D[-V, x1] // Expand
```

Out[8]= $-k\ x1 + k\ x2$

In[9]:=
```
D[-V, x2] // Expand
```

Out[9]= $k\ x1 - k\ x2$

In[10]:=
```
Tmat = {{m1, 0}, {0, m2}};
Tmat // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix} m1 & 0 \\ 0 & m2 \end{pmatrix}$$

In[12]:=
```
Vmat = {{k, -k}, {-k, k}};
Vmat // MatrixForm
```

Out[13]//MatrixForm=

$$\begin{pmatrix} k & -k \\ -k & k \end{pmatrix}$$

In[14]:=
```
rule = {m1 → m, m2 → m};
```

In[15]:=
```
mat = Vmat - Tmat ω2 /. rule;
mat // MatrixForm
```

Out[16]//MatrixForm=

$$\begin{pmatrix} k - m\,\omega 2 & -k \\ -k & k - m\,\omega 2 \end{pmatrix}$$

In[17]:=
```
sol = Solve[Det[mat] == 0, ω2]
```

Out[17]= $\left\{\{\omega 2 \to 0\}, \left\{\omega 2 \to \frac{2\ k}{m}\right\}\right\}$

In[18]:=
```
eq1 = mat.{a, b} == 0 // Thread
```
Out[18]= $\{-b\,k + a\,(k - m\,\omega 2) == 0,\ -a\,k + b\,(k - m\,\omega 2) == 0\}$

In[19]:=
```
norm = a^2 + b^2 == 1
```
Out[19]= $a^2 + b^2 == 1$

In[20]:=
```
eq2 = Join[eq1, {norm}]
```
Out[20]= $\{-b\,k + a\,(k - m\,\omega 2) == 0,\ -a\,k + b\,(k - m\,\omega 2) == 0,\ a^2 + b^2 == 1\}$

In[21]:=
```
sol1 = Solve[eq2 /. sol[[1]], {a, b}] // Last
```
Out[21]= $\left\{a \to \frac{1}{\sqrt{2}},\ b \to \frac{1}{\sqrt{2}}\right\}$

In[22]:=
```
ev1 = {a, b} /. sol1
```
Out[22]= $\left\{\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right\}$

In[23]:=
```
sol2 = Solve[eq2 /. sol[[2]], {a, b}]
```
Out[23]= $\left\{\left\{a \to -\frac{1}{\sqrt{2}},\ b \to \frac{1}{\sqrt{2}}\right\},\ \left\{a \to \frac{1}{\sqrt{2}},\ b \to -\frac{1}{\sqrt{2}}\right\}\right\}$

In[24]:=
```
ev2 = {a, b} /. Last[sol2]
```
Out[24]= $\left\{\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right\}$

In[25]:=
```
eVecs = {ev1, ev2};
eVecs // MatrixForm
```
Out[26]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

In[27]:=
```
eVecs.Vmat.Transpose[eVecs] // Simplify // MatrixForm
```
Out[27]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 2\,k \end{pmatrix}$$

In[28]:=
```
eVecs.Tmat.Transpose[eVecs] /. rule // Simplify // MatrixForm
```
Out[28]//MatrixForm=

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

Look at motion

In[29]:=
```
mode1 = ev1 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[1]]
```
Out[29]= $\left\{\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right\}$

In[30]:=
```
mode2 = ev2 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[2]]
```

Out[30]= $\left\{\frac{e^{i \sqrt{2} \sqrt{\frac{k}{m}} t}}{\sqrt{2}}, -\frac{e^{i \sqrt{2} \sqrt{\frac{k}{m}} t}}{\sqrt{2}}\right\}$

In[31]:=
```
values = {k → 1, m → 1};
```

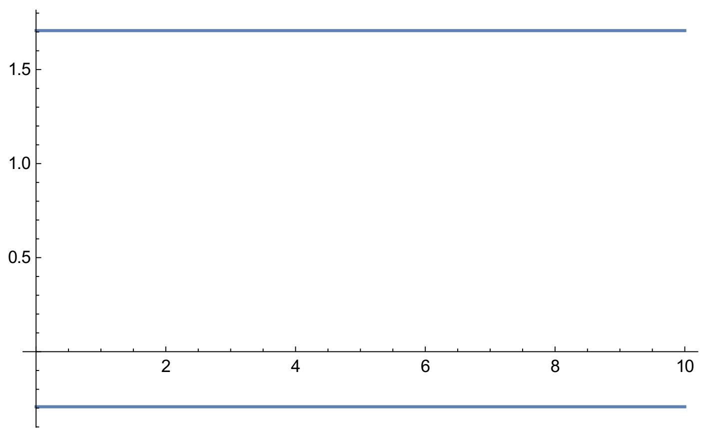
In[32]:=
```
mode1 /. values
```

Out[32]= $\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$

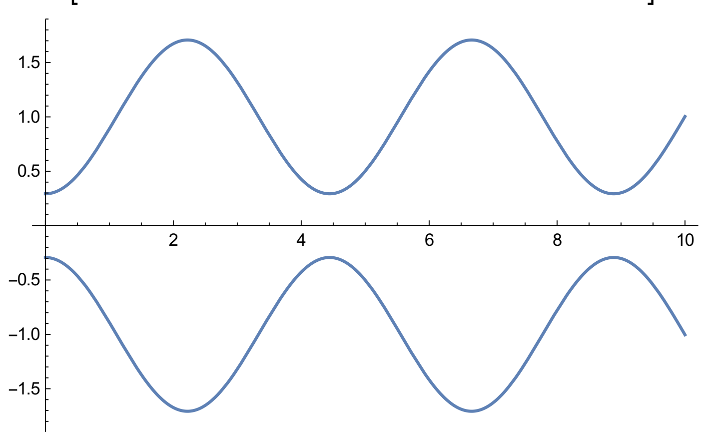
In[33]:=
```
mode1 + {-1, 1}
```

Out[33]= $\left\{-1 + \frac{1}{\sqrt{2}}, 1 + \frac{1}{\sqrt{2}}\right\}$

In[34]:=
```
Plot[mode1 + {-1, 1} /. values // Re, {t, 0, 10}]
```

Out[34]=


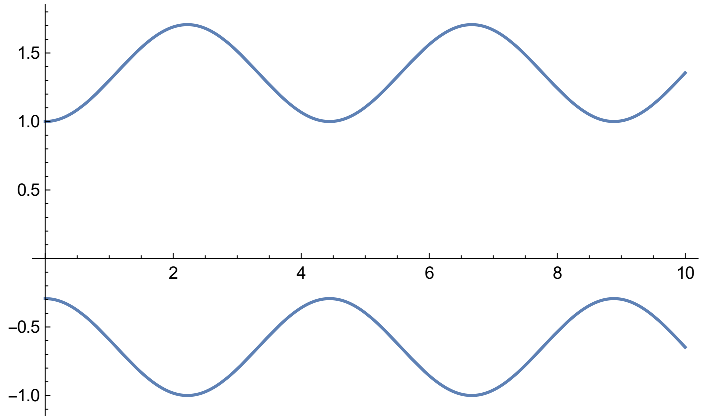
In[35]:=
```
Plot[mode2 + {-1, 1} /. values // Re, {t, 0, 10}]
```

Out[35]=


In[36]:=
```
doPlot[a_: 1 / 2] := Plot[a (mode1 + {-1, 1} /. values) + (1 - a) (mode2 + {-1, 1} /. values)
    // Re, {t, 0, 10}]
```

In[39]:=
```
doPlot[1 / 2]
```

Out[39]=



Normal Coordinates:

In[41]:=
```
nRule = {x1, x2} -> Transpose[eVecs].{n1, n2}  // Thread
```

Out[41]= $\left\{x1 \to \frac{n1}{\sqrt{2}} + \frac{n2}{\sqrt{2}},\ x2 \to \frac{n1}{\sqrt{2}} - \frac{n2}{\sqrt{2}}\right\}$

In[42]:=
```
V
```

Out[42]= $\frac{1}{2} k\ (x1 - x2)^2$

In[43]:=
```
V /. rule /. nRule // Simplify
```

Out[43]= $k\ n2^2$

In[44]:=
```
nRule2 = {v1, v2} -> Transpose[eVecs].{nv1, nv2}  // Thread
```

Out[44]= $\left\{v1 \to \frac{nv1}{\sqrt{2}} + \frac{nv2}{\sqrt{2}},\ v2 \to \frac{nv1}{\sqrt{2}} - \frac{nv2}{\sqrt{2}}\right\}$

In[45]:=
```
Vmat2 = {{0, 0}, {0, 2 k}};
Vmat2 // MatrixForm
```

Out[46]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 2\,k \end{pmatrix}$$

In[47]:=
```
T
```

Out[47]= $\frac{m1\ v1^2}{2} + \frac{m2\ v2^2}{2}$

In[48]:=
```
T /. rule /. nRule2 // Expand
```

Out[48]= $\frac{m\ nv1^2}{2} + \frac{m\ nv2^2}{2}$

In[49]:=
```
Tmat2 = {{m, 0}, {0, m}};
Tmat2 // MatrixForm
```

Out[50]//MatrixForm=
$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

In[51]:=
```
mat2 = Vmat2 - Tmat2 ω2;
mat2 // MatrixForm
```

Out[52]//MatrixForm=
$$\begin{pmatrix} -m\,\omega 2 & 0 \\ 0 & 2\,k - m\,\omega 2 \end{pmatrix}$$

In[53]:=
```
sol2 = Solve[Det[mat2] == 0, ω2]
```

Out[53]= $\left\{\{\omega 2 \to 0\},\ \left\{\omega 2 \to \frac{2\,k}{m}\right\}\right\}$

In[54]:=
```
eq1 = mat2 .{a, b} == 0 // Thread
```

Out[54]= $\{-a\,m\,\omega 2 == 0,\ b\,(2\,k - m\,\omega 2) == 0\}$

In[55]:=
```
norm = a^2 + b^2 == 1
```

Out[55]= $a^2 + b^2 == 1$

In[56]:=
```
eq2 = Join[eq1, {norm}]
```

Out[56]= $\{-a\,m\,\omega 2 == 0,\ b\,(2\,k - m\,\omega 2) == 0,\ a^2 + b^2 == 1\}$

In[57]:=
```
sol1 = Solve[eq2 /. sol2[[1]], {a, b}] // Last
```

Out[57]= $\{a \to 1,\ b \to 0\}$

In[58]:=
```
ev1 = {a, b} /. sol1
```

Out[58]= $\{1, 0\}$

In[59]:=
```
sol2 = Solve[eq2 /. sol2[[2]], {a, b}]
```

Out[59]= $\{\{a \to 0,\ b \to -1\},\ \{a \to 0,\ b \to 1\}\}$

In[60]:=
```
ev2 = {a, b} /. Last[sol2]
```

Out[60]= $\{0, 1\}$

In[61]:=
```
eVecs = {ev1, ev2};
eVecs // MatrixForm
```

Out[62]//MatrixForm=
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[63]:=
```
eVecs.Vmat2.Transpose[eVecs] // Simplify // MatrixForm
```

Out[63]//MatrixForm=
$$\begin{pmatrix} 0 & 0 \\ 0 & 2\,k \end{pmatrix}$$

In[64]:=
```
eVecs.Tmat2.Transpose[eVecs] // Simplify // MatrixForm
```
Out[64]//MatrixForm=

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

SHO
Repeat with m2→∞, or m1→0

Problem #Taylor: p418

In[132]:=

418 Chapter 11 Coupled Oscillators and Normal Modes

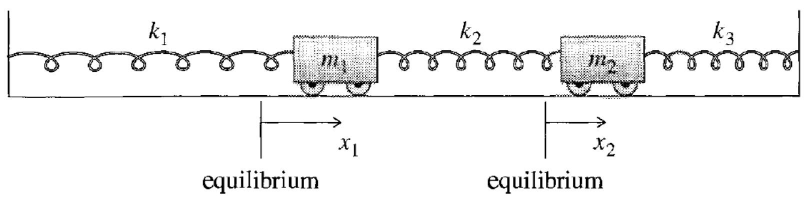


Figure 11.1 Two carts attached to fixed walls by the springs labeled k_1 and k_3, and to each other by k_2. The carts' positions x_1 and x_2 are measured from their respective equilibrium positions.

In[133]:=
```
Clear["Global`*"]
```

In[134]:=
```
T = 1/2 m1 v1^2 + 1/2 m2 v2^2;
T // Expand // Factor
```
Out[135]=

$$\frac{1}{2}\left(m1\ v1^2 + m2\ v2^2\right)$$

In[136]:=
```
V = 1/2 k1 x1^2 + 1/2 k2 (x1 - x2)^2 + 1/2 k3 x2^2;
V // Expand // Factor
```
Out[137]=

$$\frac{1}{2}\left(k1\ x1^2 + k2\ x1^2 - 2\ k2\ x1\ x2 + k2\ x2^2 + k3\ x2^2\right)$$

In[138]:=
```
Tmat = {{m1, 0}, {0, m2}};
Tmat // MatrixForm
```
Out[139]//MatrixForm=

$$\begin{pmatrix} m1 & 0 \\ 0 & m2 \end{pmatrix}$$

In[140]:=
```
Vmat = {{k1 + k2, -k2}, {-k2, k2 + k3}};

Vmat // MatrixForm
```

Out[141]//MatrixForm=

$$\begin{pmatrix} k1 + k2 & -k2 \\ -k2 & k2 + k3 \end{pmatrix}$$

In[142]:=
```
rule = {m1 → m, m2 → m, k1 → k, k2 → k, k3 → k};
```

In[143]:=
```
Tmat = Tmat /. rule;
Tmat // MatrixForm
```

Out[144]//MatrixForm=

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

In[145]:=
```
Vmat = Vmat /. rule;
Vmat // MatrixForm
```

Out[146]//MatrixForm=

$$\begin{pmatrix} 2\,k & -k \\ -k & 2\,k \end{pmatrix}$$

In[147]:=
```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```

Out[148]//MatrixForm=

$$\begin{pmatrix} 2\,k - m\,\omega 2 & -k \\ -k & 2\,k - m\,\omega 2 \end{pmatrix}$$

In[149]:=
```
sol = Solve[Det[mat] == 0, ω2]
```

Out[149]= $\left\{\left\{\omega 2 \to \frac{k}{m}\right\}, \left\{\omega 2 \to \frac{3\,k}{m}\right\}\right\}$

In[150]:=
```
eq1 = mat . {a, b} == 0 // Thread
```

Out[150]= $\{-b\,k + a\,(2\,k - m\,\omega 2) == 0, -a\,k + b\,(2\,k - m\,\omega 2) == 0\}$

In[151]:=
```
norm = a^2 + b^2 == 1
```

Out[151]= $a^2 + b^2 == 1$

In[152]:=
```
eq2 = Join[eq1, {norm}]
```

Out[152]= $\{-b\,k + a\,(2\,k - m\,\omega 2) == 0, -a\,k + b\,(2\,k - m\,\omega 2) == 0, a^2 + b^2 == 1\}$

In[153]:=
```
sol1 = Solve[eq2 /. sol[[1]], {a, b}] // Last
```

Out[153]= $\left\{a \to \frac{1}{\sqrt{2}}, b \to \frac{1}{\sqrt{2}}\right\}$

In[154]:=
```
ev1 = {a, b} /. sol1
```

Out[154]= $\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$

In[155]:=
```
sol2 = Solve[eq2 /. sol[[2]], {a, b}]
```

Out[155]= $\left\{\left\{a \to -\frac{1}{\sqrt{2}}, b \to \frac{1}{\sqrt{2}}\right\}, \left\{a \to \frac{1}{\sqrt{2}}, b \to -\frac{1}{\sqrt{2}}\right\}\right\}$

In[156]:=
```
ev2 = {a, b} /. Last[sol2]
```

Out[156]= $\left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}$

In[157]:=
```
eVecs = {ev1, ev2};
eVecs // MatrixForm
```

Out[158]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

In[159]:=
```
eVecs.Vmat.Transpose[eVecs] // Simplify // MatrixForm
```

Out[159]//MatrixForm=

$$\begin{pmatrix} k & 0 \\ 0 & 3\,k \end{pmatrix}$$

In[160]:=
```
eVecs.Tmat.Transpose[eVecs] // Simplify // MatrixForm
```

Out[160]//MatrixForm=

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

Look at motion

In[161]:=
```
mode1 = ev1 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[1]]
```

Out[161]= $\left\{\frac{e^{i\sqrt{\frac{k}{m}}\,t}}{\sqrt{2}}, \frac{e^{i\sqrt{\frac{k}{m}}\,t}}{\sqrt{2}}\right\}$

In[162]:=
```
mode2 = ev2 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[2]]
```

Out[162]= $\left\{\frac{e^{i\sqrt{3}\sqrt{\frac{k}{m}}\,t}}{\sqrt{2}}, -\frac{e^{i\sqrt{3}\sqrt{\frac{k}{m}}\,t}}{\sqrt{2}}\right\}$

In[163]:=
```
values = {k → 1, m → 1};
```

In[164]:=
```
mode1 /. values
```

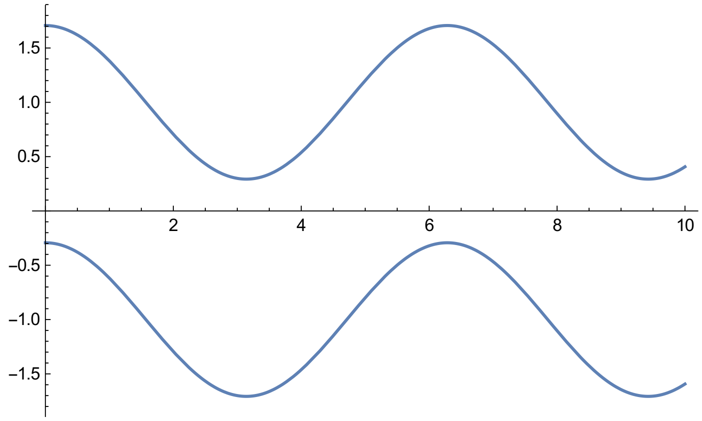
Out[164]= $\left\{\frac{e^{i\,t}}{\sqrt{2}}, \frac{e^{i\,t}}{\sqrt{2}}\right\}$

In[165]:=
```
mode1 + {-1, 1}
```

Out[165]= $\left\{-1 + \frac{e^{i\sqrt{\frac{k}{m}}\,t}}{\sqrt{2}}, 1 + \frac{e^{i\sqrt{\frac{k}{m}}\,t}}{\sqrt{2}}\right\}$

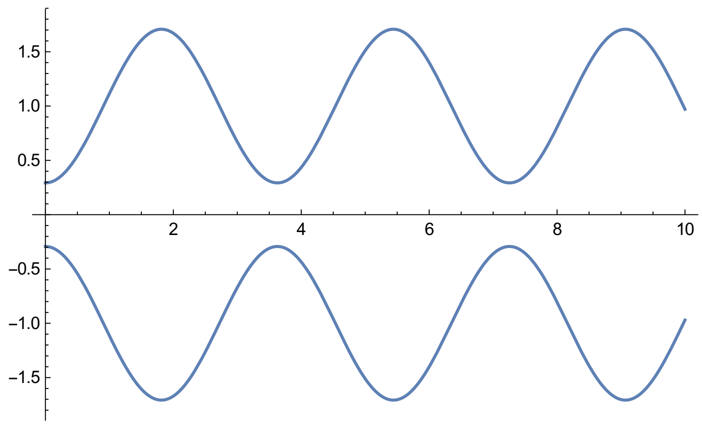
In[166]:=
```
Plot[mode1 + {-1, 1} /. values // Re, {t, 0, 10}]
```

Out[166]=



In[167]:=
```
Plot[mode2 + {-1, 1} /. values // Re, {t, 0, 10}]
```

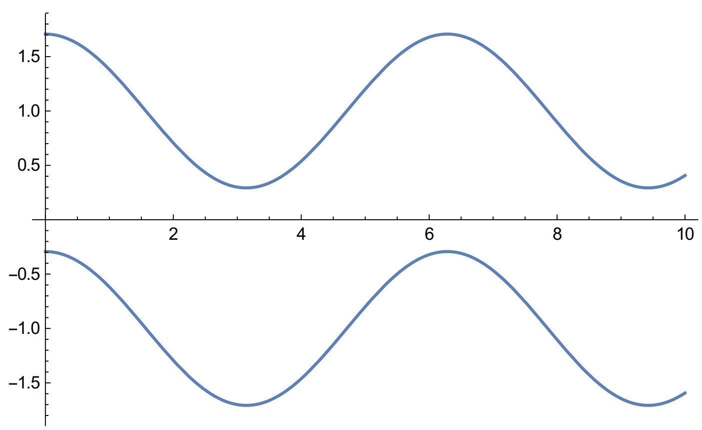
Out[167]=



In[168]:=
```
doPlot[a_: 1 / 2] := Plot[a (mode1 + {-1, 1} /. values) + (1 - a) (mode2 + {-1, 1} /. values)
    // Re, {t, 0, 10}]
```

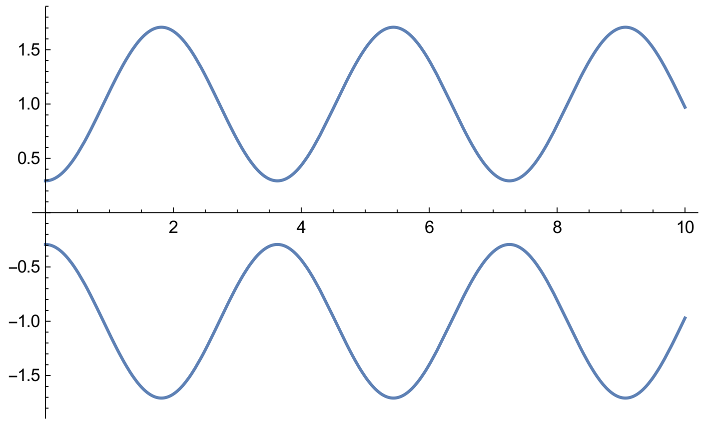
In[169]:=
```
doPlot[1]
```

Out[169]=



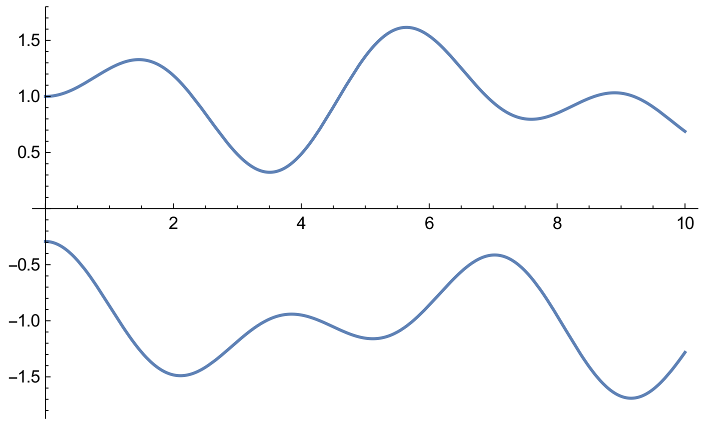
In[170]:= doPlot[0]

Out[170]=



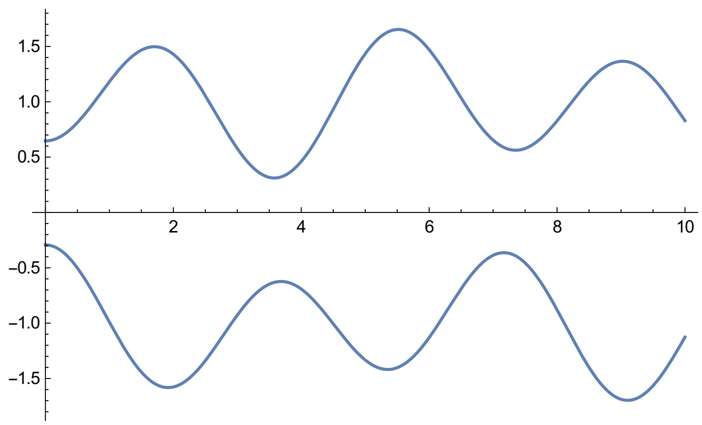
In[171]:= doPlot[1 / 2]

Out[171]=



In[172]:= doPlot[1 / 4]

Out[172]=



Normal Coordinates:

In[173]:= nRule = {x1, x2} -> Transpose[eVecs].{n1, n2} // Thread

Out[173]= $\left\{x1 \to \frac{n1}{\sqrt{2}} + \frac{n2}{\sqrt{2}},\ x2 \to \frac{n1}{\sqrt{2}} - \frac{n2}{\sqrt{2}}\right\}$

In[174]:=
```
V
```
Out[174]= $\frac{k1\,x1^2}{2} + \frac{1}{2} k2\,(x1 - x2)^2 + \frac{k3\,x2^2}{2}$

In[175]:=
```
V /. rule /. nRule // Simplify
```
Out[175]= $\frac{1}{2} k \left(n1^2 + 3\,n2^2\right)$

In[176]:=
```
nRule2 = {v1, v2} -> Transpose[eVecs].{nv1, nv2}  // Thread
```
Out[176]= $\left\{v1 \to \frac{nv1}{\sqrt{2}} + \frac{nv2}{\sqrt{2}},\ v2 \to \frac{nv1}{\sqrt{2}} - \frac{nv2}{\sqrt{2}}\right\}$

In[177]:=
```
Vmat2 = {{k, 0}, {0, 3 k}};
Vmat2 // MatrixForm
```
Out[178]//MatrixForm=
$\begin{pmatrix} k & 0 \\ 0 & 3\,k \end{pmatrix}$

In[179]:=
```
T
```
Out[179]= $\frac{m1\,v1^2}{2} + \frac{m2\,v2^2}{2}$

In[180]:=
```
T /. rule /. nRule2 // Expand
```
Out[180]= $\frac{m\,nv1^2}{2} + \frac{m\,nv2^2}{2}$

In[181]:=
```
Tmat2 = {{m, 0}, {0, m}};
Tmat2 // MatrixForm
```
Out[182]//MatrixForm=
$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$

In[183]:=
```
mat2 = Vmat2 - Tmat2 ω2;
mat2 // MatrixForm
```
Out[184]//MatrixForm=
$\begin{pmatrix} k - m\,\omega2 & 0 \\ 0 & 3\,k - m\,\omega2 \end{pmatrix}$

In[185]:=
```
sol2 = Solve[Det[mat2] == 0, ω2]
```
Out[185]= $\left\{\left\{\omega2 \to \frac{k}{m}\right\}, \left\{\omega2 \to \frac{3\,k}{m}\right\}\right\}$

In[186]:=
```
eq1 = mat2 .{a, b} == 0 // Thread
```
Out[186]= $\{a\,(k - m\,\omega2) == 0,\ b\,(3\,k - m\,\omega2) == 0\}$

In[187]:=
```
norm = a^2 + b^2 == 1
```
Out[187]= $a^2 + b^2 == 1$

In[188]:=
```
eq2 = Join[eq1, {norm}]
```
Out[188]= $\{a\,(k - m\,\omega2) == 0,\ b\,(3\,k - m\,\omega2) == 0,\ a^2 + b^2 == 1\}$

```
In[189]:= sol1 = Solve[eq2 /. sol2[[1]], {a, b}] // Last
Out[189]= {a → 1, b → 0}

In[190]:= ev1 = {a, b} /. sol1
Out[190]= {1, 0}

In[191]:= sol2 = Solve[eq2 /. sol2[[2]], {a, b}]
Out[191]= {{a → 0, b → -1}, {a → 0, b → 1}}

In[192]:= ev2 = {a, b} /. Last[sol2]
Out[192]= {0, 1}

In[193]:= eVecs = {ev1, ev2};
          eVecs // MatrixForm
Out[194]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
In[195]:= eVecs.Vmat2.Transpose[eVecs] // Simplify // MatrixForm
Out[195]//MatrixForm=
```

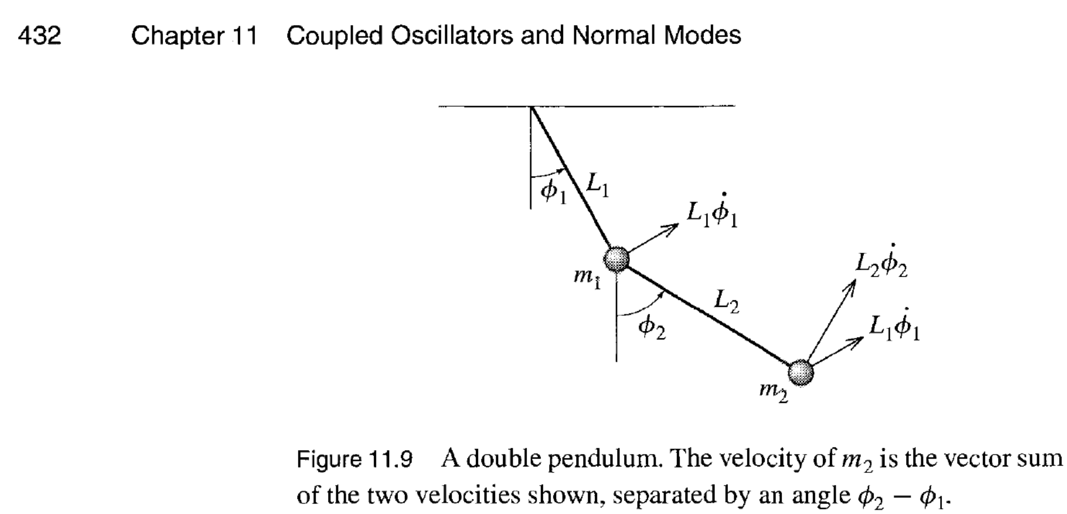
$$\begin{pmatrix} k & 0 \\ 0 & 3\,k \end{pmatrix}$$

```
In[196]:= eVecs.Tmat2.Transpose[eVecs] // Simplify // MatrixForm
Out[196]//MatrixForm=
```

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

Problem #Taylor: p418

In[197]:=




```
In[276]:= Clear["Global`*"]
```

In[277]:= $T = \frac{1}{2}$ (m1 + m2) $L1^2$ $d\phi1^2$ + $\frac{1}{2}$ m2 $L2^2$ $d\phi2^2$ + m2 L1 L2 dϕ1 dϕ2 ;

T // Expand // Factor

Out[278]= $\frac{1}{2}\left(d\phi1^2\ L1^2\ m1 + d\phi1^2\ L1^2\ m2 + 2\ d\phi1\ d\phi2\ L1\ L2\ m2 + d\phi2^2\ L2^2\ m2\right)$

In[279]:= $V = \frac{1}{2}$ (m1 + m2) g L1 $\phi1^2$ + $\frac{1}{2}$ m2 g L2 $\phi2^2$;

V // Expand // Factor

Out[280]= $\frac{1}{2}\ g\left(L1\ m1\ \phi1^2 + L1\ m2\ \phi1^2 + L2\ m2\ \phi2^2\right)$

In[281]:= rules = {m1 → m, m2 → m, L1 → L, L2 → L};

In[282]:= T /. rules // Factor

Out[282]= $\frac{1}{2}\left(2\ d\phi1^2 + 2\ d\phi1\ d\phi2 + d\phi2^2\right)\ L^2\ m$

In[283]:= V /. rules // Factor

Out[283]= $\frac{1}{2}\ g\ L\ m\left(2\ \phi1^2 + \phi2^2\right)$

In[284]:= Tmat = {{2, 1}, {1, 1}};

Tmat // MatrixForm

Out[285]//MatrixForm=

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

In[286]:= Vmat = {{2, 0}, {0, 1}};

Vmat // MatrixForm

Out[287]//MatrixForm=

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

In[288]:= mat = Vmat - Tmat ω2;

mat // MatrixForm

Out[289]//MatrixForm=

$$\begin{pmatrix} 2 - 2\ \omega2 & -\omega2 \\ -\omega2 & 1 - \omega2 \end{pmatrix}$$

In[290]:= sol = Solve[Det[mat] == 0, ω2]

Out[290]= $\left\{\left\{\omega2 \to 2 - \sqrt{2}\right\}, \left\{\omega2 \to 2 + \sqrt{2}\right\}\right\}$

In[291]:= eq1 = mat .{a, b} == 0 // Thread

Out[291]= $\{a\ (2 - 2\ \omega2) - b\ \omega2 == 0,\ b\ (1 - \omega2) - a\ \omega2 == 0\}$

In[292]:= norm = a^2 + b^2 == 1

Out[292]= $a^2 + b^2 == 1$

In[293]:=
```
eq2 = Join[eq1, {norm}]
```
Out[293]= $\{a\ (2-2\ \omega 2)-b\ \omega 2==0,\ b\ (1-\omega 2)-a\ \omega 2==0,\ a^2+b^2==1\}$

In[294]:=
```
sol1 = Solve[eq2 /. sol[[1]], {a, b}] // Last // FullSimplify
```
Out[294]= $\left\{a\to\frac{1}{\sqrt{3}},\ b\to\sqrt{\frac{2}{3}}\right\}$

In[295]:=
```
ev1 = {a, b} /. sol1
```
Out[295]= $\left\{\frac{1}{\sqrt{3}},\ \sqrt{\frac{2}{3}}\right\}$

In[296]:=
```
sol2 = Solve[eq2 /. sol[[2]], {a, b}] // FullSimplify
```
Out[296]= $\left\{\left\{a\to-\frac{1}{\sqrt{3}},\ b\to\sqrt{\frac{2}{3}}\right\},\ \left\{a\to\frac{1}{\sqrt{3}},\ b\to-\sqrt{\frac{2}{3}}\right\}\right\}$

In[297]:=
```
ev2 = {a, b} /. Last[sol2]
```
Out[297]= $\left\{\frac{1}{\sqrt{3}},\ -\sqrt{\frac{2}{3}}\right\}$

In[298]:=
```
eVecs = {ev1, ev2};
eVecs // MatrixForm
```
Out[299]//MatrixForm=
$$\begin{pmatrix}\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\\ \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}}\end{pmatrix}$$

In[300]:=
```
eVecs.Vmat.Transpose[eVecs] // Simplify // MatrixForm
```
Out[300]//MatrixForm=
$$\begin{pmatrix}\frac{4}{3} & 0\\ 0 & \frac{4}{3}\end{pmatrix}$$

In[301]:=
```
eVecs.Tmat.Transpose[eVecs] // Simplify // MatrixForm
```
Out[301]//MatrixForm=
$$\begin{pmatrix}\frac{2}{3}\left(2+\sqrt{2}\right) & 0\\ 0 & -\frac{2}{3}\left(-2+\sqrt{2}\right)\end{pmatrix}$$

Look at motion

In[302]:=
```
mode1 = ev1 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[1]]
```
Out[302]= $\left\{\frac{e^{i\sqrt{2-\sqrt{2}}\ t}}{\sqrt{3}},\ \sqrt{\frac{2}{3}}\ e^{i\sqrt{2-\sqrt{2}}\ t}\right\}$

In[303]:=
```
mode2 = ev2 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[2]]
```

Out[303]= $\left\{\frac{e^{i\sqrt{2+\sqrt{2}}\,t}}{\sqrt{3}}, -\sqrt{\frac{2}{3}}\,e^{i\sqrt{2+\sqrt{2}}\,t}\right\}$

In[304]:=
```
mode1
```

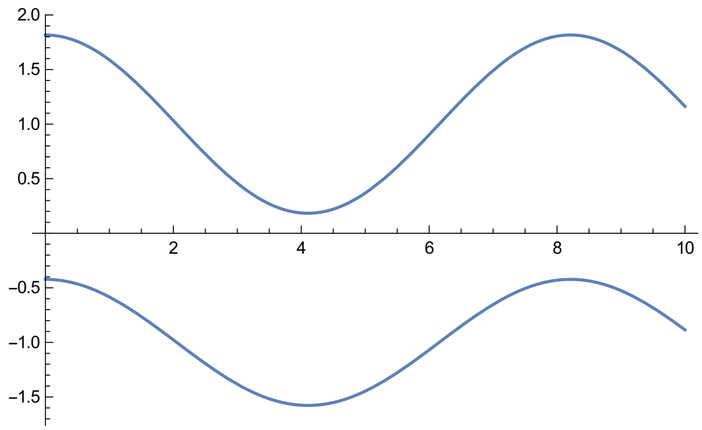
Out[304]= $\left\{\frac{e^{i\sqrt{2-\sqrt{2}}\,t}}{\sqrt{3}}, \sqrt{\frac{2}{3}}\,e^{i\sqrt{2-\sqrt{2}}\,t}\right\}$

In[305]:=
```
mode1 + {-1, 1}
```

Out[305]= $\left\{-1+\frac{e^{i\sqrt{2-\sqrt{2}}\,t}}{\sqrt{3}}, 1+\sqrt{\frac{2}{3}}\,e^{i\sqrt{2-\sqrt{2}}\,t}\right\}$

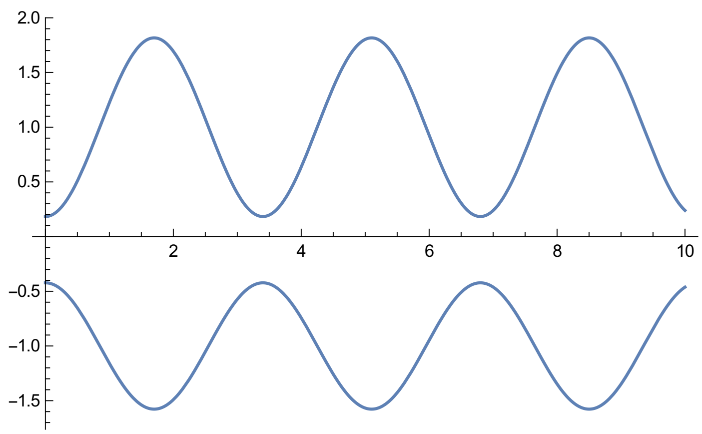
In[306]:=
```
Plot[mode1 + {-1, 1} // Re, {t, 0, 10}]
```

Out[306]=



In[307]:=
```
Plot[mode2 + {-1, 1} // Re, {t, 0, 10}]
```

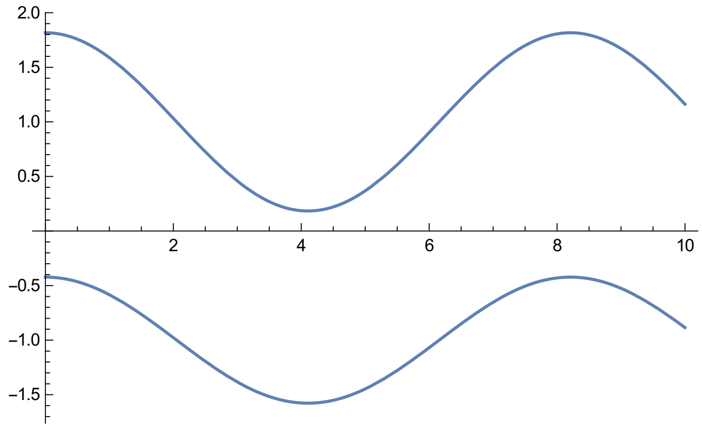
Out[307]=



In[308]:=
```
doPlot[a_: 1 / 2] := Plot[a (mode1 + {-1, 1}) + (1 - a) (mode2 + {-1, 1})
    // Re, {t, 0, 10}]
```

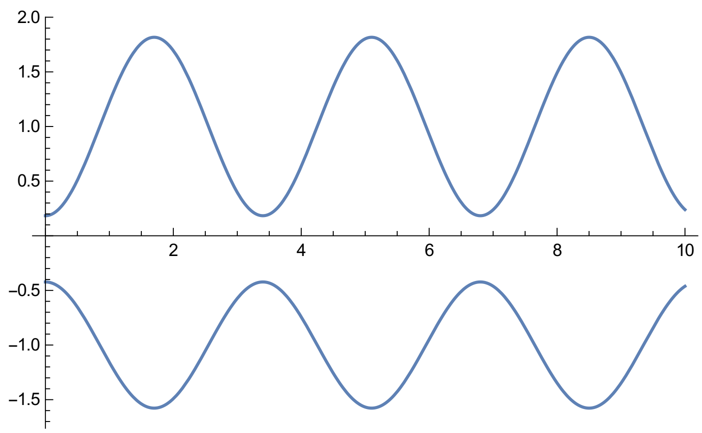
In[309]:= **doPlot[1]**

Out[309]=



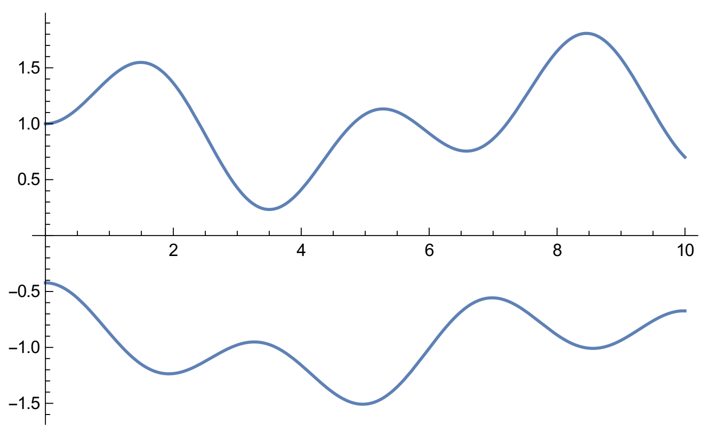
In[310]:= **doPlot[0]**

Out[310]=



In[311]:= **doPlot[1 / 2]**

Out[311]=



In[312]:= **doPlot[1 / 4]**

Out[312]=