Newton's Law

$$F = ma = \dot{p}$$

Dlambert's Principle

$$(\dot{p} - F_A)\,\delta q = 0$$

Constraint Forces do no work

Lagrange Equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0 \qquad L = T - V$$

Hamilton's Equations

Symmetric in q and p

$$\frac{\partial H}{\partial p} = +\dot{q} \qquad \frac{\partial H}{\partial q} = -\dot{p} \qquad H(q\,p) = \dot{q}\,p - L(q\,\dot{q})$$

Least Action Principle

Compact Statement

$$\delta S = 0 = \delta \int dt\, L(q\,\dot{q})$$

$\boxed{\text{Lagrangian}}$ $L(q, \dot{q})$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

$$\frac{\partial L}{\partial q} = \dot{P}$$

$$\frac{\partial L}{\partial \dot{q}} = P$$

$\boxed{\text{Hamiltonian}}$ $H(q, P)$

$$H(q\,P) = \dot{q}P - L(q, \dot{q})$$

$$\frac{\partial H}{\partial P} = +\dot{q}$$

$$\frac{\partial H}{\partial q} = -\dot{P}$$

$\boxed{\text{SHO Lagrangian}}$ $L = T - V = \frac{m\dot{q}^2}{2} - \frac{K}{2}q^2$

Eqs $\Rightarrow m\ddot{q} + Kq = 0$ $\quad q = e^{i\omega t} \Rightarrow \omega = \sqrt{\frac{K}{m}}$

One 2nd-order D.E.Q.

$\boxed{\text{SHO Hamiltonian}}$ $H = T + V = \frac{P^2}{2m} + \frac{Kq^2}{2}$

$$\frac{\partial H}{\partial P} = \frac{P}{m} = \dot{q} \qquad \frac{\partial H}{\partial q} = Kq = -\dot{P}$$

$$P = m\dot{q} \qquad -Kq = \dot{P}$$

$$P = mv \qquad -Kx = F \quad \text{Hooke's Law}$$

SHO || Hamiltonian

$F = -Kx \qquad V = \frac{1}{2}Kx^2 \qquad T = \frac{1}{2}mv^2$

$L(q\dot{q}) = T - V = \frac{m}{2}\dot{q}^2 - \frac{K}{2}q^2$

$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \boxed{m\ddot{q} + Kq = 0}$ Lag. Eq.

$H = p\dot{q} - L = \frac{p^2}{m} - \frac{p^2}{2m} + \frac{K}{2}q^2 = \frac{p^2}{2m} + \frac{K}{2}q^2$

$H = T + V$ [If t invarnt !!!]

$$\boxed{\frac{\partial H}{\partial p} = +\dot{q} \qquad \frac{\partial H}{\partial q} = -\dot{p}}$$

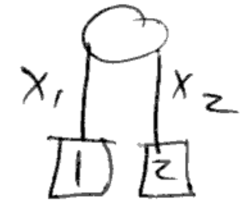
$\frac{\partial H}{\partial p} = \frac{p}{m} = \dot{q} \Rightarrow p = m\dot{q} = mv$

$\frac{\partial H}{\partial q} = Kq = -\dot{p} \Rightarrow \dot{p} = F = -Kx$ Hooke's Law

$\dot{p} = -Kq \Rightarrow \dot{(m\dot{q})} = m\ddot{q} = -Kq$

$\boxed{m\ddot{q} + Kq = 0}$ Equiv to Lagrangian

Atwood



$$T = \frac{1}{2}(m_1+m_2)\dot{x}^2 =$$

$$V = -(m_1-m_2)\, g\, x$$

$$P = \frac{\partial L}{\partial \dot{x}} = (m_1+m_2)\dot{x}$$

$$H = T + V = \frac{p^2}{2(m_1+m_2)} - (m_1-m_2)\, g\, q$$

$$\frac{\partial H}{\partial p} = \frac{p}{m_1+m_2} = \dot{q} \Longrightarrow p = (m_1+m_2)\dot{q}$$

$$\frac{\partial H}{\partial q} = -(m - m_2)\, g = -\dot{p}$$

$$\therefore \quad F = \dot{p} = (m_1-m_2)\, g$$

$$\dot{p} = (m_1+m_2)\ddot{q} = (m_1-m_2)\, g$$

$$\ddot{q} = \left(\frac{m_1-m_2}{m_1+m_2}\right) g$$