Ch 14 Scattering

Example #1 p. 562

$$N_{sc} = N_{inc}\, n_{tar}\, \sigma$$

$$= (60)\left(\frac{50}{150\ ft^2}\right)\left(\frac{1}{2}\ ft^2\right) = 10$$

(60): incident bullets

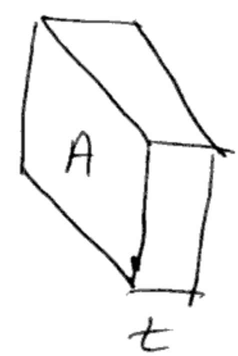
$\frac{50}{150\ ft^2}$: $\frac{\text{crows}}{\text{Area}}$

$\frac{1}{2}\ ft^2$: Cross section (area)

Example 14.2

$$N_{sc} = N_{inc}\, n_{tar}\, \sigma$$

$N_{inc} \to 10^4$, $\sigma \to 1.5 \times 10^{-28}\ m^2$



$$V = At \qquad n_{tar} = \frac{\#}{\text{Area}}$$

$$\rho = \frac{M}{V} = \frac{M}{At}$$

$$m = \text{one Al nucleus} = 27\ \text{AMU} = 27 \times (1.66 \times 10^{-27}\ kg)$$

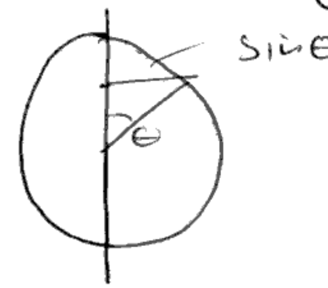
$$n_{tar} = \frac{\rho t}{m} = \frac{M}{m}\frac{t}{At} = \frac{M}{mA} = \frac{\#}{\text{Area}}$$

$$n_{tar} = \frac{(2.7 \times 10^{3}\ \frac{kg}{m^3})(10^{-4}\ m)}{[27 \times 1.66 \times 10^{-27}\ kg]} = 6 \times 10^{24}\ m^{2}$$

$$\Rightarrow N_{sc} = 9$$

Solid Angle

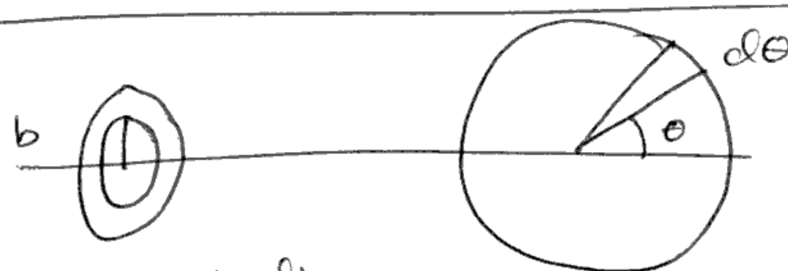
$$\Delta \Omega = \frac{A}{r^2} \longrightarrow \sin\theta \, d\theta \, d\phi$$



$$N_{sc}(\text{in } d\Omega) = N_{inc} \, n_{tar} \frac{d\sigma}{d\Omega} d\Omega$$

$$\int d\Omega = \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi = 2 \times 2\pi = 4\pi$$

14.5

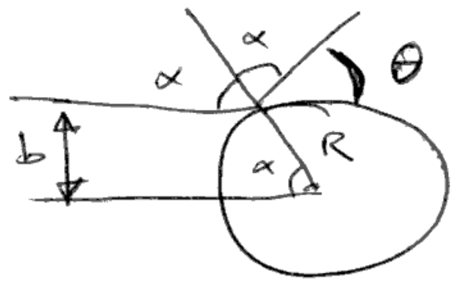


$$d\sigma = 2\pi b \, db$$

$$d\Omega = 2\pi \sin\theta \, d\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{2\pi b \, db}{2\pi \sin\theta \, d\theta} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$

Ex 14.5



$2\alpha + \theta = \pi$

$\theta = \pi - 2\alpha$

$$b = R\sin\alpha = R\sin\left(\frac{\pi - \theta}{2}\right) = R\cos(\theta/2)$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\frac{db}{d\theta} = \frac{R\cos(\theta/2)}{\sin\theta}\,\frac{R\sin(\theta/2)}{2} = \frac{R^2}{4}$$

w/ $\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) = \frac{\sin(\theta)}{2}$

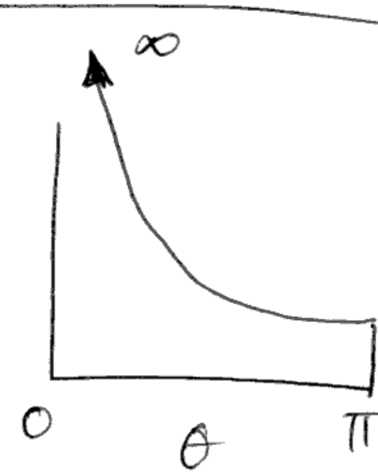
Note: $\frac{d\sigma}{d\Omega} = \text{const} = \text{isotropic}$

$$\sigma = \int \frac{d\sigma}{d\Omega}\,d\Omega = \int \frac{R^2}{4}\sin\theta\,d\theta\,d\phi$$

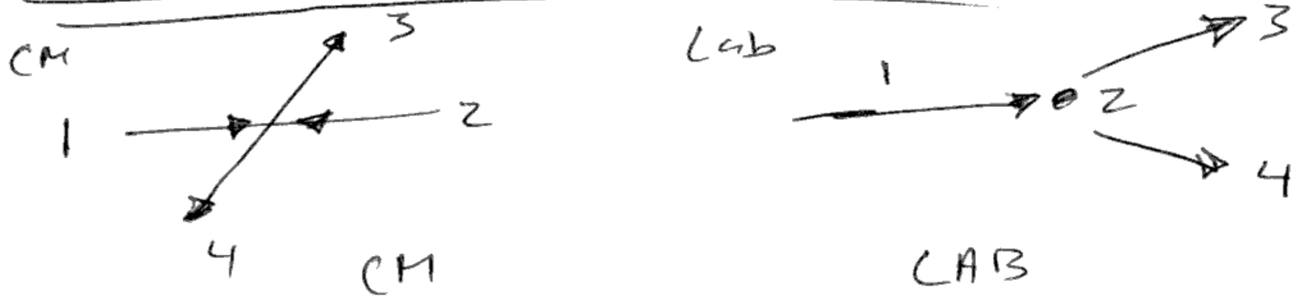
$$= \frac{R^2}{4}\cdot 4\pi = \pi R^2$$

Rutherford Scattering

$$\frac{d\sigma}{d\Omega} = \left[\frac{Kq\,Q}{4E\sin^2(\theta/2)}\right]^2$$



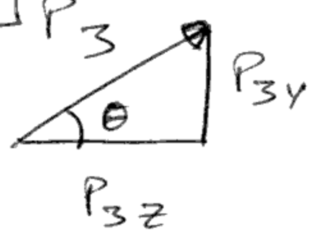
Lab to CM frame



CM	LAB
$P_1 = P(0\,0\,1)$	$P_1' = P(0\,0\,2)$
$P_2 = P(0\,0\,-1)$	$P_2' = P(0\,0\,0)$
$P_3 = P(0\;S\;C)$	$P_3' = P(0\;S\;1{+}C)$
$P_4 = P(0\;-S\;-C)$	$P_4' = P(0\;-S\;1{-}C)$

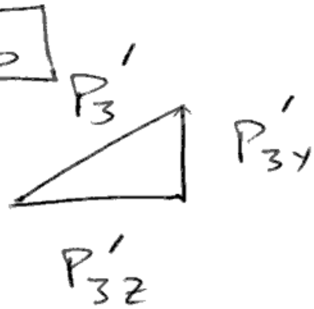
→ Add $P(0\,0\,1)$ to get ↗

CMS



$$\tan\theta = \frac{P_{3y}}{P_{3z}} = \frac{\sin\theta}{\cos\theta} \equiv \tan\theta$$

Lab



$$\tan\theta' = \frac{P_{3y}'}{P_{3z}'} = \frac{\sin\theta}{1+\cos\theta} = \tan\left(\frac{\theta}{2}\right)$$

$$\boxed{\theta_{Lab} = \theta' = \frac{\theta_{cm}}{2}}$$

$$P_3' \circ P_4' = P^2\left[-s^2 + (1+c)(1-c)\right] = P^2\left[1 - s^2 - c^2\right] = 0$$

P_3' and P_4' are <u>orthogonal</u> 90°