Air Resistance

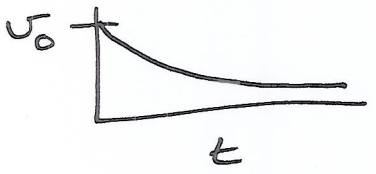
$$F = ma = m\dot{v}$$

$$F = bv + cv^2$$

$$-bv = m\dot{v} \qquad \text{Guess} \quad v = Ae^{\lambda t}$$

$$-bv = m\lambda v \quad \Leftarrow \quad \dot{v} = \lambda v$$

$$\lambda = -\frac{b}{m} \qquad A = v_0 \Rightarrow \quad v = v_0 e^{-\frac{b}{m}t}$$



Add Gravity

$$mg - bv = m\dot{v} \qquad \dot{v} = 0 \Rightarrow \text{Terminal Velocity}$$

$$\Rightarrow v_T = \frac{mg}{b}$$

$$\frac{mg}{b} - v = \frac{m}{b}\dot{v}$$

$$-u \Rightarrow \quad u = v - \frac{mg}{b} \qquad \dot{u} = \dot{v}$$

$$-u = \frac{m}{b}\dot{u} \quad \Rightarrow \quad u = u_0 e^{-\frac{b}{m}t}$$

$$v = u + \frac{mg}{b} = u + v_T = u_0 e^{-\frac{b}{m}t} + v_T$$

At $t = 0$ $\quad v = v_0 = u_0 + v_T \Rightarrow u_0 = v_0 + v_T$

$$\therefore \quad v = (v_0 - v_T)e^{-\frac{b}{m}t} + v_T = v_0 e^{-\frac{b}{m}t} + v_T\left(1 - e^{-\frac{b}{m}t}\right)$$