Euler's Theorem

For cont. Tx. $A \Rightarrow$ same as rot. about Fixed axis. (For $n=$ odd)

$$X' = AX$$

$R' = AR = R$ for special vector R

$AR = \lambda R$ w/ $\lambda = 1$

$(A - \lambda \mathbb{1})R = 0$

$\Rightarrow$ Real orthog. matrix A: $(A^T = A^{-1})$ specifying physical motion $\Rightarrow \lambda = +1$

$(A - 1)A^T = (1 - A^T)$ For $AA^T = 1$

$|A - 1|\,|A^T| = |1 - A^T|$ $|A| = |A^T| = +1$ for proper orthog. rot

$$|A - 1| = |1 - A|$$

$$|A - 1| = (-1)^n |A - 1|$$

only true if $|A - \mathbb{1}| = 0$

$\Rightarrow \exists \lambda$ st. $|A - \lambda \mathbb{1}| = 0$

$$|A| = \lambda_1 \lambda_2 \lambda_3 = \lambda_1 \lambda_2 (1) = 1$$

$\therefore \lambda_1 \lambda_2 = 1$

$\Rightarrow$ ~~$\lambda_{1,2} = \pm 1$ but~~

what is mag. of λ :

$$R^{+\prime} R' = (AR)^{+} (AR) = R^{+} R$$

$$= \lambda^{*} R^{+} \lambda R$$

$$\Rightarrow \lambda \lambda^{*} = 1 = |\lambda|^{2}$$

$\therefore$ $|\lambda_1| = 1 = |\lambda_2|$ $\quad \lambda_1 \lambda_2 = 1$

$$\Rightarrow \lambda_1 = e^{+i\phi} \qquad \lambda_2 = e^{-i\phi}$$

Consider rot about z.

$$A = \begin{pmatrix} c & s & \\ -s & c & \\ & & 1 \end{pmatrix}$$

$\mathrm{Tr}\, A = 1 + 2c = \lambda_1 + \lambda_2 + \lambda_3$

$|A| = c^2 + s^2 = 1$

If $\lambda_3 = 1$, $\lambda_1 + \lambda_2 = 2c_\phi$

$$e^{i\phi} + e^{-i\phi} =$$

Infinitesimal Rotations:

$$A = 1 + \varepsilon$$

$$A^{-1} = 1 - \varepsilon = A^T$$

$$A A^{-1} = 1 - \varepsilon^2 \sim 1$$

$$\therefore \quad A^T = (1+\varepsilon)^T = 1 + \varepsilon^T = 1 - \varepsilon$$

$$\Rightarrow \quad \varepsilon = -\varepsilon^T$$

$$\Rightarrow \quad \varepsilon = \begin{pmatrix} 0 & d\Omega_3 & -d\Omega_2 \\ & 0 & d\Omega_1 \\ & & 0 \end{pmatrix}$$

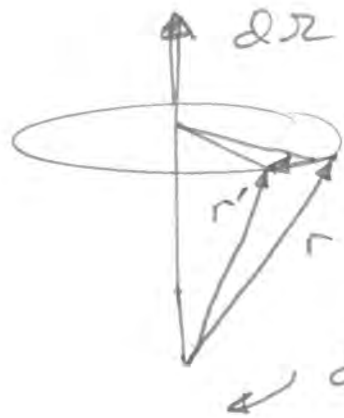
$$d\vec{X} = \vec{X}' - \vec{X} = A\vec{X} - \vec{X} = \varepsilon\vec{X} = \left(\vec{X} \times d\overleftrightarrow{\Omega}\right)$$

Try: $\vec{X} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\quad X_1' = X_1 + d\Omega_3 X_2 - d\Omega_2 X_3$

$$dX_1 = X_1' - X_1 = X_2\, d\Omega_3 - X_3\, d\Omega_2$$

$$\therefore \quad dX_i = \varepsilon_{ijk} X_j\, d\Omega_k$$

$$\boxed{d\vec{X} = \vec{X} \times d\overleftrightarrow{\Omega}}$$



$d\vec{r} = r' - r = \vec{r} \times d\vec{\Omega}$

$d\phi \sim d\Omega$

$$d\vec{\Omega} = \hat{n}\, d\phi$$

$C = D \times F \qquad C_i = \varepsilon_{ijk} D_j F_k$

Axial ~~Polar~~ vector   Invariant under parity

Coriolis Force:

$$(dG)_{space} = (dG)_{body} + (dG)_{Rot}$$

rotation of body

movement relative to body coords.

$$V_s = V_r + \omega \times r = \left(\frac{d}{dt} + \omega \times\right) r$$

space coords

Earth coords

rotation of earth

Take $\left(\frac{d}{dt}\right)_s = \left(\frac{d}{dt}\right)_r + \omega \times$

$\cancel{a_s = a_r +}$

$$\left(\frac{dV_s}{dt}\right)_s = \frac{dV_r}{dt} + \omega \times V_r + \frac{d(\omega \times r)}{dt} + \omega \times (\omega \times r)$$

$$a_s = a_r + 2(\omega \times v_r) + \omega \times (\omega \times r)$$

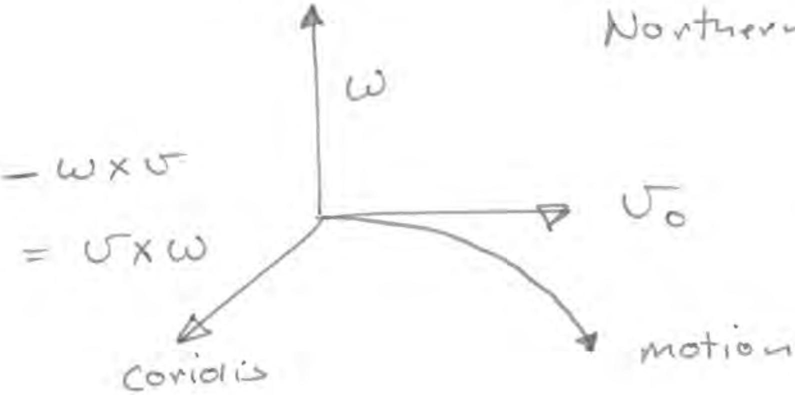
$F = m a_s \Rightarrow$

centrifugal stationary

coriolis

$$m a_r = \underbrace{\underbrace{(m a_s)}_{F} - 2(m)(\omega \times v_r) - m\omega \times (\omega \times r)}_{F_{eff}}$$

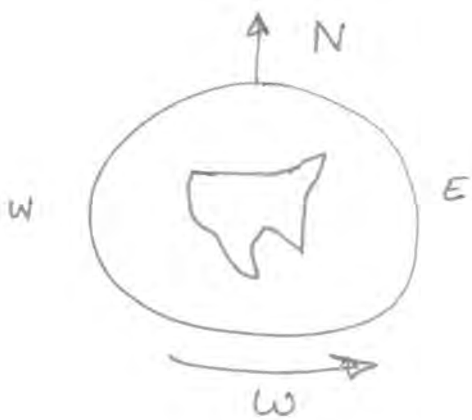
Northern hemisphere

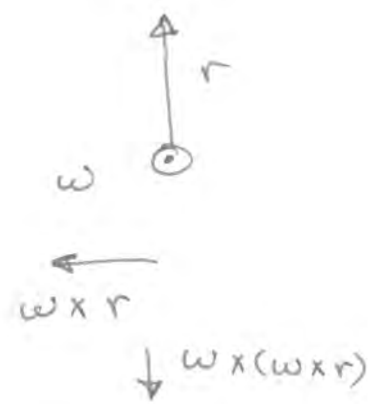


Stationary: $|\omega \times (\omega \times r)| = m\omega^2 r \sin\theta$

$= \dfrac{m v^2}{r} \sin\theta$ centrifugal Force

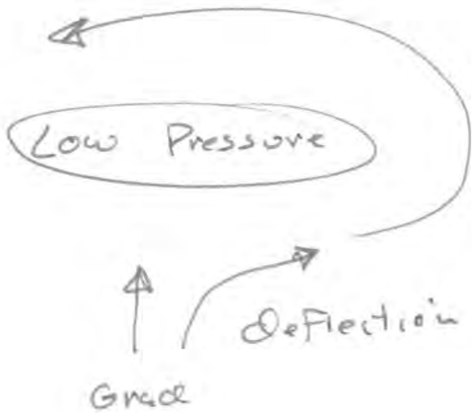
coriolis: $-2m(\omega \times v_r)$





Effective Force

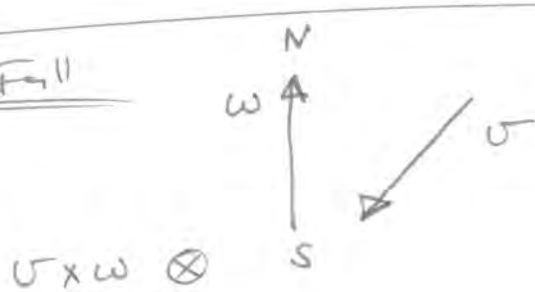
Winds



Circulation

in N Hemisphere

Free Fall



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