Problem 1

```
Clear["Global`*"]
```

```
f[x_] == c[n] Sin[n x]
```

f[x_] == c[n] Sin[n x]

```
rhs = Integrate[ 1 Sin[37 x], {x, 0, π}]
```

$\frac{2}{37}$

```
lhs = Integrate[ c[37] Sin[37 x]^2, {x, 0, 2 π}]
```

π c[37]

```
eq = lhs == rhs
```

$\pi\, c[37] == \frac{2}{37}$

```
Solve[eq, c[37]]
```

$\left\{\left\{c[37] \to \frac{2}{37\,\pi}\right\}\right\}$

Problem 3

```
Clear["Global`*"]
```

```
Lag[qi_,L_ ,Qi_,t_:t] :=
D[ D[L,(D[qi[t],t])],t]-D[L,qi[t]]==Qi;
```

```
Lag[qi_List,L_,Qi_List,t_:t] :=
(D[ D[L,(D[qi[[#]][t],t])],t]-D[L,qi[[#]][t]]==Qi[[#]]
)&  /@  Range[1,Length[qi]];
```

```
Protect[Lag];
```

Problem 3: Sphere Rolling on a Fixed Sphere   *part c is wrong*

Remarks and outline

Solution

```
Clear["Global`*"]
```

Part a

```
Tcm = 1/2 m (r′[t]^2 + r[t]^2 θ1′[t]^2);

Trot = 0;

T=Tcm+Trot;

V= m g r[t] Cos[θ1[t]];

L=T-V
```

$-g\, m \,\text{Cos}[\theta 1[t]]\, r[t] + \frac{1}{2} m \left(r'[t]^2 + r[t]^2\, \theta 1'[t]^2\right)$

```
constraints= { r[t]-a-b==0 }
```

$\{-a - b + r[t] == 0\}$

```
constraintRule = DSolve[constraints , {r}, t]
```

$\{\{r \to \text{Function}[\{t\}, a + b]\}\}$

```
vars = {r[t], θ1[t]};

Qeq = Sum[λ[j] * D[constraints[[j, 1]], vars[[#]]], {j, 1, 1}] & /@ {1, 2}
```

$\{\lambda[1], 0\}$

```
(eqMotion = Lag[{r, θ1}, L, Qeq]) ;
Column[eqMotion]
```

$g\, m\, \text{Cos}[\theta 1[t]] - m\, r[t]\, \theta 1'[t]^2 + m\, r''[t] == \lambda[1]$

$-g\, m\, r[t]\, \text{Sin}[\theta 1[t]] + 2\, m\, r[t]\, r'[t]\, \theta 1'[t] + m\, r[t]^2\, \theta 1''[t] == 0$

Part b

NEED TO FIX ORDER HERE

```
({θSol, λ1Sol} = Flatten[
    Solve[Flatten[eqMotion //.constraintRule], {θ1″[t], λ[1]}] /.Rule → Equal]);
Column[
 {θSol,
  λ1Sol}]
```

$\theta 1''[t] == \frac{g\, \text{Sin}[\theta 1[t]]}{a+b}$

$\lambda[1] == g\, m\, \text{Cos}[\theta 1[t]] - a\, m\, \theta 1'[t]^2 - b\, m\, \theta 1'[t]^2$

```
eqθ = (θSol[[1]] - θSol[[2]])
```

$-\frac{g\, \text{Sin}[\theta 1[t]]}{a + b} + \theta 1''[t]$

DO INDEFINITE INTEGRAL HERE, AND AFTER SUBSTITUTE IN LIMITS

```
Clear[intθTMP]
```

```
intθTMP[t_] = Integrate[eqθ θ1'[t], t]
```

$$\frac{g\,\mathrm{Cos}[\theta1[t]]}{a+b}+\frac{1}{2}\,\theta1'[t]^2$$

```
intθ = intθTMP[t] - intθTMP[0]
```

$$-\frac{g\,\mathrm{Cos}[\theta1[0]]}{a+b}+\frac{g\,\mathrm{Cos}[\theta1[t]]}{a+b}-\frac{1}{2}\,\theta1'[0]^2+\frac{1}{2}\,\theta1'[t]^2$$

```
intθ = intθ /. {θ1[0] → 0, θ1'[0] → 0}
```

$$-\frac{g}{a+b}+\frac{g\,\mathrm{Cos}[\theta1[t]]}{a+b}+\frac{1}{2}\,\theta1'[t]^2$$

```
solθ = Solve[intθ == 0, {θ1'[t]}][[1]] // Simplify
```

$$\left\{\theta1'[t]\to-\frac{\sqrt{2}\,\sqrt{g-g\,\mathrm{Cos}[\theta1[t]]}}{\sqrt{a+b}}\right\}$$

Part c

```
eq1 = λ1Sol /. λ[1] → 0 //. solθ
```

$$0 == g\,m\,\mathrm{Cos}[\theta1[t]]-\frac{2\,a\,m\,(g-g\,\mathrm{Cos}[\theta1[t]])}{a+b}-\frac{2\,b\,m\,(g-g\,\mathrm{Cos}[\theta1[t]])}{a+b}$$

```
sol2=Solve[eq1,Cos[θ1[t]]]  //Simplify//Flatten
```

$$\left\{\mathrm{Cos}[\theta1[t]]\to\frac{2}{3}\right\}$$

```
sol3 =
 Solve[sol2 //. {Rule → Equal}, θ1[t], InverseFunctions → True] /. {C[1] → 0} // Normal //
  Flatten
```

$$\left\{\theta1[t]\to-\mathrm{ArcCos}\left[\frac{2}{3}\right],\ \theta1[t]\to\mathrm{ArcCos}\left[\frac{2}{3}\right]\right\}$$

```
(θ1[t] / Degree) //. sol3[[2]] // N
```

48.1897

Problem 4: Bead on a Rotating Hoop

Remarks and outline

Solution

```
Lag[qi_,L_ ,Qi_,t_:t] :=
D[ D[L,(D[qi[t],t])],t]-D[L,qi[t]]==Qi;

Lag[qi_List,L_,Qi_List,t_:t] :=
(D[ D[L,(D[qi[[#]][t],t])],t]-D[L,qi[[#]][t]]==Qi[[#]]
)&  /@  Range[1,Length[qi]];

Protect[Lag];

Clear["Global`*"]
```

Part a

```
x2rRule =
  {x, y, z} → {(r Sin[θ[#]] Cos[ω #]) &, (r Sin[θ[#]] Sin[ω #]) &, (r Cos[θ[#]]) &} // Thread;
  ψRule = {θ → (π - ψ[#] &)};
```

```
T = 1/2 m (x'[t]^2 + y'[t]^2 + z'[t]^2) //. x2rRule /. ψRule // Simplify
```

$$\frac{1}{2} m r^2 \left(\omega^2 \text{Sin}[\psi[t]]^2 + \psi'[t]^2\right)$$

```
V = m g z[t] //. x2rRule /. ψRule // Simplify
```

$$-g m r \text{Cos}[\psi[t]]$$

```
 L = (T - V)
```

$$g m r \text{Cos}[\psi[t]] + \frac{1}{2} m r^2 \left(\omega^2 \text{Sin}[\psi[t]]^2 + \psi'[t]^2\right)$$

```
eq1 = Lag[ψ, L, 0, t]  // Simplify
```

$$m r \left(\left(g - r\,\omega^2 \text{Cos}[\psi[t]]\right) \text{Sin}[\psi[t]] + r\,\psi''[t]\right) == 0$$

```
eq2 = eq1[[1]]/(m r) /. ψ''[t] → 0
```

$$\left(g - r\,\omega^2 \text{Cos}[\psi[t]]\right) \text{Sin}[\psi[t]]$$

```
Solve[eq2/Sin[ψ[t]] == 0 , Cos[ψ[t]]]
```

$$\left\{\left\{\text{Cos}[\psi[t]] \to \frac{g}{r\,\omega^2}\right\}\right\}$$