Physics 3344 | Professor Olness
Homework 10 | November 20, 2018

1) Three equal masses are connected by three identical springs and constrained to move on a circular frictionless rim of radius b. Set up the small oscillations Lagrangian and find the normal modes by:

a) Using symmetry considerations;

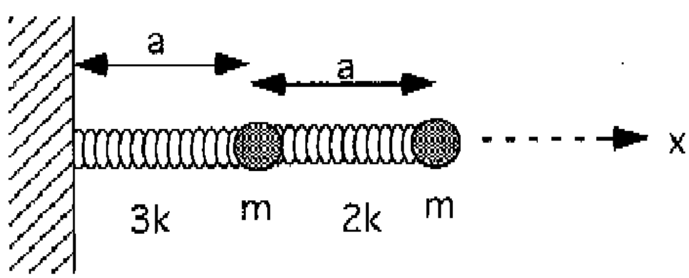
b) Diagonalizing the kinetic and potential energy matrices.

2) A rigid uniform bar of mass M and length L is supported in equilibrium in a horizontal position by two identical springs attached one to each end. Assume its motion is constrained to a vertical plane. You may neglect any horizontal motion, but do not neglect the moment of inertia of the mass.

a) Find the normal modes and eigenfrequencies of small oscillations of the system .

b) If initially one end of the bar is displaced, the other remaining in its equilibrium position, and the system is released from rest, find the motion.

3. 2 objects, each of mass m, are attached to each other by a spring, and the left mass is also attached by a spring to a fixed wall. The springs are of equilibrium length a. The figure shows a top view. The masses are on a frictionless surface, and can only move along the x-axis . The left spring has spring constant 3k, and the right spring has spring constant 2k.




(a) Find the Lagrangian for this system.

(b) Find the normal modes and their frequencies.

4)

A pendulum of length $L$ and mass $m$ is connected to a block also of mass $m$ that is free to move horizontally on a frictionless surface. The block is connected to a wall with a spring of spring constant $k$. For the special case where

$$\sqrt{\frac{k}{m}} = \sqrt{\frac{g}{L}} = \omega_0 \tag{1}$$

determine:

(a) The frequencies of the normal modes of this system for small oscillations around the equilibrium positions.

(b) The motion of each of the normal modes.