Problem #1)

```
Clear["Global`*"]
```

```
T = 1/2 m b^2 (dθ1^2 + dθ2^2 + dθ3^2)
```

$$\frac{1}{2} b^2 \left(d\theta 1^2 + d\theta 2^2 + d\theta 3^2\right) m$$

```
V = 1/2 k b^2 ((θ1 - θ2)^2 + (θ2 - θ3)^2 + (θ3 - θ1)^2) // Expand
```

$$b^2 k \theta 1^2 - b^2 k \theta 1 \theta 2 + b^2 k \theta 2^2 - b^2 k \theta 1 \theta 3 - b^2 k \theta 2 \theta 3 + b^2 k \theta 3^2$$

```
D[T, dθ1]
```

$$b^2 d\theta 1 m$$

```
Tmat = m b^2 DiagonalMatrix[{1, 1, 1}];
Tmat // MatrixForm
```

$$\begin{pmatrix} b^2 m & 0 & 0 \\ 0 & b^2 m & 0 \\ 0 & 0 & b^2 m \end{pmatrix}$$

```
D[V, θ1]
```

$$2 b^2 k \theta 1 - b^2 k \theta 2 - b^2 k \theta 3$$

```
Vmat = k b^2 {{2, -1, -1}, {-1, 2, -1}, {-1, -1, 2}};
Vmat // MatrixForm
```

$$\begin{pmatrix} 2 b^2 k & -b^2 k & -b^2 k \\ -b^2 k & 2 b^2 k & -b^2 k \\ -b^2 k & -b^2 k & 2 b^2 k \end{pmatrix}$$

```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```

$$\begin{pmatrix} 2 b^2 k - b^2 m \omega 2 & -b^2 k & -b^2 k \\ -b^2 k & 2 b^2 k - b^2 m \omega 2 & -b^2 k \\ -b^2 k & -b^2 k & 2 b^2 k - b^2 m \omega 2 \end{pmatrix}$$

```
sol = Solve[Det[mat] == 0, ω2]
```

$$\left\{\{\omega 2 \to 0\}, \left\{\omega 2 \to \frac{3 k}{m}\right\}, \left\{\omega 2 \to \frac{3 k}{m}\right\}\right\}$$

```
eq1 = mat .{a, b, c} == 0 // Thread;
eq1 // Column
```

$$-b^3\,k - b^2\,c\,k + a\left(2\,b^2\,k - b^2\,m\,\omega 2\right) == 0$$
$$-a\,b^2\,k - b^2\,c\,k + b\left(2\,b^2\,k - b^2\,m\,\omega 2\right) == 0$$
$$-a\,b^2\,k - b^3\,k + c\left(2\,b^2\,k - b^2\,m\,\omega 2\right) == 0$$

```
norm = a^2 + b^2 + c^2 == 1
```

$$a^2 + b^2 + c^2 == 1$$

```
eq2 = Join[eq1, {norm}];
eq2 // Column
```

$$-b^3\,k - b^2\,c\,k + a\left(2\,b^2\,k - b^2\,m\,\omega 2\right) == 0$$
$$-a\,b^2\,k - b^2\,c\,k + b\left(2\,b^2\,k - b^2\,m\,\omega 2\right) == 0$$
$$-a\,b^2\,k - b^3\,k + c\left(2\,b^2\,k - b^2\,m\,\omega 2\right) == 0$$
$$a^2 + b^2 + c^2 == 1$$

```
sol1 = Solve[eq2 /. sol[[1]], {a, b, c}] // Last
```

Solve: Equations may not give solutions for all "solve" variables.

$$\left\{a \to \frac{1}{\sqrt{3}},\ b \to \frac{1}{\sqrt{3}},\ c \to \frac{1}{\sqrt{3}}\right\}$$

```
ev1 = {a, b, c} /. sol1
```

$$\left\{\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}}\right\}$$

```
sol2 = Solve[eq2 /. sol[[2]], {a, b, c}] /. a -> 0
```

Solve: Equations may not give solutions for all "solve" variables.

$$\left\{\{b \to 0,\ c \to -1\},\ \{b \to 0,\ c \to 1\},\ \left\{b \to -\frac{1}{\sqrt{2}},\ c \to \frac{1}{\sqrt{2}}\right\},\ \left\{b \to \frac{1}{\sqrt{2}},\ c \to -\frac{1}{\sqrt{2}}\right\}\right\}$$

```
ev2 = {a, b, c} /. Last[sol2]
```

$$\left\{a,\ \frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right\}$$

```
ev2 = {a, b, c} /. Last[sol2] /. a -> 0
```

$$\left\{0,\ \frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right\}$$

```
eVecs = Orthogonalize[{ev1, ev2, {1, 2, 3}}] // Simplify
```

$$\left\{\left\{\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}}\right\},\ \left\{0,\ \frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right\},\ \left\{-\sqrt{\frac{2}{3}},\ \frac{1}{\sqrt{6}},\ \frac{1}{\sqrt{6}}\right\}\right\}$$

```
(* a cross check for Chris *)
Transpose[eVecs] == Inverse[eVecs]
eVecs.Transpose[eVecs]
```

True

$$\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$$

```
Vdiag = eVecs.Vmat.Transpose[eVecs] // Simplify;
Vdiag // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 3\,b^2\,k & 0 \\ 0 & 0 & 3\,b^2\,k \end{pmatrix}$$

```
Tdiag = eVecs.Tmat.Transpose[eVecs] // Simplify;
Tdiag // MatrixForm
```

$$\begin{pmatrix} b^2\,m & 0 & 0 \\ 0 & b^2\,m & 0 \\ 0 & 0 & b^2\,m \end{pmatrix}$$

Problem #2)

```
Clear["Global`*"]

T = 1/2 m v2 + 1/2 Ibar ωBar^2;
ωBar = 2/L (dx1 - dx2);
Ibar = 1/12 m L^2;
v2 = dx1^2 + dx2^2;
T // Expand // Factor
```

$$\frac{1}{3}\left(2\,dx1^2 - dx1\,dx2 + 2\,dx2^2\right)\,m$$

```
V = 1/2 m g (x1 + x2) + 1/2 k (x1^2 + x2^2) // Expand
```

$$\frac{g\,m\,x1}{2} + \frac{k\,x1^2}{2} + \frac{g\,m\,x2}{2} + \frac{k\,x2^2}{2}$$

```
D[T, dx1] // Expand
```

$$\frac{4\,dx1\,m}{3} - \frac{dx2\,m}{3}$$

```
D[T, dx2] // Expand
```

$$-\frac{dx1\, m}{3}+\frac{4\, dx2\, m}{3}$$

```
Tmat = m/3 {{2, -1/2}, {-1/2, 2}};
Tmat // MatrixForm
```

$$\begin{pmatrix} \frac{2\, m}{3} & -\frac{m}{6} \\ -\frac{m}{6} & \frac{2\, m}{3} \end{pmatrix}$$

```
D[V, x1]
```

$$\frac{g\, m}{2}+k\, x1$$

```
D[V, x2]
```

$$\frac{g\, m}{2}+k\, x2$$

```
Vmat = k/2 {{1, 0}, {0, 1}};
```

```
Vmat // MatrixForm
```

$$\begin{pmatrix} \frac{k}{2} & 0 \\ 0 & \frac{k}{2} \end{pmatrix}$$

```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```

$$\begin{pmatrix} \frac{k}{2}-\frac{2\, m\, \omega 2}{3} & \frac{m\, \omega 2}{6} \\ \frac{m\, \omega 2}{6} & \frac{k}{2}-\frac{2\, m\, \omega 2}{3} \end{pmatrix}$$

```
sol = Solve[Det[mat] == 0, ω2]
```

$$\left\{\left\{\omega 2 \to \frac{3\, k}{5\, m}\right\}, \left\{\omega 2 \to \frac{k}{m}\right\}\right\}$$

```
eq1 = mat .{a, b} == 0 // Thread
```

$$\left\{\frac{b\, m\, \omega 2}{6}+a\left(\frac{k}{2}-\frac{2\, m\, \omega 2}{3}\right)==0,\ \frac{a\, m\, \omega 2}{6}+b\left(\frac{k}{2}-\frac{2\, m\, \omega 2}{3}\right)==0\right\}$$

```
norm = a^2 + b^2 == 1
```

$$a^2+b^2==1$$

```
eq2 = Join[eq1, {norm}]
```

$$\left\{\frac{b\, m\, \omega 2}{6}+a\left(\frac{k}{2}-\frac{2\, m\, \omega 2}{3}\right)==0,\ \frac{a\, m\, \omega 2}{6}+b\left(\frac{k}{2}-\frac{2\, m\, \omega 2}{3}\right)==0,\ a^2+b^2==1\right\}$$

```
Solve[eq2 /. sol[[1]], {a, b}]
```

$$\left\{\left\{a \to -\frac{1}{\sqrt{2}},\ b \to \frac{1}{\sqrt{2}}\right\},\ \left\{a \to \frac{1}{\sqrt{2}},\ b \to -\frac{1}{\sqrt{2}}\right\}\right\}$$

```
sol1 = Solve[eq2 /. sol[[1]], {a, b}] // Last
```

$$\left\{a \to \frac{1}{\sqrt{2}},\ b \to -\frac{1}{\sqrt{2}}\right\}$$

```
ev1 = {a, b} /. sol1
```

$$\left\{\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right\}$$

```
sol2 = Solve[eq2 /. sol[[2]], {a, b}]
```

$$\left\{\left\{a \to -\frac{1}{\sqrt{2}},\ b \to -\frac{1}{\sqrt{2}}\right\},\ \left\{a \to \frac{1}{\sqrt{2}},\ b \to \frac{1}{\sqrt{2}}\right\}\right\}$$

```
ev2 = {a, b} /. Last[sol2]
```

$$\left\{\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right\}$$

```
eVecs = {ev1, ev2}
```

$$\left\{\left\{\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right\},\ \left\{\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right\}\right\}$$

```
eVecs.Vmat.Transpose[eVecs] // Simplify // MatrixForm
```

$$\begin{pmatrix} \frac{k}{2} & 0 \\ 0 & \frac{k}{2} \end{pmatrix}$$

```
eVecs.Tmat.Transpose[eVecs] // Simplify // MatrixForm
```

$$\begin{pmatrix} \frac{5\,m}{6} & 0 \\ 0 & \frac{m}{2} \end{pmatrix}$$

Part B Look at motion

```
values = {k → 1, m → 1};
```

```
sol
```

$$\left\{\left\{\omega 2 \to \frac{3\,k}{5\,m}\right\},\ \left\{\omega 2 \to \frac{k}{m}\right\}\right\}$$

```
mode1 = ev1 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[1]]
```

$$\left\{\frac{e^{i \sqrt{\frac{3}{5}} \sqrt{\frac{k}{m}}\, t}}{\sqrt{2}}, -\frac{e^{i \sqrt{\frac{3}{5}} \sqrt{\frac{k}{m}}\, t}}{\sqrt{2}}\right\}$$

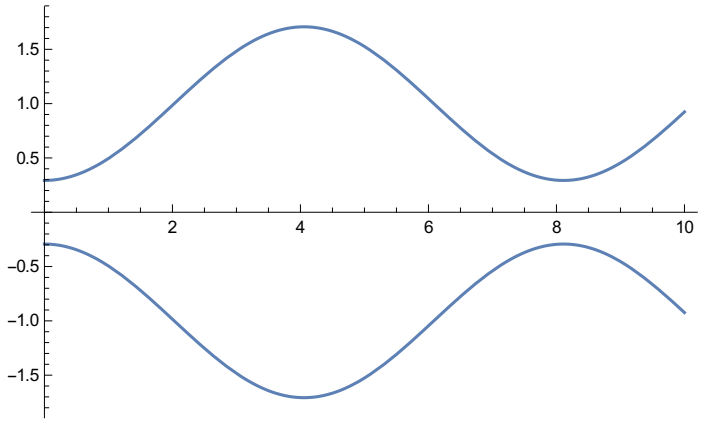
```
mode2 = ev2 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[2]]
```

$$\left\{\frac{e^{i \sqrt{\frac{k}{m}}\, t}}{\sqrt{2}}, \frac{e^{i \sqrt{\frac{k}{m}}\, t}}{\sqrt{2}}\right\}$$

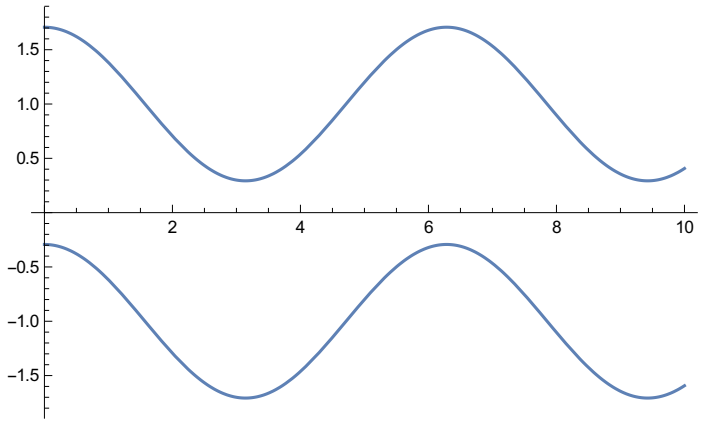
```
mode1 + {-1, 1} /. values
```

$$\left\{-1 + \frac{e^{i \sqrt{\frac{3}{5}}\, t}}{\sqrt{2}}, 1 - \frac{e^{i \sqrt{\frac{3}{5}}\, t}}{\sqrt{2}}\right\}$$

```
Plot[mode1 + {-1, 1} /. values // Re, {t, 0, 10}]
```




```
Plot[mode2 + {-1, 1} /. values // Re, {t, 0, 10}]
```




```
{x1, x2} == eVecs.{n1, n2}
```

$$\{x1, x2\} == \left\{\frac{n1}{\sqrt{2}} - \frac{n2}{\sqrt{2}}, \frac{n1}{\sqrt{2}} + \frac{n2}{\sqrt{2}}\right\}$$

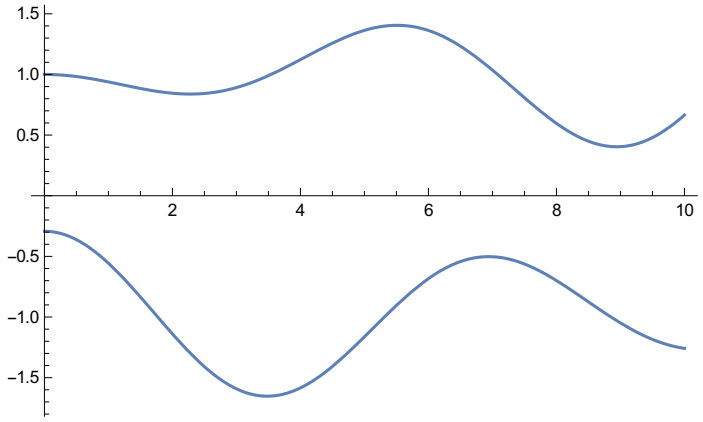
```
{x1, x2} == eVecs.{n1, n2} /. {n1 → 1/2, n2 → 1/2}
```

$$\{x1, x2\} == \left\{0, \frac{1}{\sqrt{2}}\right\}$$

```
mix[a_] := a mode1 + (1 - a) mode2 /. values // Re
mix[1/2] /. {t → 0} // Simplify
```

$$\left\{\frac{1}{\sqrt{2}}, 0\right\}$$

```
doPlot[a_: 1/2] := Plot[a (mode1 + {-1, 1} /. values) + (1 - a) (mode2 + {-1, 1} /. values)
    // Re, {t, 0, 10}]
doPlot[1/2]
```



Problem #3)

```
Clear["Global`*"]
```

```
T = 1/2 m v1^2 + 1/2 m v2^2
```

$$\frac{m\, v1^2}{2} + \frac{m\, v2^2}{2}$$

```
V = 1/2 (3 k) x1^2 + 1/2 (2 k) (x1 - x2)^2 // Expand
```

$$\frac{5\, k\, x1^2}{2} - 2\, k\, x1\, x2 + k\, x2^2$$

```
D[T, v1]
```

$$m\, v1$$

```
Tmat = m DiagonalMatrix[{1, 1}];
Tmat // MatrixForm
```

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

```
D[V, x1]
```

$$5\,k\,x1 - 2\,k\,x2$$

```
D[V, x2]
```

$$-2\,k\,x1 + 2\,k\,x2$$

```
Vmat = k {{5, -2}, {-2, 2}};
Vmat // MatrixForm
```

$$\begin{pmatrix} 5\,k & -2\,k \\ -2\,k & 2\,k \end{pmatrix}$$

```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```

$$\begin{pmatrix} 5\,k - m\,\omega 2 & -2\,k \\ -2\,k & 2\,k - m\,\omega 2 \end{pmatrix}$$

```
sol = Solve[Det[mat] == 0, ω2]
```

$$\left\{\left\{\omega 2 \to \frac{k}{m}\right\}, \left\{\omega 2 \to \frac{6\,k}{m}\right\}\right\}$$

```
eq1 = mat .{a, b} == 0 // Thread;
eq1 // Column
```

$$-2\,b\,k + a\,(5\,k - m\,\omega 2) == 0$$
$$-2\,a\,k + b\,(2\,k - m\,\omega 2) == 0$$

```
norm = a^2 + b^2 == 1
```

$$a^2 + b^2 == 1$$

```
eq2 = Join[eq1, {norm}];
eq2 // Column
```

$$-2\,b\,k + a\,(5\,k - m\,\omega 2) == 0$$
$$-2\,a\,k + b\,(2\,k - m\,\omega 2) == 0$$
$$a^2 + b^2 == 1$$

```
sol1 = Solve[eq2 /. sol[[1]], {a, b}] // Last
```

$$\left\{a \to \frac{1}{\sqrt{5}}, b \to \frac{2}{\sqrt{5}}\right\}$$

```
ev1 = {a, b} /. sol1
```

$$\left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\}$$

```
sol2 = Solve[eq2 /. sol[[2]], {a, b}]
```

$$\left\{\left\{a \to -\frac{2}{\sqrt{5}}, b \to \frac{1}{\sqrt{5}}\right\}, \left\{a \to \frac{2}{\sqrt{5}}, b \to -\frac{1}{\sqrt{5}}\right\}\right\}$$

```
ev2 = {a, b} /. Last[sol2]
```

$$\left\{\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right\}$$

```
eVecs = {ev1, ev2} // Simplify
```

$$\left\{\left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\}, \left\{\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right\}\right\}$$

```
eVecs.eVecs
```

$$\{\{1, 0\}, \{0, 1\}\}$$

```
Vdiag = eVecs.Vmat.Transpose[eVecs] // Simplify ;
Vdiag // MatrixForm
```

$$\begin{pmatrix} k & 0 \\ 0 & 6\,k \end{pmatrix}$$

```
Tdiag = eVecs.Tmat.Transpose[eVecs] // Simplify;
Tdiag // MatrixForm
```

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

Problem #4)

```
Clear["Global`*"]
```

```
T = 1/2 m v1^2 + 1/2 m (v1 + L dθ)^2 // Expand
```

$$\frac{1}{2} d\theta^2\, L^2\, m + d\theta\, L\, m\, v1 + m\, v1^2$$

```
V = m g L (1 - Cos[θ]) + 1/2 k x^2 // Expand
```

$$g\, L\, m + \frac{k\, x^2}{2} - g\, L\, m\, \text{Cos}[\theta]$$

```
Series[Cos[θ], {θ, 0, 2}] // Normal
```

$$1 - \frac{\theta^2}{2}$$

```
V = V /. {Cos[θ] → 1 - θ^2/2} // Simplify
```

$$\frac{1}{2}\left(k\,x^2 + g\,L\,m\,\theta^2\right)$$

```
D[T, v1]
```

$$d\theta\,L\,m + 2\,m\,v1$$

```
D[T, dθ]
```

$$d\theta\,L^2\,m + L\,m\,v1$$

```
Tmat = m {{2, L}, {L, L^2}};
Tmat // MatrixForm
```

$$\begin{pmatrix} 2\,m & L\,m \\ L\,m & L^2\,m \end{pmatrix}$$

```
D[V, x]
```

$$k\,x$$

```
D[V, θ]
```

$$g\,L\,m\,\theta$$

```
Vmat = {{k, 0}, {0, g L m}};
Vmat // MatrixForm
```

$$\begin{pmatrix} k & 0 \\ 0 & g\,L\,m \end{pmatrix}$$

```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```

$$\begin{pmatrix} k - 2\,m\,\omega 2 & -L\,m\,\omega 2 \\ -L\,m\,\omega 2 & g\,L\,m - L^2\,m\,\omega 2 \end{pmatrix}$$

```
rules1 = Solve[k/m == g/L, k][[1]]
```

$$\left\{k \to \frac{g\,m}{L}\right\}$$

```
mat = mat //. rules1 // Simplify;
mat // MatrixForm
```

$$\begin{pmatrix} \frac{m\,(g - 2\,L\,\omega 2)}{L} & -L\,m\,\omega 2 \\ -L\,m\,\omega 2 & L\,m\,(g - L\,\omega 2) \end{pmatrix}$$

```
sol = Solve[Det[mat] == 0, ω2] // Simplify
```

$$\left\{\left\{\omega 2 \to -\frac{\left(-3 + \sqrt{5}\right)\,g}{2\,L}\right\}, \left\{\omega 2 \to \frac{\left(3 + \sqrt{5}\right)\,g}{2\,L}\right\}\right\}$$

```
eq1 = mat .{a, b} == 0 // Thread;
eq1 // Column
```

$$-b\,L\,m\,\omega 2+\frac{a\,m\,(g-2\,L\,\omega 2)}{L}==0$$
$$-a\,L\,m\,\omega 2+b\,L\,m\,\left(g-L\,\omega 2\right)==0$$

```
norm = a^2 + b^2 == 1
```

$$a^2+b^2==1$$

```
eq2 = Join[eq1, {norm}];
eq2 // Column
```

$$-b\,L\,m\,\omega 2+\frac{a\,m\,(g-2\,L\,\omega 2)}{L}==0$$
$$-a\,L\,m\,\omega 2+b\,L\,m\,\left(g-L\,\omega 2\right)==0$$
$$a^2+b^2==1$$

```
sol1 = Solve[eq2 /. sol[[1]], {a, b}][[1]] // Simplify
```

$$\left\{a\to \frac{i\sqrt{2}\,\left(-1+\sqrt{5}\right)\,L}{\left(-3+\sqrt{5}\right)\,\sqrt{-2-\left(3+\sqrt{5}\right)\,L^2}},\ b\to -\frac{i\sqrt{2}}{\sqrt{-2-\left(3+\sqrt{5}\right)\,L^2}}\right\}$$

```
ev1 = {a, b} /. sol1
```

$$\left\{\frac{i\sqrt{2}\,\left(-1+\sqrt{5}\right)\,L}{\left(-3+\sqrt{5}\right)\,\sqrt{-2-\left(3+\sqrt{5}\right)\,L^2}},\ -\frac{i\sqrt{2}}{\sqrt{-2-\left(3+\sqrt{5}\right)\,L^2}}\right\}$$

```
sol2 = Solve[eq2 /. sol[[2]], {a, b}][[1]] // Simplify
```

$$\left\{a\to \frac{\left(-2+\sqrt{5}\right)\,\left(3+\sqrt{5}\right)\,\sqrt{7+3\sqrt{5}}\,L}{2\sqrt{7+3\sqrt{5}+\left(3+\sqrt{5}\right)\,L^2}},\ b\to -\frac{\sqrt{7+3\sqrt{5}}}{\sqrt{7+3\sqrt{5}+\left(3+\sqrt{5}\right)\,L^2}}\right\}$$

```
ev2 = {a, b} /. sol2
```

$$\left\{\frac{\left(-2+\sqrt{5}\right)\,\left(3+\sqrt{5}\right)\,\sqrt{7+3\sqrt{5}}\,L}{2\sqrt{7+3\sqrt{5}+\left(3+\sqrt{5}\right)\,L^2}},\ -\frac{\sqrt{7+3\sqrt{5}}}{\sqrt{7+3\sqrt{5}+\left(3+\sqrt{5}\right)\,L^2}}\right\}$$

eVecs = {ev1, ev2} // Simplify

$$\left\{\left\{\frac{i\sqrt{2}\left(-1+\sqrt{5}\right)L}{\left(-3+\sqrt{5}\right)\sqrt{-2-\left(3+\sqrt{5}\right)L^2}},-\frac{i\sqrt{2}}{\sqrt{-2-\left(3+\sqrt{5}\right)L^2}}\right\},\right.$$

$$\left.\left\{\frac{\left(-2+\sqrt{5}\right)\left(3+\sqrt{5}\right)\sqrt{7+3\sqrt{5}}\,L}{2\sqrt{7+3\sqrt{5}+\left(3+\sqrt{5}\right)L^2}},-\frac{\sqrt{7+3\sqrt{5}}}{\sqrt{7+3\sqrt{5}+\left(3+\sqrt{5}\right)L^2}}\right\}\right\}$$

eVecs.eVecs // FullSimplify

$$\left\{\left\{L\left(\frac{2L}{3-\sqrt{5}+2L^2}-\frac{i\left(-1+\sqrt{5}\right)}{2\sqrt{-1-L^2\left(3+L^2\right)}}\right),\frac{\left(-1+\sqrt{5}\right)L}{3-\sqrt{5}+2L^2}+\frac{i}{\sqrt{-1-L^2\left(3+L^2\right)}}\right\},\right.$$

$$\left\{-\left(\left(2L\left(iL\left(3+\sqrt{5}+2L^2\right)+\left(1+\sqrt{5}\right)\sqrt{-1-L^2\left(3+L^2\right)}\right)\right)\Big/\right.\right.$$

$$\left.\left.\left(\sqrt{-3+\sqrt{5}-2L^2}\left(3+\sqrt{5}+2L^2\right)^{3/2}\right)\right),\frac{1}{1-\frac{1}{2}\left(-3+\sqrt{5}\right)L^2}-\frac{i\left(-1+\sqrt{5}\right)L}{2\sqrt{-1-L^2\left(3+L^2\right)}}\right\}\right\}$$

Vdiag = eVecs.Vmat.Transpose[eVecs] // FullSimplify ;
Vdiag // MatrixForm

$$\begin{pmatrix} k+\frac{-2k+2gLm}{2+\left(3+\sqrt{5}\right)L^2} & \frac{iL\left(-kL+gm\right)}{\sqrt{-1-L^2\left(3+L^2\right)}} \\ \frac{iL\left(-kL+gm\right)}{\sqrt{-1-L^2\left(3+L^2\right)}} & \frac{L\left(\left(3+\sqrt{5}\right)kL+\left(7+3\sqrt{5}\right)gm\right)}{7+3\sqrt{5}+\left(3+\sqrt{5}\right)L^2} \end{pmatrix}$$

Vdiag = eVecs.Vmat.Transpose[eVecs] /. rules1 // FullSimplify ;
Vdiag // MatrixForm

$$\begin{pmatrix} \frac{\left(5+\sqrt{5}\right)gLm}{2+\left(3+\sqrt{5}\right)L^2} & 0 \\ 0 & \frac{2\left(5+2\sqrt{5}\right)gLm}{7+3\sqrt{5}+\left(3+\sqrt{5}\right)L^2} \end{pmatrix}$$

```
Tdiag = eVecs.Tmat.Transpose[eVecs] /. rules1 // FullSimplify;
Tdiag // MatrixForm
```

$$\begin{pmatrix} \frac{2\left(5+2\sqrt{5}\right)L^2 m}{2+\left(3+\sqrt{5}\right)L^2} & 0 \\ 0 & \frac{\left(5+\sqrt{5}\right)L^2 m}{7+3\sqrt{5}+\left(3+\sqrt{5}\right)L^2} \end{pmatrix}$$