****

Orbits: Kepler: $1/r^2$

```
Clear["Global`*"]
```

```
power = 1.0
```

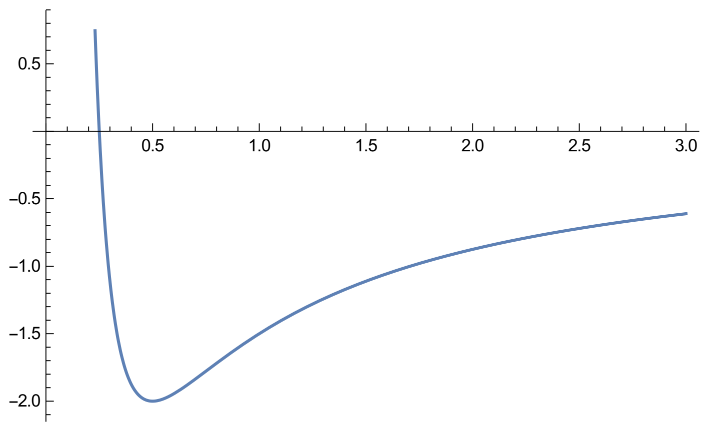
```
1.
```

```
values={m->1, el->1, k->2,energy->-1};
```

```
Veff=((1/2) el^2/(m r^2) + (-k/r^power))
```

$$\frac{el^2}{2\,m\,r^2}-\frac{k}{r^{1.}}$$

```
Plot[ Veff /.values, {r,0.1,3},AxesOrigin->{0,0}]
```




```
T=energy-Veff
```

$$\text{energy}-\frac{el^2}{2\,m\,r^2}+\frac{k}{r^{1.}}$$

```
eq1= (1/2) m r'[t]^2 - T ==0
```

$$-\text{energy}+\frac{el^2}{2\,m\,r^2}-\frac{k}{r^{1.}}+\frac{1}{2}\,m\,r'[t]^2==0$$

```
dr2=r'[t]^2 /.Solve[eq1,r'[t]][[1]] //Apart
```

$$-\frac{1.\,el^2}{m^2\,r^{2.}}+\frac{2.\,\left(1.\,k+1.\,\text{energy}\,r^{1.}\right)}{m\,r^{1.}}$$

```
Plot[dr2 /.values,{r,0.1,4}];
```

```
root1=FindRoot[0==dr2 /.values //Evaluate,{r,0.02}]
```

$\{r\to 0.292893\}$

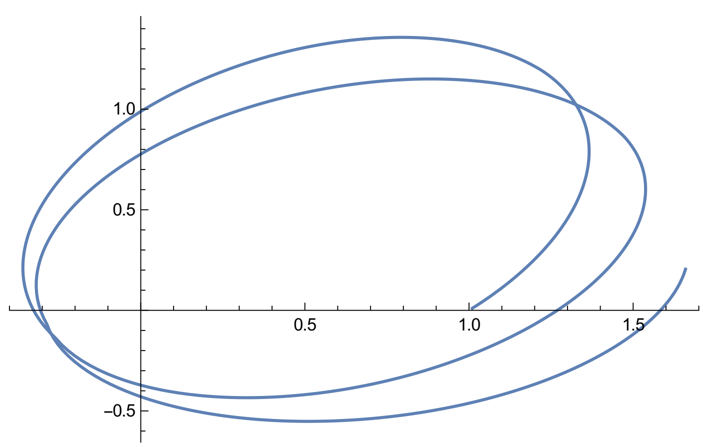
```
root2=FindRoot[0==dr2 /.values //Evaluate,{r,1}]
```

{r → 1.70711}

```
{rmax,rmin}=  {root1,root2} //Map[Last,#,{2}]& //Flatten
```

{0.292893, 1.70711}

```
r0=1;
theta0=0;
t0=0;
timeStep=0.01;
rSign=+1;
xarray={};
print=False;

Do[
      dr0= (rSign Sqrt[ dr2] timeStep) //.values //.r->r0;
      r1= r0 + dr0;
      dtheta0=el/( m r0^2) //.values;
      theta1= theta0 + dtheta0 timeStep;

      test=  dr2 //.values //.r->r1;
      (* If[test<=0 ,r1= r0 - 1 dr0; rSign=-rSign;]; *)
      If[test<=0 ,r1= r0 - 1 dr0; rSign=-rSign;];

      t0=t0+timeStep;
      r0=r1;
      theta0=theta1;

      xarray=Append[xarray,{r1 Cos[theta1], r1 Sin[theta1] }];

      If[print,Print[t0,"   ",r0,"   ",dr0,"   ",theta0]; ]
  ,{i,1,1000}]

  ListPlot[xarray,PlotJoined->True]
```



****

Orbits: Quadratic: r^2

```
Clear["Global`*"]
```

In[1]:=
```
power = -2.0
```

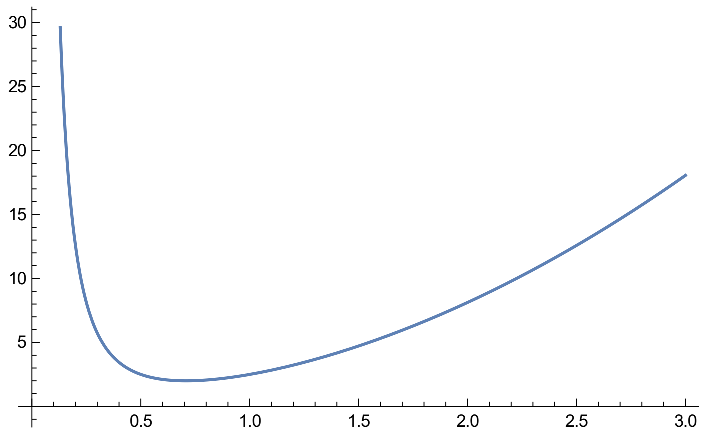
Out[1]= $-2.$

In[15]:=
```
values={m->1, el->1, k->-2,energy->+10};
```

In[16]:=
```
Veff=((1/2) el^2/(m r^2) + (-k/r^power))
```

Out[16]= $\frac{el^2}{2\, m\, r^2} - k\, r^{2.}$

In[17]:=
```
Plot[ Veff /.values, {r,0.1,3},AxesOrigin->{0,0}]
```

Out[17]=


In[18]:=
```
T=energy-Veff
```

Out[18]= $\text{energy} - \frac{el^2}{2\, m\, r^2} + k\, r^{2.}$

In[19]:=
```
eq1= (1/2) m r'[t]^2 - T ==0
```

Out[19]= $-\text{energy} + \frac{el^2}{2\, m\, r^2} - k\, r^{2.} + \frac{1}{2}\, m\, r'[t]^2 == 0$

In[20]:=
```
dr2=r'[t]^2 /.Solve[eq1,r'[t]][[1]] //Apart
```

Out[20]= $\frac{2.\, k\, r^{2.}}{m} + \frac{2.\, \left(-0.5\, el^2 + 1.\, \text{energy}\, m\, r^2\right)}{m^2\, r^2}$

In[21]:=
```
Plot[dr2 /.values,{r,0.1,4}];
```

In[22]:=
```
root1=FindRoot[0==dr2 /.values //Evaluate,{r,0.02}]
```

Out[22]= $\{r \to 0.224745\}$

In[23]:=
```
root2=FindRoot[0==dr2 /.values //Evaluate,{r,1}]
```

Out[23]= {r → 2.22474}

In[24]:=
```
{rmax,rmin}=  {root1,root2} //Map[Last,#,{2}]& //Flatten
```

Out[24]= {0.224745, 2.22474}

In[70]:=
```
r0=1;
theta0=0;
t0=0;
timeStep=0.001;
rSign=+1;
xarray={};
print=False;

Do[
       dr0= (rSign Sqrt[ dr2] timeStep) //.values //.r->r0;
       r1= r0 + dr0;
       dtheta0=el/( m r0^2) //.values;
       theta1= theta0 + dtheta0 timeStep;

       test=  dr2 //.values //.r->r1;
       (* If[test<=0 ,r1= r0 - 1 dr0; rSign=-rSign;]; *)
       If[test<=0 ,r1= r0 - 1 dr0; rSign=-rSign;];

       t0=t0+timeStep;
       r0=r1;
       theta0=theta1;

       xarray=Append[xarray,{r1 Cos[theta1], r1 Sin[theta1] }];

       If[print,Print[t0,"   ",r0,"   ",dr0,"   ",theta0]; ]
   ,{i,1,30000}]

   ListPlot[xarray,PlotJoined->True]
```

Out[80]=