IMPORTANT:

- This exam is OPEN book (hard-copy only), OPEN notes (hard-copy only).
- No other sources are allowed. CLOSED INTERNET!
- You may use a calculator, but NOT a cell phone calculator.
- You need to do your work in front of the computer please.
- Show your work. I need to see all steps.
- When you complete the exam, please upload your work promptly.

SUGGESTIONS:

1) Show all work. Explain if it might help with partial credit.
2) Use symbols as much as possible. It makes it easier to allocate partial credit.
3) Do not waste an excessive amount of time on any one problem or individual part. Note the point allocation, and budget your time accordingly.
4) Write clearly so that we can try to reconstruct your steps. Make figures of reasonable size.
5) Make sure that you do all the problems.

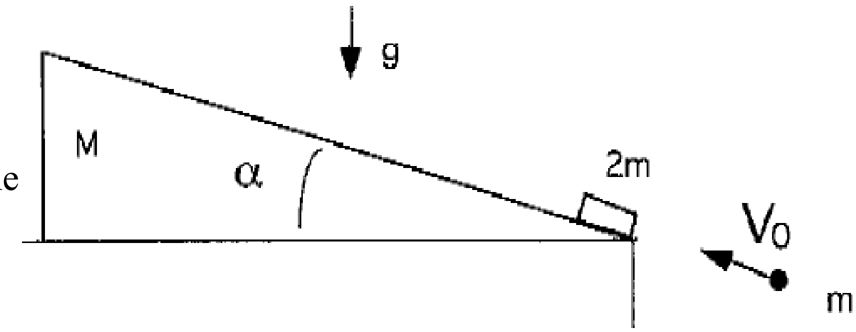
#	Points	Score:
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	**30**	
8	**30**	
9	**30**	
10	**30**	
Total	**240**	

Problem 1: (20 Pts)

A slider of mass 2m is initially at rest at the bottom edge of a wedge which is stationary and has angle α. Assume there is NO friction. At t=0 a bullet of mass m and velocity v0 traveling parallel to the incline of the wedge hits and sticks to the slider.

a) What is the maximum height above the table reached by the slider?

b) At what time does the slider reach the maximum height?



Problem 2: (20 Pts)

A rigid body consists of 5 masses fastened as follows:

m at (1,0,0)

2m at (0,1,2) and (0,2,1),

3m at (3,1,0) and (1,3,0).

Find the inertia tensor. *[That is all! Do* ***NOT*** *solve for eigenvalues or eigenvectors.]*

Problem 3: (20 Pts) Consider the below matrices.

Can these represent rotations of a solid object???

a) If yes, **what** is the rotation angle???

b) If not, clearly explain **why** they can NOT be rotations?

$$M_1 = \begin{pmatrix} \frac{1}{2} & +\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \; M_2 = \begin{pmatrix} \frac{1}{2} & +\frac{\sqrt{3}}{2} \\ +\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \; M_3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$M_4 = \begin{pmatrix} -1 & 0 & \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \; M_5 = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \; M_6 = \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Problem 4: (20 Pts) For the below matrix, find the eigenvalues and the associated eigenvectors. Be sure to indicate which eigenvalues go with each eigenvectors.

I want all 3 of them!

Hint: Look closely at this matrix; one of the eigenvalues is 5, so you only have to solve a quadratic, NOT a cubic.

$$M = \begin{pmatrix} 5 & 0 & 0 \\ 0 & \frac{27}{3} & \frac{8}{3} \\ 0 & \frac{18}{3} & \frac{27}{3} \end{pmatrix}$$

Problem 5: (20 Pts)

Consider m=1kg of water on the surface of the earth.

a) Find the relative force on m due to the sun compared to the moon.

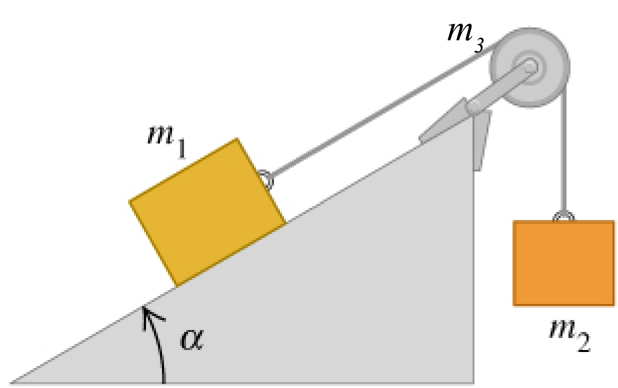
b) Presumably you found the force of the sun is larger than the moon; but, why are the tided influence more by the moon rather than the sun? Explain **clearly** and use any formulas you think might help.

Problem 6: (20 Pts)

a) For the below M and K matrices, find the frequencies of motion, and the normal modes. If you like, for simplicity, you can set m=k=1.

$$M = \begin{pmatrix} m & 2m & 0 \\ 2m & m & 0 \\ 0 & 0 & m \end{pmatrix} \quad K = \begin{pmatrix} 2k & k & -k \\ k & 2k & k \\ -k & k & 2k \end{pmatrix}$$

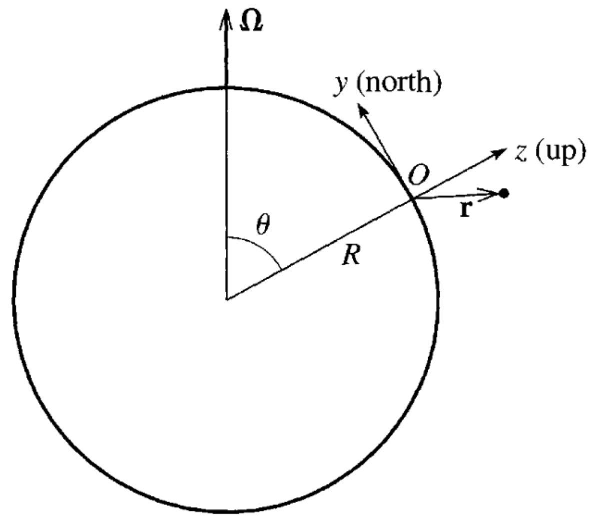
Problem 7: (30 Pts) Consider an Atwood machine with mass m_1 on a frictionless ramp with a MASSIVE pulley of mass m_3, radius r, and moment of inertia of $I_3 = (2/5)\, m_3\, r^2$, and a hanging mass m_2. The string is wound around the pulley and does not slip. The system starts from rest. For coordinates, measure the distance of m_1 along the ramp to be x_1, and the rotation of the pulley to be θ.



Part a) Compute the Lagrangian L, and obtain the associated Euler-Lagrange equations of motion in terms of $\{x_1, x_1', x_2, x_2', \theta, \theta'\}$ using a Lagrange multiplier λ_1 for the constraint the total length of string is constant, and λ_2 for the constraint that the pulley does not slip.. **SHOW WORK!**

Part b) Find the acceleration of the m_1. Also solve the Lagrange multipliers λ. **SHOW WORK**

Problem 8: (30 Pts) SpaceX drops a payload from space down to Dallas (θ =57 degrees co-latititude). Assume it's initial motion is in a direct line to the center of the earth, and the velocity is constant v.

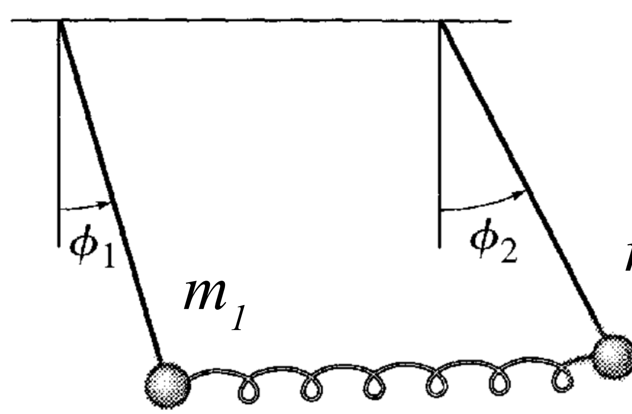


Part a) Compute the magnitude and direction of the Coriolis and centrifugal forces.

Part b) If the object fall at a constant velocity v=100m/s from a height of 10km, find the approximate displacement each due to the Coriolis and centrifugal forces.

Part c) For an object moving in some direction $v_2$near Dallas, is there any v_2 where the direction of the Coriolis and centrifugal forces could be aligned. If so, what is the direction, OR if not, why not?

Problem 9: (30 Points)
The figure shows a system of two coupled pendulums, each of length L and masses m_1 and m_2. Assume the motion is confined to a plane, and the angles ϕ_1 and ϕ_2 are small. The equilibrium position of the spring (with constant k) is for $\phi_1 = \phi_2 = 0$.
(Note, you can ignore gravity.)



Part a: Write down the total kinetic energy T, the potential energy V, and the Lagrangian L.

Part b: Now, let's set m_1=m_2=m and compute the Lagrange equations of motion.

Part c: Find and describe the normal modes.

<u>Problem 10:</u> (30 Points)

A newly invented particle the SMUon (no relation to the muon) is found to be created by cosmic rays in the upper atmosphere and travel at (essentially) the speed of light toward the surface of the earth.

a) The lifetime of the SMUon particle in its reference frame is found to be 0.5μs. WITHOUT the effects of special relativity, how far will the SMUon travel before it decays.

b) Now consider relativistic effects on the observed lifetime of the SMUon. To make a SMUon collider, let us suppose the it must typically travel 10km. Compute the relativistic γ necessary.

c) Use the relativistic γ the relativistic β.

d) An observer is at rest on an asteroid. From the perspective of that observer, two space ships approach each other each traveling at $v = 0.8c$. When they pass, what is their relative speed from the perspective of an observer on either ship?

<u>Let's now look at the Electron Ion Collider (EIC).</u>

Useful information: m_p=1 GeV, m_e=1/2000 GeV.
(We'll work in GeV units and make these approximations.)
Recall "s" is the invariant length-squared of the energy momentum vector, similar to $r^2=x^2+y^2$.

e) Let's collide an electron and proton in collider mode.
The proton has an energy of 275GeV and the electron has an energy of 18GeV.
Compute the CMS energy rs=√s, where rs is the square-root of s, it has units of energy (GeV), and represent the total energy momentum available to "spend" on producing particles.
<u>I want a number!</u>
[Hint, for this part, you can neglect the masses of BOTH the proton and electron, so the energy and momentum components are equal: i.e., p^μ =(p1,0,0,0,p1), e^μ =(p2,0,0,0,-p2).]

f) Let's collide an electron and proton when the proton is at rest.
Compute the CMS energy rs=√s. <u>I want a number!</u>
[Hint, for this part, you can neglect the masses of the electron, but NOT the proton, so the energy and momentum components are equal: i.e., p^μ =(m_p,0,0,0,0), e^μ =(p2,0,0,0,-p2).]

g) Let's collide an electron and proton when the electron is at rest.
Compute the CMS energy rs=√s. <u>I want a number!</u>
[Hint, for this part, you can neglect the masses of the proton, but NOT the electron, so the energy and momentum components are equal: i.e., p^μ =(p1,0,0,0,p1), e^μ =(m_e,0,0,0,0).]