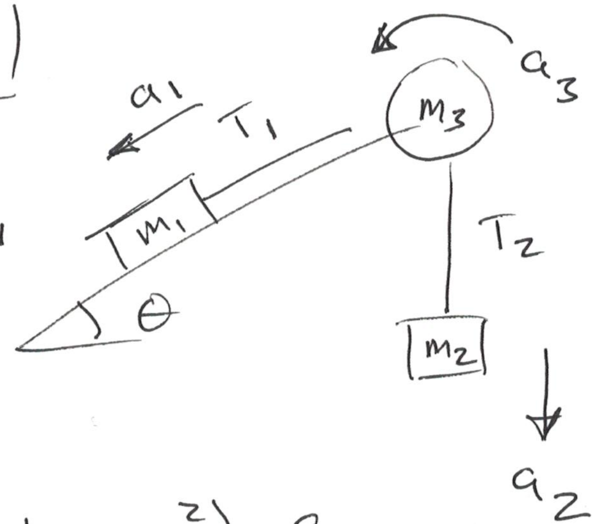
Exam # 1 Problem # 2



① $m_1 g \sin\theta - T_1 - \mu N = m_1 a_1$

② $m_1 g \cos\theta - N = 0$

③ $m_2 g - T_2 = m_2 a_2$

④ $r T_1 - r T_2 = I_3 \alpha_3 \equiv \left(\frac{1}{2} m_3 r^2\right) \frac{a_3}{r}$

⑤ $a_1 = -a_2$ ⑥ $a_3 = +a_1$

Check your convention and directions

ⓑ $a_1 = a_2 = a_3 = 0$

$T_1 = T_2 = m_2 g$ $N = m_1 g \cos\theta$

$\mu = \tan\theta - \frac{m_2}{m_1} \sec\theta \equiv \frac{m_1 \sin\theta - m_2}{m_1 \cos\theta}$

#3 (a) $F = ma = m\dot{v}$

$$m\dot{v} = mg - bv$$

$$\dot{v} = 0 \rightarrow \quad v_T = \frac{mg}{b} \quad \text{or} \quad mg = v_T b$$

(b) $m\dot{v} = mg - bv = -b(v - v_T)$

Let $u = (v - v_T) \quad du = dv$

$$m\dot{u} = m\frac{du}{dt} = -b(v - v_t) = -bu$$

$$\frac{m\,du}{u} = -b\,dt$$

$$\int_{u_0}^{u} \frac{du}{u} = \int_0^t dt \, \frac{-b}{m}$$

$$\ln\left[\frac{u}{u_0}\right] = -\frac{b}{m}t \Rightarrow u = u_0 e^{-\frac{b}{m}t}$$

$$\Rightarrow (v - v_T) = (v_0 - v_T) e^{-\frac{b}{m}t}$$

$$\boxed{v = v_T + (v_0 - v_T) e^{-\frac{b}{m}t}}$$

Check $\quad t = 0 \quad v = v_0$

$\quad t = \infty \quad v = v_T$

#4 Line Integral $\qquad W = \int F \cdot dr$

$$\vec{F} = (2x - y)\,\hat{x} + (x + 3y)\,\hat{y}$$

(a) $$\int_1^0 F_x(x,0)\, dx + \int_0^1 F_y(0,y)\, dy$$

$$= \int_1^0 (2x - 0)\, dx + \int_0^1 (0 + 3y)\, dy = \frac{1}{2}$$

(b) $$x = (1-t)^2 \qquad y = t^3$$

$$dx = -2(1-t)\, dt \qquad dy = 3t^2\, dt$$

$$\int_P^Q F_x(x,y)\, dx + \int_P^Q F_y(x\,y)\, dy$$

$$\int_0^1 dx\,(2x - y) + dy\,(x + 3y)$$

$$\int_0^1 dt\, \overbrace{[-2(1-t)]}^{dx}\, \overbrace{[2(1-t)^2 - t^3]}^{2x\ -\ y}$$
$$+ \underbrace{[3t^2]}_{dy}\, \underbrace{[(1-t)^2 + 3t^3]}_{x\ +\ 3y}$$

$$= \int_0^1 dt\, [-4 + 12t - 9t^2 + 0t^3 + t^4 + 9t^5]$$

$$= -4 + \frac{12}{2} - \frac{9}{3} + \frac{0}{4} + \frac{1}{5} + \frac{9}{6}$$

$$= 7/10$$