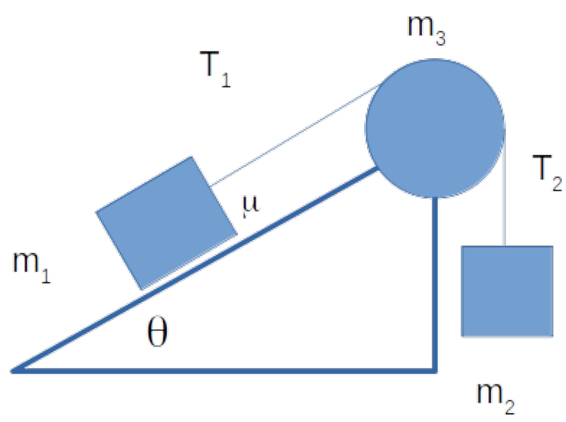
2) Mass on incline :

In[86]:=
```
Clear["Global`*"]
```



Part a)

In[87]:=
```
eqs = {m1 g Sin[θ] - t1 - u normal == m1 a1,
  normal == m1 g Cos[θ],
  m2 g - t2 == m2 a2,
  r t1 - r t2 == 1/2 m3 r^2 a3/r,
  a1 == -a2,
  a3 == a1}
```

Out[87]=
```
{-t1 - normal u + g m1 Sin[θ] == a1 m1, normal == g m1 Cos[θ],
 g m2 - t2 == a2 m2, r t1 - r t2 == (a3 m3 r)/2, a1 == -a2, a3 == a1}
```

In[88]:=
```
Solve[eqs, {t1, t2, normal, a1, a2, a3}] // Simplify
```

Out[88]=
```
{{t1 -> -(g m1 (-2 m2 + (2 m2 + m3) u Cos[θ] - (2 m2 + m3) Sin[θ]))/(2 m1 + 2 m2 + m3),
  t2 -> -(g m2 (-2 m1 - m3 + 2 m1 u Cos[θ] - 2 m1 Sin[θ]))/(2 m1 + 2 m2 + m3),
  normal -> g m1 Cos[θ], a1 -> -(2 g (m2 + m1 u Cos[θ] - m1 Sin[θ]))/(2 m1 + 2 m2 + m3),
  a2 -> (2 g (m2 + m1 u Cos[θ] - m1 Sin[θ]))/(2 m1 + 2 m2 + m3), a3 -> -(2 g (m2 + m1 u Cos[θ] - m1 Sin[θ]))/(2 m1 + 2 m2 + m3)}}
```

Part b)

In[89]:=
```
eqs2 = eqs /. {a1 → 0, a2 → 0, a3 → 0}
```

Out[89]=
```
{-t1 - normal u + g m1 Sin[θ] == 0, normal == g m1 Cos[θ], g m2 - t2 == 0, r t1 - r t2 == 0, True, True}
```

In[90]:=
```
eqs2 // TableForm
```

Out[90]//TableForm=
```
-t1 - normal u + g m1 Sin[θ] == 0
normal == g m1 Cos[θ]
g m2 - t2 == 0
r t1 - r t2 == 0
True
True
```

In[96]:=
```
sol = Solve[eqs2, {t1, t2, normal, u}][[1]] // Simplify
```

Out[96]= $\left\{t1 \to g\,m2,\ t2 \to g\,m2,\ normal \to g\,m1\,\mathrm{Cos}[\theta],\ u \to -\frac{m2\,\mathrm{Sec}[\theta]}{m1} + \mathrm{Tan}[\theta]\right\}$

In[97]:=
```
sol // FullSimplify
```

Out[97]= $\left\{t1 \to g\,m2,\ t2 \to g\,m2,\ normal \to g\,m1\,\mathrm{Cos}[\theta],\ u \to -\frac{m2\,\mathrm{Sec}[\theta]}{m1} + \mathrm{Tan}[\theta]\right\}$

In[101]:=
```
u /. sol // Together // TrigExpand
```

Out[101]= $-\frac{m2\,\mathrm{Sec}[\theta]}{m1} + \mathrm{Tan}[\theta]$

In[103]:= $\frac{m1\,\mathrm{Sin}[\theta] - m2}{m1\,\mathrm{Cos}[\theta]}$ // FullSimplify

Out[103]= $-\frac{m2\,\mathrm{Sec}[\theta]}{m1} + \mathrm{Tan}[\theta]$

3) Falling: linear resistance

In[ ]:=
```
Clear["Global`*"]
```

a) Vertical falling:

In[ ]:=
```
eq1 = m v'[t] == m g - b v[t] - c v[t]^2 /. {c → 0}
```

Out[ ]=
```
m v′[t] == g m - b v[t]
```

In[ ]:=
```
bc = v[0] == v0
```

Out[ ]=
```
v[0] == v0
```

In[ ]:= eq2 = eq1 /. {v'[t] → 0}

Out[ ]= $0 == g\,m - b\,v[t]$

In[ ]:= solT = Solve[eq2, v[t]][[1]]

Out[ ]= $\left\{v[t] \to \frac{g\,m}{b}\right\}$

In[ ]:= vTer = v[t] /. solT

Out[ ]= $\frac{g\,m}{b}$

In[ ]:= vSol = Solve[vT == vTer, b][[1]]

Out[ ]= $\left\{b \to \frac{g\,m}{vT}\right\}$

b) Vertical falling:

In[ ]:= eqs = Join[{eq1, bc}]

Out[ ]= $\{m\,v'[t] == g\,m - b\,v[t],\ v[0] == v0\}$

In[ ]:= dsol1 = DSolve[eqs, v[t], t][[1]] // ExpandAll // Simplify

Out[ ]= $\left\{v[t] \to \frac{e^{-\frac{b\,t}{m}}\left(\left(-1 + e^{\frac{b\,t}{m}}\right) g\,m + b\,v0\right)}{b}\right\}$

In[ ]:= Limit[v[t] /. dsol1, t → Infinity, GenerateConditions → False]

Out[ ]= $\frac{g\,m}{b}$

In[ ]:= dsol2 = dsol1 /. vSol // Simplify

Out[ ]= $\left\{v[t] \to e^{-\frac{g\,t}{vT}}\left(v0 + \left(-1 + e^{\frac{g\,t}{vT}}\right) vT\right)\right\}$

In[ ]:= v[t] /. dsol2 /. t → 0

Out[ ]= v0

In[ ]:= Limit[v[t] /. dsol2, t → Infinity, GenerateConditions → False]

Out[ ]= vT

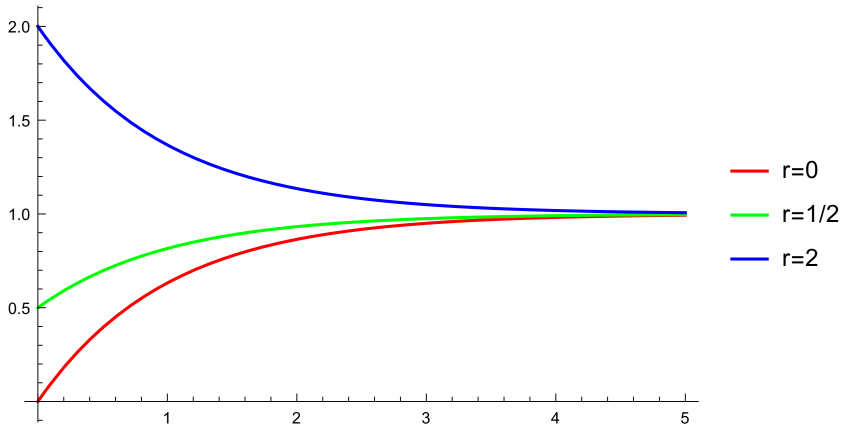
In[ ]:= (* Set v0 = to a ratio "r" of vTerminal : *)
values = {g → vT, v0 → r vT, vT → 1};

In[ ]:= velocity[t_, r_] = v[t] /. dsol2 //. values

Out[ ]= $e^{-t}\left(-1 + e^{t} + r\right)$

In[∘]:=
```
Plot[{
   velocity[t, 0],
   velocity[t, 0.5],
   velocity[t, 2]},
  {t, 0, 5},
  PlotStyle → {Red, Green, Blue},
  PlotLegends → {"r=0", "r=1/2", "r=2"},
  PlotRange → All
]
```

Out[∘]=



a) Vertical falling: quadratic resistance

In[∘]:=
```
eq4 = m v'[t] == m g - b v[t] - c v[t]^2 /. {b → 0}
```

Out[∘]= $m\, v'[t] == g\, m - c\, v[t]^2$

In[∘]:=
```
eq5 = eq4 /. {v'[t] → 0}
```

Out[∘]= $0 == g\, m - c\, v[t]^2$

In[∘]:=
```
solTc = Solve[eq5, v[t]][[1]]
```

Out[∘]= $\left\{v[t] \to -\frac{\sqrt{g}\ \sqrt{m}}{\sqrt{c}}\right\}$

---

4) Line Integral:

In[∘]:=
```
Clear["Global`*"]
```

Part a

In[ ]:=
```
Integrate[2 x, {x, 1, 0}] + Integrate[3 y, {y, 0, 1}]
```
Out[ ]= $\frac{1}{2}$

Part b

In[ ]:=
```
Integrate[2 x, {x, 1, 0}] + Integrate[3 y, {y, 0, 1}]
```
Out[ ]= $\frac{1}{2}$

In[ ]:=
```
rules = {fx → 2 x - y, fy → x + 3 y, x → (1 - t)^2, y → t^3, dx → (-2) (1 - t) dt, dy → 3 t^2 dt}
```
Out[ ]= $\{fx \to 2x - y,\ fy \to x + 3y,\ x \to (1-t)^2,\ y \to t^3,\ dx \to -2\,dt\,(1-t),\ dy \to 3\,dt\,t^2\}$

In[ ]:=
```
integrand = fx dx + fy dy //. rules
```
Out[ ]= $-2\,dt\,(1-t)\left(2(1-t)^2 - t^3\right) + 3\,dt\,t^2\left((1-t)^2 + 3t^3\right)$

In[ ]:=
```
integrand2 = integrand/dt // Expand
```
Out[ ]= $-4 + 12t - 9t^2 + t^4 + 9t^5$

In[ ]:=
```
Integrate[integrand2, {t, 0, 1}]
```
Out[ ]= $\frac{7}{10}$

In[104]:=
```
-4 + 12/2 - 9/3 + 1/5 + 9/6
```
Out[104]= $\frac{7}{10}$

5) Planet Hollywood

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
values = {m -> 7.35 × 10^22, r → 1.74 × 10^6, bigG -> 6.67 × 10^-11}
```
Out[ ]= $\{m \to 7.35 \times 10^{22},\ r \to 1.74 \times 10^{6},\ bigG \to 6.67 \times 10^{-11}\}$

Part a)

In[ ]:= eq1 = m1 g == $\frac{\text{bigG m m1}}{r^2}$

Out[ ]= g m1 == $\frac{\text{bigG m m1}}{r^2}$

In[ ]:= solG = Solve[eq1, g][[1]]

Out[ ]= $\left\{g \to \frac{\text{bigG m}}{r^2}\right\}$

g /. solG /. values

Out[ ]= 1.61925

$\frac{g}{9.8}$ /. solG /. values

Out[ ]= 0.16523

Part b)

In[ ]:= u = $\frac{-\text{bigG m m1}}{r}$;

u /. values /. {m1 → 1}

Out[ ]= -2.8175×10^6

Part c)

In[ ]:= eq3 = u + ke == 0 /. $\left\{ke \to \frac{1}{2}\text{ m1 } v^2\right\}$

Out[ ]= $-\frac{\text{bigG m m1}}{r} + \frac{\text{m1 } v^2}{2} == 0$

In[ ]:= solV = Solve[eq3, v][[2]]

Out[ ]= $\left\{v \to \frac{\sqrt{2}\ \sqrt{\text{bigG}}\ \sqrt{m}}{\sqrt{r}}\right\}$

In[ ]:= vEscape = v /. solV /. values

Out[ ]= 2373.82

In[ ]:= UnitConvert[Quantity[vEscape, $\frac{\text{"Meters"}}{\text{"Seconds"}}$], $\frac{\text{"Miles"}}{\text{"Seconds"}}$]

Out[ ]= 1.47502 mi/s

```
In[ ]:= UnitConvert[Quantity[vEscape, "Meters"/"Seconds"], "Kilometers"/"Seconds"]
```

Out[ ]= 2.37382 km/s

Part d)

In[ ]:= eq4 = $\frac{\text{bigG m m1}}{r^2}$ == m1 $\frac{v^2}{r}$

Out[ ]= $\frac{\text{bigG m m1}}{r^2} == \frac{\text{m1 } v^2}{r}$

In[ ]:= eq5 = v == $\frac{2\pi r}{\text{period}}$

Out[ ]= $v == \frac{2\pi r}{\text{period}}$

```
In[ ]:= UnitConvert[Quantity[24, "Hours"], "Seconds"]
```

Out[ ]= 86 400 s

```
In[ ]:= ruleP = {period → 86 400}
```

Out[ ]= {period → 86 400}

```
In[ ]:= solR = Solve[{eq4, eq5}, {r, v}][[2]]
```

Out[ ]= $\left\{r \to \frac{\text{bigG}^{1/3}\, m^{1/3}\, \text{period}^{2/3}}{(2\pi)^{2/3}},\ v \to \frac{\text{bigG}^{1/3}\, m^{1/3}\, (2\pi)^{1/3}}{\text{period}^{1/3}}\right\}$

```
In[ ]:= r0 = r /. solR /. values /. ruleP
```

Out[ ]= 9.7505×10^6

Part EXTRA) Use mass of earth

In[ ]:= r1 = r /. solR /. $\{m \to 6 \times 10^{24}\}$ /. values /. ruleP

Out[ ]= 4.22975×10^7

```
In[ ]:= UnitConvert[Quantity[r1, "Meters"], "Miles"]
```

Out[ ]= 26 282.5 mi