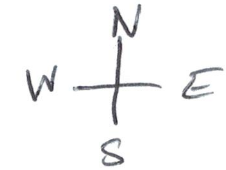
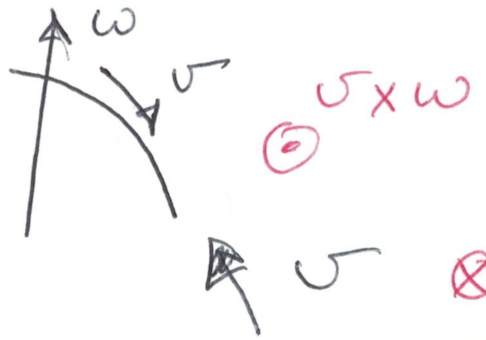
Exam #1

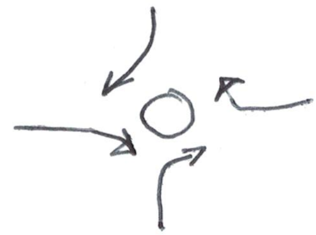
$F_{cor} = 2m\,v \times \omega$



#1

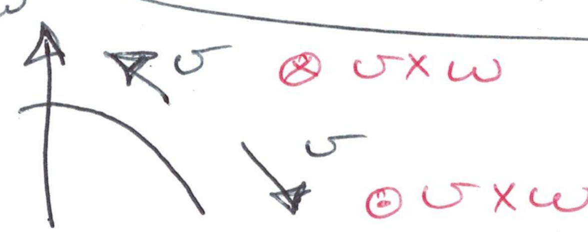
(a)



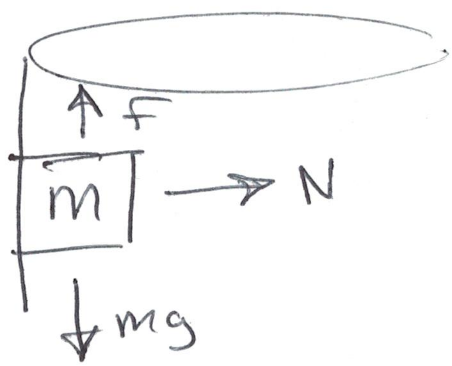


counter clockwise

(b) Yes, if direction is reversed, water will go clockwise



#2



In inertial frame

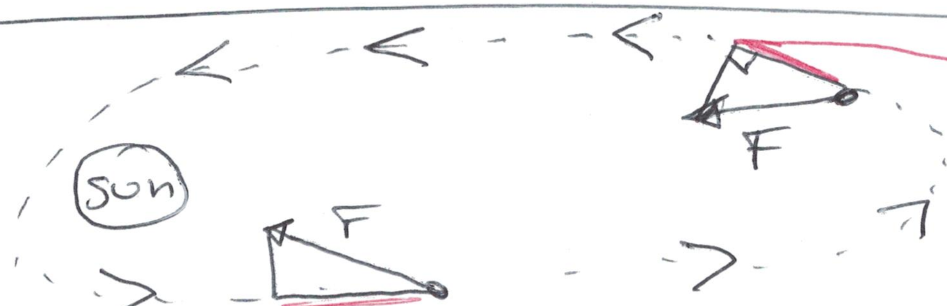
$$F = ma = \frac{mv^2}{r}$$

$$N = \frac{mv^2}{r}$$

or $$N - \frac{mv^2}{r} = 0$$

#3

(a)



This component accelerates mass

This component decellerates mass

(b) Kepler's 2nd Law will hold becaus $V = V(r)$

(c) $$F = \frac{GMm}{r^{2.01}}$$

orbits do not close

They precess

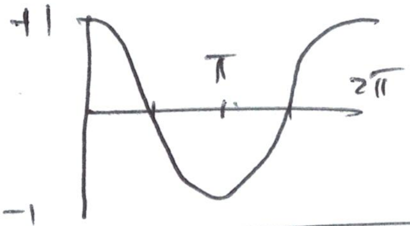
#4 $\quad F(x) = \sum_n b_n \sin(nx)$

$$\int_0^{2\pi} \sin(17x)\, f(x) = \sum_n \int_0^{2\pi} b_n \sin(nx) \sin(17x)$$

$$\int_\pi^{2\pi} (\sin 17x)(37)\, dx = b_{17} \int_0^{2\pi} \sin^2(17x)$$

$\leftrightarrow \frac{1}{2}$

$$37\left[-\frac{\cos(17x)}{17}\right]_\pi^{2\pi} = b_{17} \cdot \frac{1}{2} \cdot 2\pi = \pi b_{17}$$



$$37\left[\frac{-1-(-1)}{17}\right] = \frac{-2}{17} = \pi b_{17}$$

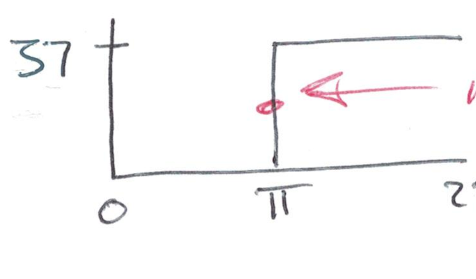
$$\boxed{b_{17} = \frac{-2 \cdot 37}{17\pi}}$$

(b) $\sum_n \int b_n \sin(nx) \sin(17x)$

because orthogonal, Keep only $n=17$

(c) Need $\cos(nx)$ also

(d) $F(x) = \sum_n b_n \sin(nx)$ is continuous !

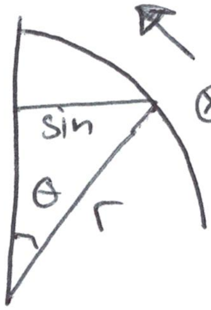


must be Average Value

$= \frac{1}{2} \cdot 37 = \frac{37}{2}$

#5 Geodesic on Sphere:

$$\dot{\phi} = \frac{d\phi}{d\theta}$$



$ds^2 = r^2 d\theta^2$

$ds^2 = r^2 \sin^2\theta \, d\phi$

$$L = \int \sqrt{ds^2} = \int r \sqrt{1 + \sin^2\theta \, \dot{\phi}^2} \, d\theta$$

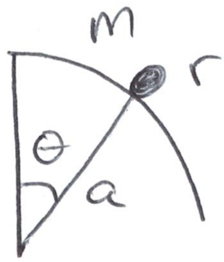
$$\frac{\partial L}{\partial \phi} = 0 \Rightarrow \frac{\partial L}{\partial \dot{\phi}} = \frac{\sin^2\theta \, \dot{\phi}}{\sqrt{1 + \sin^2\theta \, \dot{\phi}^2}} = \alpha$$

Let's pick starting point at North Pole $\theta = 0$

$\sin\theta = 0 \Rightarrow \alpha = 0 \Rightarrow \dot{\phi} = 0$

$\therefore$ Geodesic is along any line of longitude

#6



$T = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2$

$V = -mgr\cos\theta$

$F \Rightarrow r - a = 0$

$L = T - V$

$\frac{\partial L}{\partial r} = m r \dot{\theta}^2 + mg\cos\theta \qquad \frac{\partial L}{\partial \dot{r}} = m\dot{r} \qquad \frac{\partial F}{\partial r} = 1$

$\frac{\partial L}{\partial \theta} = -mgr\sin\theta \qquad \frac{\partial L}{\partial \dot{\theta}} = m r^2 \dot{\theta} \qquad \frac{\partial F}{\partial \theta} = 0$

$\boxed{r} \quad m\ddot{r} - m r \dot{\theta}^2 - mg\cos\theta = \lambda$

$\boxed{\theta} \quad m r^2 \ddot{\theta} + 2 m r \dot{r} \dot{\theta} - mgr\sin\theta = 0$

Substitute: $\ddot{r} = \dot{r} = 0, \quad r = a$

$\boxed{r} \quad -m a \dot{\theta}^2 - mg\cos\theta = \lambda$

$\boxed{\theta} \quad m a^2 \ddot{\theta} - mga\sin\theta = 0$

$\hookrightarrow \ddot{\theta} = \frac{g}{a}\sin\theta$

$$\int_0^{\theta} \ddot{\theta}\, d\theta = \frac{g}{a} \int_0^{\theta} \sin\theta \, d\theta$$

$$\frac{\dot{\theta}^2}{2} = \frac{g}{a}\left[-\cos\theta\right]_0^{\theta} = -\frac{g}{a}(1-\cos\theta)$$

$$\boxed{r} \quad -\lambda = \dot{\theta}^2 + \frac{g}{a}\cos\theta$$

set to zero

$$0 = -\frac{2g}{a}(1-\cos\theta) + \frac{g}{a}(\cos\theta)$$

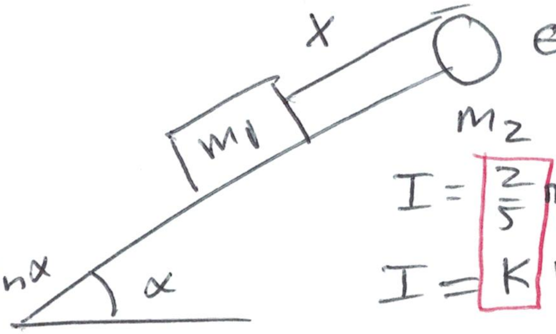
$$0 = -2(1-\cos\theta) + (\cos\theta)$$

$$0 = -2 + 3\cos\theta$$

$$\cos\theta = \frac{2}{3} \qquad \theta = \cos^{-1}\left(\frac{2}{3}\right) = 48^\circ$$

Problem #1

$$T = \frac{1}{2} m_1 \dot{X}^2 + \frac{1}{2} I \dot{\theta}^2$$



$$I = \boxed{\tfrac{2}{5}}\, m_2 r^2$$

$$I = \boxed{K}\, m_2 r^2$$

$$V = mgh = -m_1 g X \sin\alpha$$

$$L = T - V = \frac{m_1}{2} \dot{X}^2 + \frac{I}{2} \dot{\theta}^2 + m_1 g X \sin\alpha$$

$$\frac{\partial L}{\partial X} = mg \sin\alpha \qquad \frac{\partial L}{\partial \dot{X}} = m_1 \dot{X} \qquad \frac{\partial L}{\partial \theta} = 0 \qquad \frac{\partial L}{\partial \dot{\theta}} = I \dot{\theta}$$

Constraint: $X = r\theta \Rightarrow f = X - r\theta = 0$

$$\frac{\partial f}{\partial X} = +1 \qquad \frac{\partial f}{\partial \theta} = -r \qquad \boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \lambda \frac{\partial f}{\partial q}}$$

$\boxed{X}$ $\quad m_1 \ddot{X} - m_1 g \sin\alpha = \lambda$

$\boxed{\theta}$ $\quad I \ddot{\theta} - 0 = -r\lambda \quad \Rightarrow \quad K m_2 r^2 \frac{\ddot{X}}{r} = -r\lambda$

$\boxed{F}$ $\quad F \Rightarrow \ddot{X} = r\ddot{\theta} \qquad\qquad K m_2 \ddot{X} = -\lambda$

$X \Rightarrow$

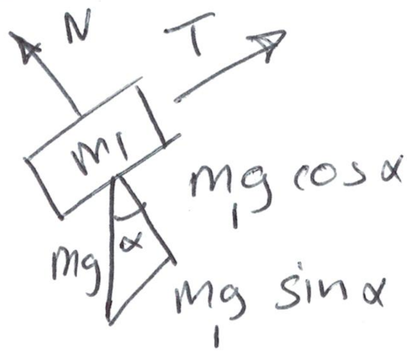
$$m_1 \ddot{X} - m_1 g \sin\alpha = \lambda = -K m_2 \ddot{X}$$

$$\ddot{X}(m_1 + K m_2) = m_1 g \sin\alpha$$

$$\ddot{X} = \frac{m_1 g \sin\alpha}{(m_1 + K m_2)} \qquad K = \frac{2}{5}$$

$$\lambda = -K m_2 \ddot{X} = -\frac{K m_1 m_2 g \sin\alpha}{(m_1 + K m_2)}$$

Force Method



$$F = ma$$

(1) $m_1 g\sin\alpha - T = m_1 a$

$$\tau = I\alpha$$

$$rT = I\alpha \qquad \alpha = \frac{a}{r}$$

$$rT = K m_2 r^2 \frac{a}{r}$$

(2) $T = K m_2 a$

(1) (2) $\rightarrow$ $m_1 g\sin\alpha - K m_2 a = m_1 a$

$$\boxed{\frac{m_1 g\sin\alpha}{m_1 + K m_2} = a}$$

$$T = K m_2 a = K\frac{m_1 m_2 g\sin\alpha}{m_1 + K m_2}$$