IMPORTANT:

- This exam is closed book, closed notes. You may use a calculator, but NOT a cell phone calculator.
- You need to do your work in front of the computer please.
- Show your work. I need to see all steps.
- You will be given basic formulas, and this is your starting point. This course is NOT about memorizing formulas, so if your answer depends on a special formula, it must be derived from the standard starting points. [This is to your benefit; if you know how to use the basic formulas, you'll do fine.]
- You'll be given a formula sheet. Same as last time, plus some extra formulas.
- When you complete the exam, please upload your work promptly.

SUGGESTIONS:

1) Show all work. Explain if it might help with partial credit.
2) Use symbols as much as possible. It makes it easier to allocate partial credit.
3) Do not waste an excessive amount of time on any one problem or individual part. Note the point allocation, and budget your time accordingly.
4) Write clearly so that we can try to reconstruct your steps. Make figures of reasonable size.
5) Make sure that you do all the problems.

Problem 1: (20 Pts) When a toilet is flushed or a sink is drained, the water begins to rotate about the drain on the way down. Assuming no initial rotation and a flow initially directly straight toward the drain, explain what causes the rotation and which direction it has in the northern hemisphere. **[The more complete the explanation, the more points!!!]** (Note that this is a small effect and in most toilets the rotation is caused by directional water jets.)
Would the direction of rotation reverse if water were forced up the drain? **Explain your answer!!!**

For this problem, put your text into Canvas, and your diagram on the paper to be scanned.

Problem 2: (20 Pts) In one amusement park ride, riders enter a large vertical barrel and stand against the wall on its horizontal floor. The barrel is spun up and the floor drops away. Riders feel as if they are pinned to the wall by a force something like the gravitational force. This is a fictitious force sensed and used by the riders to explain events in the rotating frame of reference of the barrel.
Explain in an inertial frame of reference (Earth is nearly one) what pins the riders to the wall, and identify all of the real forces acting on them.

For this problem, put your text into Canvas, and your diagram on the paper to be scanned.

Problem 3: (20 Pts)
a) Draw a free body diagram for a satellite in an elliptical orbit showing why its speed increases as it approaches its parent body and decreases as it moves away. Draw the figure to be scanned, and explain in text.
b) Suppose Newton's gravitational law $F=GMm/r^2$ is modified by mysterious forces into $F=GMm/r^{2.1}$. [That is, the exponent is 2.1 instead of 2.0]. How would this affect Kepler's law relating area swept per unit time. How would this affect the shape of the orbit? Draw the figure to be scanned, and explain in text.

For this problem, put your text into Canvas, and your diagram on the paper to be scanned.

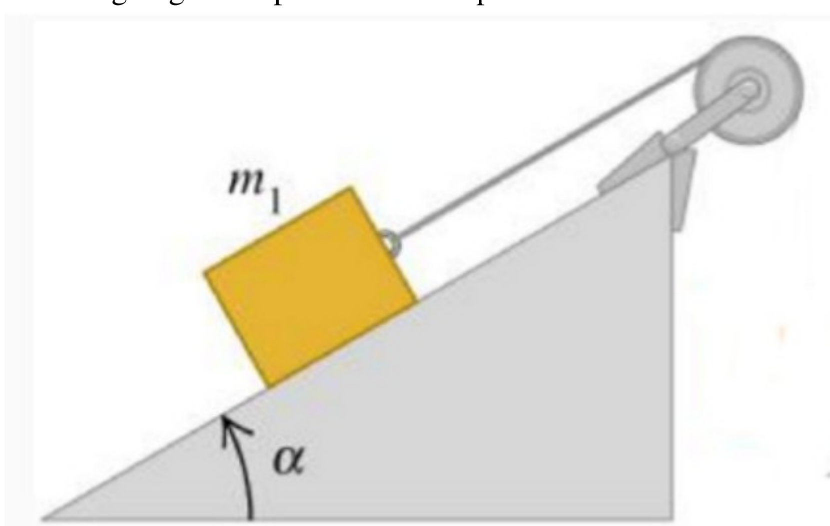
Problem 4: (20 Pts) Consider the unit step function on the interval $[0,2\pi]$ defined such that $f(x)=0$ for $[0,\pi]$, and $f(x)=37$ for $[\pi,2\pi]$. We will express this as a sine series $f(x) = \sum b_n \text{Sin}[n\, x]$.

a) Compute the 17^{th} coefficient b_{17} using orthogonality and normalization properties.
Hint:I've not given you a formula for b_n, so start by multiplying both sides by ….
IMPORTANT: If you've memorized the formulas, that is great,
but I want the derivation—zero credit if you just write down the answer.
b) Why is it important that the Sin[n x] functions are orthogonal?
c) Why are the Sin[n x] functions not a complete set? What is missing?
d) What is the value of the Fourier series at exactly $x=\pi$.

Problem 5: (20 Pts) We want to find the shortest distance between two points on a sphere of radius r=1. Use Spherical-Polar coordinates (r,θ,ϕ) to first compute the length $L(r,\theta,\phi)$ of the line segment. Then use the Euler-Lagrange equations to find the minimum path. *Hint 1: r is a constant. Hint 2: you get most of the points for correctly setting up the problem, so be clear about your coordinates and diagrams.*

Problem 6: (20 Pts) A point mass m slides without friction on a fixed cylinder of radius a. The only external force is gravity. If the mass starts from rest on top of the larger cylinder, find (using Lagrange multipliers) the point at which the mass falls off the cylinder. *Hint: you get most of the points for correctly setting up the problem, so be clear about your coordinates and diagrams. Note, this is simpler than the homework problem as you the point mass does not have any rotational component.*

Problem 7: (20 Pts) Consider an Atwood machine with mass m_1 on a frictionless ramp with a MASSIVE pulley of mass m_2, radius r, and moment of inertia of $I_2 = (3/5)\, m_2\, r^2$. The string is wound around the pulley and does not slip. The system starts from rest. For coordinates, measure the distance of m_1 along the ramp to be x, and the rotation of the pulley to be θ.
Part a) Compute the Lagrangian L, and obtain the associated Euler-Lagrange equations of motion in terms of $\{x, x', \theta, \theta'\}$ using a Lagrange multiplier λ. SHOW WORK!
Part b) Find the acceleration of the m_1. Also solve the Lagrange multiplier λ and compare this to the tension T in the string. SHOW WORK!



ADVANCED PLACEMENT PHYSICS 1 EQUATIONS, EFFECTIVE 2015

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x\left(x - x_0\right)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\left|\vec{F}_f\right| \le \mu\left|\vec{F}_n\right|$$

$$a_c = \frac{v^2}{r}$$

$$\vec{p} = m\vec{v}$$

$$\Delta\vec{p} = \vec{F}\,\Delta t$$

$$K = \frac{1}{2}mv^2$$

$$\Delta E = W = F_{\parallel}d = Fd\cos\theta$$

$$P = \frac{\Delta E}{\Delta t}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$x = A\cos(2\pi f t)$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$\tau = r_{\perp}F = rF\sin\theta$$

$$L = I\omega$$

$$\Delta L = \tau\,\Delta t$$

$$K = \frac{1}{2}I\omega^2$$

$$\left|\vec{F}_s\right| = k\left|\vec{x}\right|$$

$$U_s = \frac{1}{2}kx^2$$

$$\rho = \frac{m}{V}$$

a = acceleration
A = amplitude
d = distance
E = energy
f = frequency
F = force
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
V = volume
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
ρ = density
τ = torque
ω = angular speed

$$\Delta U_g = mg\,\Delta y$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$$

$$\vec{g} = \frac{\vec{F}_g}{m}$$

$$U_G = -\frac{Gm_1 m_2}{r}$$

ELECTRICITY

$$\left|\vec{F}_E\right| = k\left|\frac{q_1 q_2}{r^2}\right|$$

$$I = \frac{\Delta q}{\Delta t}$$

$$R = \frac{\rho\ell}{A}$$

$$I = \frac{\Delta V}{R}$$

$$P = I\,\Delta V$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

A = area
F = force
I = current
ℓ = length
P = power
q = charge
R = resistance
r = separation
t = time
V = electric potential
ρ = resistivity

WAVES

$$\lambda = \frac{v}{f}$$

f = frequency
v = speed
λ = wavelength

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

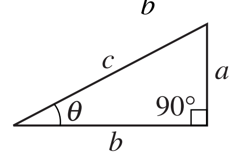
Rectangular solid
$V = \ell wh$

Cylinder
$V = \pi r^2\ell$
$S = 2\pi r\ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Right triangle
$c^2 = a^2 + b^2$
$\sin\theta = \frac{a}{c}$
$\cos\theta = \frac{b}{c}$
$\tan\theta = \frac{a}{b}$



$$S[q, q'] = \int L[q, q']dq$$
$$L = T - U$$
$$\delta S[q, q'] = 0$$

$$\sin\theta^2 + \cos\theta^2 = 1$$

$$\boxed{\frac{d}{dt}\frac{dL}{dq'} - \frac{dL}{dq} = \lambda\frac{df}{dq}}$$

f is constraint equation

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$R_{CM} = \frac{r_1 m_1 + r_2 m_2}{m_1 + m_2}$$

$$r(\phi) = \frac{c}{1 + \epsilon\cos\phi}$$

- the Coriolis force

  $$\mathbf{F}_{\text{Coriolis}} = -2m\boldsymbol{\Omega} \times \mathbf{v}_{\text{r}}$$

- the centrifugal force

  $$\mathbf{F}_{\text{centrifugal}} = -m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$$

- and the Euler force

  $$\mathbf{F}_{\text{Euler}} = -m\frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} \times \mathbf{r}$$

$$\mathbf{F}_{\text{cor}} = 2m\dot{\mathbf{r}} \times \boldsymbol{\Omega} \qquad \text{and} \qquad \mathbf{F}_{\text{cf}} = m(\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{\Omega}. \qquad \text{[Eqs. (9.35) \& (9.36)]}$$