IMPORTANT:

- This exam is OPEN book (hardcopy only), OPEN notes (hardcopy only). CLOSED INTERNET! You may use a calculator, but NOT a cell phone calculator.
- You need to do your work in front of the computer please.
- Show your work. I need to see all steps.
- When you complete the exam, please upload your work promptly.

SUGGESTIONS:

1) Show all work. Explain if it might help with partial credit.
2) Use symbols as much as possible. It makes it easier to allocate partial credit.
3) Do not waste an excessive amount of time on any one problem or individual part. Note the point allocation, and budget your time accordingly.
4) Write clearly so that we can try to reconstruct your steps. Make figures of reasonable size.
5) Make sure that you do all the problems.

Problem 1: (30 Pts) Consider the below matrices. Can these represent rotations of a solid object???

a) If yes, **what** is the rotation angle???

b) If not, clearly explain **why** they can NOT be rotations?

$$M_1 = \begin{pmatrix} \cos\theta & +\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad M_2 = \begin{pmatrix} \cos\theta & +\sin\theta \\ +\sin\theta & \cos\theta \end{pmatrix} \quad M_3 = \begin{pmatrix} \frac{\sqrt{3}}{2} & +\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \quad M_4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$M_5 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & +\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \quad M_6 = \begin{pmatrix} -1 & 0 & \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad M_7 = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \quad M_8 = \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Problem 2: (20 Pts) For the below matrix, find the eigenvalues and the associated eigenvectors. Be sure to indicate which eigenvalues go with each eigenvectors. I want all 3 of them! **Hint**: Look closely at this matrix; one of the eigenvalues is 4, so you only have to solve a quadratic, NOT a cubic.

$$M = \begin{pmatrix} 4 & 0 & 0 \\ 0 & \frac{11}{2} & \frac{3}{2} \\ 0 & \frac{3}{2} & \frac{11}{2} \end{pmatrix}$$

Problem 3: (20 Pts)

The below matrix represents a rotation about some axis vector v. Find direction vector v, and the angle of the rotation. [Hint: I only want the axis vector; not the other eigenvectors!]

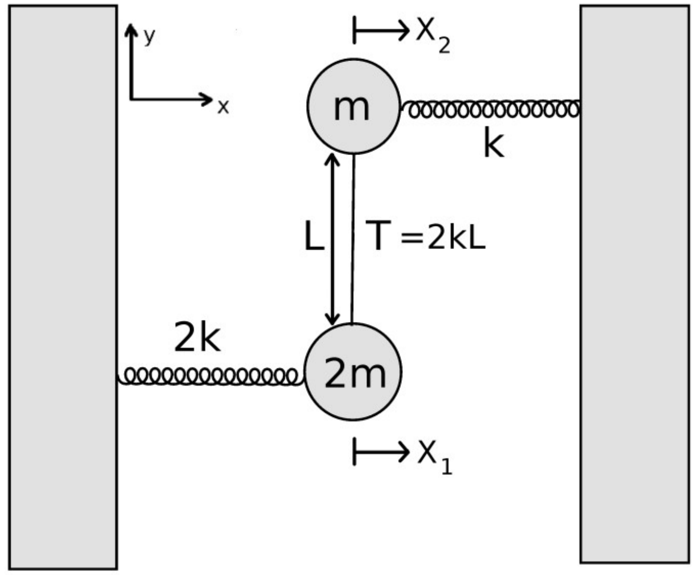
$$M = \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{-\sqrt{3}}{2} \\ 0 & 1 & 0 \end{pmatrix}$$

Problem 4: (30 Pts)

a) For the below M and K matrices, find the frequencies of motion, and the normal modes.
If you like, for simplicity, you can set m=k=1. [Hint: the numbers should come out relatively simple.]

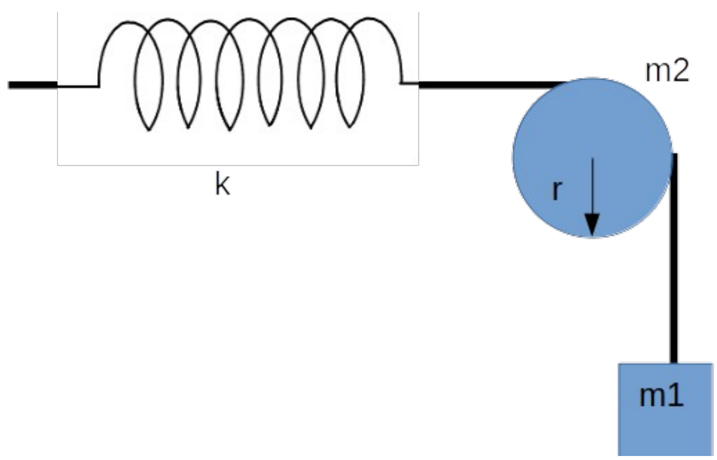
$$M = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \quad K = \begin{pmatrix} 2k & k & -k \\ k & 2k & k \\ -k & k & 2k \end{pmatrix}$$

Problem 5: (30 Points) Scattering.
The figure shows a system of masses. The mass of 2m is connected to an immobile wall with a spring of constant 2k, while the mass of m is connected to an immobile wall with a spring of constant k. The masses are then coupled to each other with a massless elastic band of length L, under tension force T = 2kL, so the effective spring constant here is (2k). The masses are constrained to move in the x direction only. At equilbrium the masses have the same x position and the springs are uncompressed. There is no friction or gravity. The displacements from equilibrium are small enough (x_1, x_2 << L), so that the tension in the band stays constant. Hint: You will need to figure out how long the elastic band stretches and use (2k) for the spring constant. The Pythagorean theorem may come in handy since the new length is $L^2=L_0^2+\Delta x^2$.



Write down the coupled differential equations describing the displacement of the masses from equilibrium {x1,x2}, and find the corresponding M and K matrices. [This is all I want here; you do NOT need to find the eigenvalues or eigenvectors.]

Problem 6: (30 Points) Hamiltonian Systems:
Consider the system shown in the diagram. This has a spring of constant K, a rotating pulley disk of mass m_2 and moment of inertia $I_2=(1/2)\, m_2 r^2$, and mass m_1 under the pull of gravity. The rope moves over the pulley disk without slipping, so $x= \theta\, r$.



a) Write down the kinetic energy T and potential energy V in terms of x[t] and x'[t]. Use only the variable x[t], and covert θ[t] to x[t].
b) Write down the Lagrangian L=T-V as a function of only x[t] and x'[t].
c) Compute the momentum using the formula ∂L/∂x'[t].
d) Compute the Hamiltonian H=T+V as only a function of x[t] and p[t]; you should eliminate x'[t].
e) Compute the Hamilton equations of motion.

Problem 7: (30 Points) Scattering.
a) A beam of 10^4 neutral Higgs bosons are fired through a lead foil 0.3mm thick. Take the cross section of the lead to be 50 barns. How many Higgs bosons will be scattered? The specific gravity of lead is 11.34, and the atomic mass is 208.

b) In Example 14.5 "Hard Sphere Scattering" we found that the differential cross section (Eq.14.25) was: $d\sigma/d\Omega = R^2/4$. If we scatter 10^6 pellets from a hard sphere which satisfies this relation, find the number of pellets which scatter between an angle of 10º and 20º. [Hint, the differential cross section is a constant!] SHOW WORK!!

c) In the laboratory frame, a proton scatters from a proton at rest (equal masses) at an angle of 60º. Find the scattering angle when transformed to the Center of Mass System (CMS) frame.