Problem #1)

Problem 1: (30 Pts) Consider the below matrices. Can these represent rotations of a solid object???
a) If yes, **what** is the rotation angle???
b) If not, clearly explain **why** they can NOT be rotations?

In[ ]:=
$$M_1 = \begin{pmatrix} \cos\theta & +\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad M_2 = \begin{pmatrix} \cos\theta & +\sin\theta \\ +\sin\theta & \cos\theta \end{pmatrix} \quad M_3 = \begin{pmatrix} \frac{\sqrt{3}}{2} & +\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \quad M_4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$M_5 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & +\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \quad M_6 = \begin{pmatrix} -1 & 0 & \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad M_7 = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \quad M_8 = \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

ANSWERS :

M1: Yes, rotation by θ

M2: No, need one negative and one positive Sinθ term

M3: Yes, rotation by 30 Degrees

M4: Yes, rotation by 180 Degrees

M5: Yes 3-D rotation about the x-axis by θ

M6: No, this flips all axes. (This is a parity transformation.)

M7: No, this will stretch the axes.

M8: No, this will stretch x-axis and compress y-axis

Problem #2)

In[1]:=
```
Clear["Global`*"]
```

In[2]:=
$$m = \begin{pmatrix} 4 & 0 & 0 \\ 0 & \frac{11}{2} & \frac{3}{2} \\ 0 & \frac{3}{2} & \frac{11}{2} \end{pmatrix};$$

In[3]:=
$$m2 = \begin{pmatrix} \frac{11}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{11}{2} \end{pmatrix};$$

In[4]:=
```
one = DiagonalMatrix [{1, 1, 1}];
```

In[5]:=
```
Det[m - λ one]
```

Out[5]= $112 - 72\,\lambda + 15\,\lambda^2 - \lambda^3$

In[6]:=
```
sol = Solve[Det[m - λ one] == 0, λ]
```

Out[6]= $\{\{\lambda \to 4\}, \{\lambda \to 4\}, \{\lambda \to 7\}\}$

In[7]:=
```
Eigenvalues[m]
```
Out[7]= {7, 4, 4}

In[8]:=
```
Eigenvalues[m2]
```
Out[8]= {7, 4}

In[9]:=
```
eqs = (m - λ one).{a, b, c} == 0 // Thread
```
Out[9]= $\left\{a\,(4-\lambda) == 0,\ \frac{3\,c}{2} + b\left(\frac{11}{2}-\lambda\right) == 0,\ \frac{3\,b}{2} + c\left(\frac{11}{2}-\lambda\right) == 0\right\}$

In[10]:=
```
sol[[1]]
```
Out[10]= {λ → 4}

In[11]:=
```
sol1 = Solve[eqs /. sol[[1]], {a, b, c}][[1]]
```
••• Solve : Equations may not give solutions for all "solve" variables .

Out[11]= {c → -b}

In[12]:=
```
v1 = {a, b, c} /. sol1
```
Out[12]= {a, b, -b}

In[13]:=
```
v1 = {a, b, c} /. sol1 /. {a → 1, b → 0}
```
Out[13]= {1, 0, 0}

In[14]:=
```
sol[[3]]
```
Out[14]= {λ → 7}

In[15]:=
```
sol3 = Solve[eqs /. sol[[3]], {a, b, c}][[1]]
```
••• Solve : Equations may not give solutions for all "solve" variables .

Out[15]= {a → 0, c → b}

In[16]:=
```
v3 = {a, b, c} /. sol3
```
Out[16]= {0, b, b}

In[17]:=
```
v3 = {a, b, c} /. sol3 /. {b → 1}
```
Out[17]= {0, 1, 1}

In[18]:=
```
v2 = Cross[v1, v3]
```
Out[18]= {0, -1, 1}

In[19]:=
```
{a, b, c} /. sol1 /. {a → 0, b → -1}
```
Out[19]= {0, -1, 1}

In[20]:=
```
{v1, v2, v3}
```
Out[20]= {{1, 0, 0}, {0, -1, 1}, {0, 1, 1}}

Problem #3)

In[21]:=
```
Clear["Global`*"]
```

In[22]:=
$$m = \left\{\left\{\frac{\sqrt{3}}{2}, 0, 1/2\right\}, \left\{1/2, 0, -\frac{\sqrt{3}}{2}\right\}, \{0, 1, 0\}\right\};$$
```
one = DiagonalMatrix[{1, 1, 1}];
m // MatrixForm
```

Out[24]//MatrixForm=
$$\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \end{pmatrix}$$

In[25]:=
```
m.Transpose[m]
```

Out[25]= {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

In[26]:=
```
Eigenvalues[m]
```

Out[26]=
$$\left\{\frac{1}{4}\left(-2+\sqrt{3}+i\sqrt{16-\left(-2+\sqrt{3}\right)^2}\right), \frac{1}{4}\left(-2+\sqrt{3}-i\sqrt{16-\left(-2+\sqrt{3}\right)^2}\right), 1\right\}$$

In[27]:=
```
eqs = (m - λ one).{a, b, c} == 0 /. {λ → 1} // Thread
```

Out[27]=
$$\left\{\left(-1+\frac{\sqrt{3}}{2}\right) a + \frac{c}{2} == 0, \frac{a}{2} - b - \frac{\sqrt{3}\, c}{2} == 0, b - c == 0\right\}$$

In[28]:=
```
sol = Solve[eqs, {a, b, c}][[1]]
```

Solve : Equations may not give solutions for all "solve " variables .

Out[28]=
$$\left\{b \to -\left(-2+\sqrt{3}\right) a, c \to -\left(-2+\sqrt{3}\right) a\right\}$$

In[29]:=
```
vec1 = {a, b, c} /. sol /. {a → 1}
```

Out[29]=
$$\left\{1, 2-\sqrt{3}, 2-\sqrt{3}\right\}$$

In[30]:=
$$\frac{\text{vec1}}{2-\sqrt{3}}$$

Out[30]=
$$\left\{\frac{1}{2-\sqrt{3}}, 1, 1\right\}$$

In[31]:=
$$\frac{\text{vec1}}{2-\sqrt{3}} \text{ // Simplify}$$

Out[31]=
$$\left\{2+\sqrt{3}, 1, 1\right\}$$

In[32]:=
```
vec1 // Normalize // Simplify
```
Out[32]=
$$\left\{\frac{1}{\sqrt{15-8\sqrt{3}}}, \frac{2-\sqrt{3}}{\sqrt{15-8\sqrt{3}}}, \frac{2-\sqrt{3}}{\sqrt{15-8\sqrt{3}}}\right\}$$

In[33]:=
```
eq = Tr[m] == 1 + 2 Cos[θ]
```
Out[33]=
$$\frac{\sqrt{3}}{2} == 1 + 2\,\mathrm{Cos}[\theta]$$

In[34]:=
```
Solve[eq, Cos[θ]][[1]]
```
Out[34]=
$$\left\{\mathrm{Cos}[\theta] \to \frac{1}{4}\left(-2+\sqrt{3}\right)\right\}$$

In[35]:=
```
tsol = Solve[eq, θ][[1]]
```
Out[35]=
$$\left\{\theta \to \boxed{-\mathrm{ArcCos}\left[\frac{1}{4}\left(-2+\sqrt{3}\right)\right] + 2\pi\,\mathbb{c}_1 \quad \text{if} \quad \mathbb{c}_1 \in \mathbb{Z}}\right\}$$

In[36]:=
```
tsol1 = Solve[eq, θ][[1]] /. {C[_] :> 0}
```
Out[36]=
$$\left\{\theta \to -\mathrm{ArcCos}\left[\frac{1}{4}\left(-2+\sqrt{3}\right)\right]\right\}$$

In[37]:=
```
tsol2 = Solve[eq, θ][[1]] /. {C[_] :> 1}
```
Out[37]=
$$\left\{\theta \to 2\pi - \mathrm{ArcCos}\left[\frac{1}{4}\left(-2+\sqrt{3}\right)\right]\right\}$$

In[38]:=
$$\theta 0 = \frac{\theta}{\mathrm{Degree}} \text{ /. tsol1 // N}$$
Out[38]=
-93.841

In[39]:=
$$\theta 0 = \frac{\theta}{\mathrm{Degree}} \text{ /. tsol2 // N}$$
Out[39]=
266.159

In[40]:=
$$1 \Big/ \left(2 - \sqrt{3}\right) \text{ // Simplify}$$
Out[40]=
$$2 + \sqrt{3}$$

In[41]:=
$$m.\left\{1 \Big/ \left(2 - \sqrt{3}\right), 1, 1\right\} \text{ // Simplify}$$
Out[41]=
$$\left\{2 + \sqrt{3}, 1, 1\right\}$$

In[42]:=
$$m.\left\{\left(2 + \sqrt{3}\right), 1, 1\right\} \text{ // Simplify}$$
Out[42]=
$$\left\{2 + \sqrt{3}, 1, 1\right\}$$

Problem #4)

In[43]:=
```
Clear["Global`*"]
```

In[44]:=
```
Tmat = m DiagonalMatrix [{1, 1, 1}];
Tmat // MatrixForm
```

Out[45]//MatrixForm=

$$\begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix}$$

In[46]:=
```
Vmat = k {{2, 1, -1}, {1, 2, 1}, {-1, 1, 2}};
Vmat // MatrixForm
```

Out[47]//MatrixForm=

$$\begin{pmatrix} 2k & k & -k \\ k & 2k & k \\ -k & k & 2k \end{pmatrix}$$

In[48]:=
```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```

Out[49]//MatrixForm=

$$\begin{pmatrix} 2k - m\,\omega 2 & k & -k \\ k & 2k - m\,\omega 2 & k \\ -k & k & 2k - m\,\omega 2 \end{pmatrix}$$

In[50]:=
```
Det[mat] // Factor
```

Out[50]= $-m\,\omega 2\,(3k - m\,\omega 2)^2$

In[51]:=
```
sol = Solve[Det[mat] == 0, ω2] // Simplify
```

Out[51]= $\left\{\{\omega 2 \to 0\}, \left\{\omega 2 \to \frac{3k}{m}\right\}, \left\{\omega 2 \to \frac{3k}{m}\right\}\right\}$

In[52]:=
```
eq1 = mat.{a, b, c} == 0 // Thread;
eq1 // Column
```

Out[53]=

```
b k - c k + a (2 k - m ω2) == 0
a k + c k + b (2 k - m ω2) == 0
- a k + b k + c (2 k - m ω2) == 0
```

In[54]:=
```
sol1 = Solve[eq1 /. sol[[1]], {a, b, c}][[1]]  // Simplify
```

••• Solve : Equations may not give solutions for all "solve" variables .

Out[54]= {b → - a, c → a}

In[55]:=
```
ev1 = {a, b, c} /. sol1 /. {a → 1} // Simplify
```

Out[55]= {1, -1, 1}

In[56]:=
```
sol2 = Solve[eq1 /. sol[[2]], {a, b, c}][[1]] // Simplify // PowerExpand
```

Solve : Equations may not give solutions for all "solve " variables .

Out[56]= $\{c \to -a + b\}$

In[57]:=
```
ev2 = {a, b, c} /. sol2 /. {a → 1} // Simplify
```

Out[57]= $\{1, b, -1 + b\}$

In[58]:=
```
ev2 = ev2 /. {b → 1}
```

Out[58]= $\{1, 1, 0\}$

In[59]:=
```
ev3 = Cross[ev1, ev2]
```

Out[59]= $\{-1, 1, 2\}$

In[60]:=
```
eVecs = Normalize /@ {ev1, ev2, ev3} // Simplify
```

Out[60]=
$$\left\{\left\{\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}, \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}, \left\{-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}\right\}\right\}$$

In[61]:=
```
Vdiag = eVecs.Vmat.Transpose[eVecs] // FullSimplify ;
Vdiag // MatrixForm
```

Out[62]//MatrixForm=
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 3k & 0 \\ 0 & 0 & 3k \end{pmatrix}$$

In[63]:=
```
Tdiag = eVecs.Tmat.Transpose[eVecs] // FullSimplify ;
Tdiag // MatrixForm
```

Out[64]//MatrixForm=
$$\begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix}$$

Part B Look at motion

In[65]:=
```
values = {m → 1, k → 1};
```

In[66]:=
```
sol /. values
```

Out[66]= $\{\{\omega 2 \to 0\}, \{\omega 2 \to 3\}, \{\omega 2 \to 3\}\}$

In[67]:=
```
mode1 = ev1 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[1]] /. values
```

Out[67]= $\{1, -1, 1\}$

In[68]:=
```
mode1 = ev1 (x0 + v0 t ) /. {ω → Sqrt[ω2]} /. sol[[1]] /. values /. {x0 → 0, v0 → 1}
```

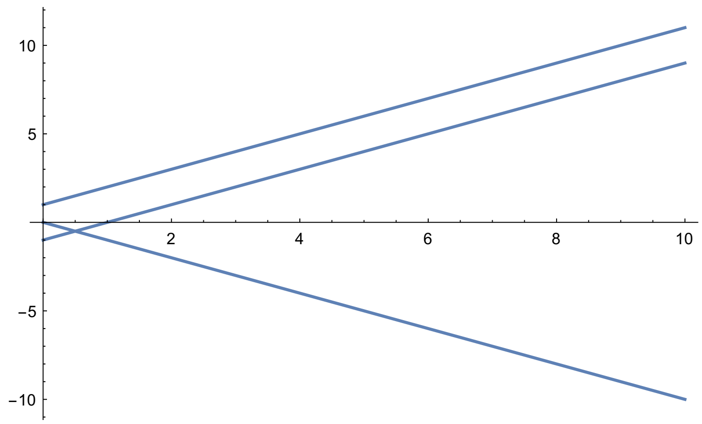
Out[68]= $\{t, -t, t\}$

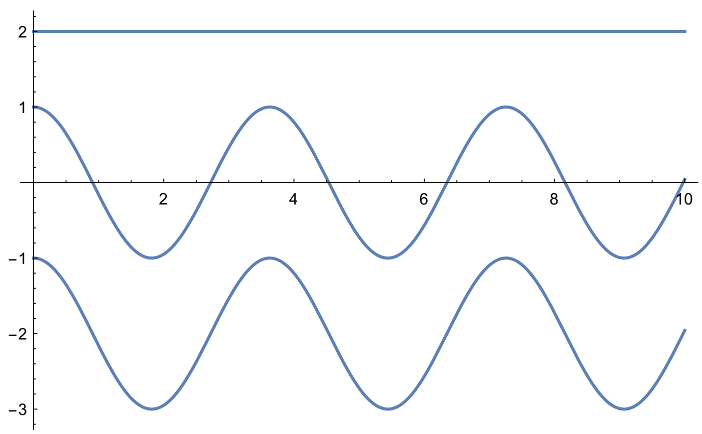
In[69]:=
```
mode2 = ev2 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[2]] /. values
```
Out[69]= $\{e^{i\sqrt{3}\,t},\ e^{i\sqrt{3}\,t},\ 0\}$

In[70]:=
```
mode3 = ev3 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[3]] /. values
```
Out[70]= $\{-e^{i\sqrt{3}\,t},\ e^{i\sqrt{3}\,t},\ 2\,e^{i\sqrt{3}\,t}\}$

In[71]:=
```
mode1 + {-1, 1} /. values
```
••• Thread : Objects of unequal length in {t, −t, t} + {−1, 1} cannot be combined .

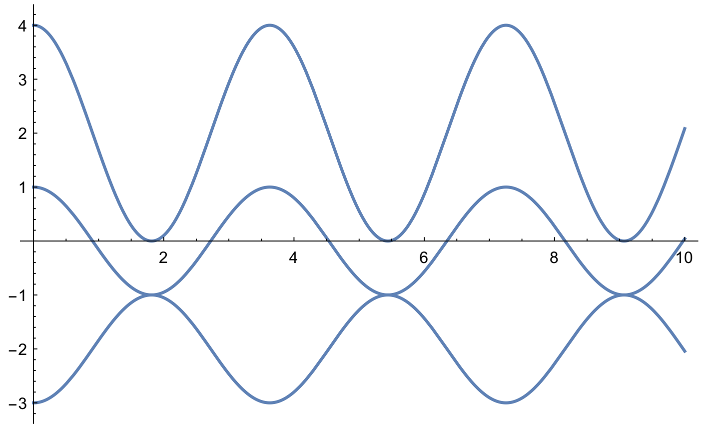
Out[71]= {-1, 1} + {t, -t, t}

In[72]:=
```
Plot[mode1 + {-1, 0, 1} /. values // Re, {t, 0, 10}]
```
Out[72]=


In[73]:=
```
Plot[mode2 + {-2, 0, 2} /. values // Re, {t, 0, 10}]
```
Out[73]=


In[74]:=
```
Plot[mode3 + {-2, 0, 2} /. values // Re, {t, 0, 10}]
```

Out[74]=



Problem #5)

In[75]:=
```
Clear["Global`*"]
```

In[76]:= $T = \frac{1}{2}(2\,m)\,x1'[t]^2 + \frac{1}{2}(1\,m)\,x2'[t]^2;$

In[77]:= $V = \frac{1}{2}(2\,k)\,x1[t]^2 + \frac{1}{2}(1\,k)\,x2[t]^2 + \frac{1}{2}(2\,k)\,(x1[t] - x2[t])^2;$

In[78]:=
```
lag = T - V
```

Out[78]= $-k\,x1[t]^2 - k\,(x1[t] - x2[t])^2 - \frac{1}{2}k\,x2[t]^2 + m\,x1'[t]^2 + \frac{1}{2}m\,x2'[t]^2$

In[79]:=
```
D[D[lag, x1'[t]], t] - D[lag, x1[t]] // Simplify
```

Out[79]= $4\,k\,x1[t] - 2\,k\,x2[t] + 2\,m\,x1''[t]$

In[80]:=
```
D[D[lag, x2'[t]], t] - D[lag, x2[t]] // Simplify
```

Out[80]= $-2\,k\,x1[t] + 3\,k\,x2[t] + m\,x2''[t]$

In[81]:=
```
Tmat = m DiagonalMatrix[{2, 1}];
Tmat // MatrixForm
```

Out[82]//MatrixForm=

$$\begin{pmatrix} 2\,m & 0 \\ 0 & m \end{pmatrix}$$

In[83]:=
```
Vmat = k {{4, -2}, {-2, 3}};
Vmat // MatrixForm
```

Out[84]//MatrixForm=

$$\begin{pmatrix} 4\,k & -2\,k \\ -2\,k & 3\,k \end{pmatrix}$$

In[85]:=
```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```

Out[86]//MatrixForm=

$$\begin{pmatrix} 4\,k - 2\,m\,\omega 2 & -2\,k \\ -2\,k & 3\,k - m\,\omega 2 \end{pmatrix}$$

In[87]:=
```
sol = Solve[Det[mat] == 0, ω2] // Simplify
```

Out[87]= $\left\{\left\{\omega 2 \to \frac{k}{m}\right\}, \left\{\omega 2 \to \frac{4\,k}{m}\right\}\right\}$

In[88]:=
```
eq1 = mat .{a, b} == 0 // Thread;
eq1 // Column
```

Out[89]=

$-2\,b\,k + a\,(4\,k - 2\,m\,\omega 2) == 0$

$-2\,a\,k + b\,(3\,k - m\,\omega 2) == 0$

In[90]:=
```
sol1 = Solve[eq1 /. sol[[1]], {a, b}][[1]]  // Simplify
```

••• Solve : Equations may not give solutions for all "solve " variables .

Out[90]= $\{b \to a\}$

In[91]:=
```
ev1 = {a, b} /. sol1 /. {a → 1} // Simplify
```

Out[91]= $\{1, 1\}$

In[92]:=
```
sol2 = Solve[eq1 /. sol[[2]], {a, b}][[1]]  // Simplify // PowerExpand
```

••• Solve : Equations may not give solutions for all "solve " variables .

Out[92]= $\{b \to -2\,a\}$

In[93]:=
```
ev2 = {a, b} /. sol2 /. {a → 1} // Simplify
```

Out[93]= $\{1, -2\}$

In[94]:=
```
eVecs = Normalize /@ {ev1, ev2} // Simplify
```

Out[94]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right\}\right\}$

In[95]:=
```
Vdiag = eVecs.Vmat.Transpose[eVecs] // FullSimplify ;
Vdiag // MatrixForm
```

Out[96]//MatrixForm=

$$\begin{pmatrix} \frac{3\,k}{2} & 0 \\ 0 & \frac{24\,k}{5} \end{pmatrix}$$

In[97]:=
```
Tdiag = eVecs.Tmat.Transpose[eVecs] // FullSimplify ;
Tdiag // MatrixForm
```

Out[98]//MatrixForm=

$$\begin{pmatrix} \frac{3\,m}{2} & 0 \\ 0 & \frac{6\,m}{5} \end{pmatrix}$$

Problem #6)

In[99]:= `Clear["Global`*"]`

In[100]:= $T = \frac{1}{2} m1\, x'[t]^2 + \frac{1}{2} (inertia)\, w'[t]^2;$

In[101]:= $V = \frac{1}{2} k\, x[t]^2 - m1\, g\, x[t];$

In[102]:= $inertia = \frac{1}{2} m2\, r^2;$

$w'[t] = \frac{x'[t]}{r};$

In[104]:= T

Out[104]= $\frac{1}{2} m1\, x'[t]^2 + \frac{1}{4} m2\, x'[t]^2$

In[105]:= V

Out[105]= $-g\, m1\, x[t] + \frac{1}{2} k\, x[t]^2$

In[106]:= lag = T - V

Out[106]= $g\, m1\, x[t] - \frac{1}{2} k\, x[t]^2 + \frac{1}{2} m1\, x'[t]^2 + \frac{1}{4} m2\, x'[t]^2$

In[107]:= `eqP = p[t] == D[lag, x'[t]] // Factor`

Out[107]= $p[t] == \frac{1}{2} (2\, m1 + m2)\, x'[t]$

In[108]:= `psol = DSolve[eqP, p[t], t][[1]]`

Out[108]= $\left\{p[t] \to \frac{1}{2} (2\, m1 + m2)\, x'[t]\right\}$

In[109]:= `xsol = Solve[eqP, x'[t]][[1]]`

Out[109]= $\left\{x'[t] \to \frac{2\, p[t]}{2\, m1 + m2}\right\}$

In[110]:= ham = T + V

Out[110]= $-g\, m1\, x[t] + \frac{1}{2} k\, x[t]^2 + \frac{1}{2} m1\, x'[t]^2 + \frac{1}{4} m2\, x'[t]^2$

In[111]:= `ham = ham /. xsol // Simplify`

Out[111]= $\frac{p[t]^2}{2\, m1 + m2} + \frac{1}{2} x[t] (-2\, g\, m1 + k\, x[t])$

In[112]:=
```
eq1 = D[ham, p[t]] == x'[t] // Simplify
```

Out[112]= $x'[t] == \frac{2\, p[t]}{2\, m1 + m2}$

In[113]:=
```
eq2 = - D[ham, x[t]] == p'[t]
```

Out[113]= $-\frac{1}{2} k\, x[t] + \frac{1}{2} (2\, g\, m1 - k\, x[t]) == p'[t]$

In[114]:=
```
p2sol = DSolve[eq1, p, t][[1]]
```

Out[114]= $\left\{p \to \text{Function}\left[\{t\}, \frac{1}{2} (2\, m1 + m2)\, x'[t]\right]\right\}$

In[115]:=
```
eqHam = eq2 /. p2sol // Simplify
```

Out[115]= $2\, g\, m1 == 2\, k\, x[t] + (2\, m1 + m2)\, x''[t]$

Check that Lagrange equations are equivalent

In[116]:=
```
D[D[lag, x'[t]], t] - D[lag, x[t]] // Simplify
```

Out[116]= $-g\, m1 + k\, x[t] + \left(m1 + \frac{m2}{2}\right) x''[t]$

In[117]:=
```
(* Same thing, just multiply by 2 and move to other side:*)
eqLag = 2 D[D[lag, x'[t]], t] == 2 D[lag, x[t]] // Simplify
```

Out[117]= $2\, g\, m1 == 2\, k\, x[t] + (2\, m1 + m2)\, x''[t]$

In[118]:=
```
eqLag == eqHam
```

Out[118]= True

Problem #7)

Problem A

In[119]:=
```
Clear["Global`*"]
```

In[120]:= $\text{ntar} = \frac{\rho\, t}{m}$ /. {$\rho \to (11.34 \times 10^{3})$, $t \to (0.3 \times 10^{-3})$, $m \to (208 * (1.66 \times 10^{-27}))$}

Out[120]= 9.85287×10^{24}

In[121]:= Nsc = Nin ntar sig /. {Nin → 10^{4}, sig → 50×10^{-28}}

Out[121]= 492.644

Problem B

In[122]:=
```
Clear["Global`*"]
```

In[123]:=
```
solidAngle = Integrate[Sin[θ], {θ, 0, π}, {ϕ, 0, 2 π}]
```

Out[123]= 4π

In[124]:=
```
coverage = Integrate[Sin[θ], {θ, θ1, θ2}, {ϕ, 0, 2 π}]
```

Out[124]= $2\pi(\text{Cos}[\theta 1] - \text{Cos}[\theta 2])$

In[125]:=
```
coverage /. {θ1 → 10 Degree, θ2 → 20 Degree}
```

Out[125]= $2\pi(\text{Cos}[10\,°] - \text{Cos}[20\,°])$

In[126]:=
```
ratio = coverage/solidAngle /. {θ1 → 10 Degree, θ2 → 20 Degree}
```

Out[126]= $\frac{1}{2}(\text{Cos}[10\,°] - \text{Cos}[20\,°])$

In[127]:=
```
ratio // N
```

Out[127]= 0.0225576

In[128]:=
```
10^6 ratio // N
```

Out[128]= 22 557.6

Problem C

In[129]:=
```
Clear["Global`*"]
```

In[130]:=
```
θlab = 60 Degree
```

Out[130]= 60 °

In[131]:=
```
θcms = 2 θlab
```

Out[131]= 120 °