Exam #3

#2

$$M = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 11/2 & 3/2 \\ 0 & 3/2 & 11/2 \end{pmatrix} \qquad \lambda = \underbrace{4, 4}_{\text{degenerate}}, 7$$

$$v = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\underbrace{4 \qquad 4}_{} \qquad 7$$

#3

$$M_2 = \begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix} \qquad M_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}$$

Note

$$\text{Trace } M_2 = 2\cos\theta \qquad \text{Trace } M_3 = 1 + 2\cos\theta$$

#4

$$\omega^2 = 0, \frac{3K}{m}, \frac{3K}{m}$$

$$v = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -1 \\ +1 \\ 2 \end{pmatrix}}_{\text{degenerate}} \longleftarrow \text{All Orthogonal}$$

#5

$$D^2 = L^2 + (x_1 - x_2)^2$$

$$V_L = \frac{1}{2}(2K) D^2 = K\left[L^2 + (x_1 - x_2)^2\right]$$

$$T = \frac{1}{2}(2m)\dot{x}_1^2 + \frac{1}{2}(m)\dot{x}_2^2$$

$$V = \frac{1}{2}(2K)x_1^2 + \frac{1}{2}(K)x_2^2 + V_L$$

$$T = \begin{pmatrix} 2m & 0 \\ 0 & m \end{pmatrix} \qquad V = K\begin{pmatrix} 4 & -2 \\ -2 & 3 \end{pmatrix}$$

6

$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2}\left(\frac{m_2}{2}\right)\dot{x}^2$$

$$V = -m_1 g x + \frac{1}{2} k x^2$$

$$P = \left(m_1 + \frac{m_2}{2}\right)\dot{x} \equiv \underline{M}\,\dot{x}$$

$$H = T + V = \frac{P^2}{2\underline{M}} + \frac{1}{2} k x^2 - m_1 g x$$

Eqs: $P = \underline{M}\,\dot{x}$ $\quad \dot{P} = -kx + m_1 g$

#7

a

$$n_{tar} = \frac{\rho t}{m} = \frac{\left(11.34 \times 10^{3}\ \frac{kg}{m^3}\right)\left(0.3 \times 10^{-3}\ m\right)}{208 \times \left(1.66 \times 10^{-27}\ kg\right)}$$

$$n_{tar} = 9.86 \times 10^{24}\ \frac{1}{m^2} \qquad N_{sc} = 492$$

$$\sigma = 50 \times 10^{-28}\ m$$

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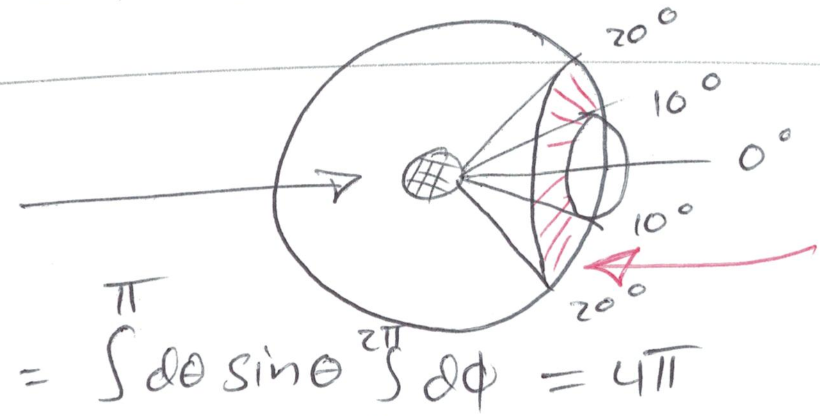
b



Find Area of red compared to total

$$\text{Total} = \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi = 4\pi$$

$$\text{Red Area} = \int_{10^\circ}^{20^\circ} d\theta \sin\theta \int_0^{2\pi} d\phi = 2\pi\left[\cos 10^\circ - \cos 20^\circ\right]$$

$$\text{Fraction} = \frac{\text{Red Area}}{\text{Total}} = 0.022 \Rightarrow N = 22{,}558$$