Physics 3344	Professor Olness
Homework 10	October 28, 2020

1) Three equal masses are connected by three identical springs and constrained to move on a circular frictionless rim of radius b. Set up the small oscillations Lagrangian and find the normal modes by:

a) Using symmetry considerations;

b) Diagonalizing the kinetic and potential energy matrices.

2) A rigid uniform bar of mass M and length L is supported in equilibrium in a horizontal position by two identical springs attached one to each end. Assume its motion is constrained to a vertical plane. You may neglect any horizontal motion, but do not neglect the moment of inertia of the mass.

a) Find the normal modes and eigenfrequencies of small oscillations of the system .

b) If initially one end of the bar is displaced, the other remaining in its equilibrium position, and the system is released from rest, find the motion.

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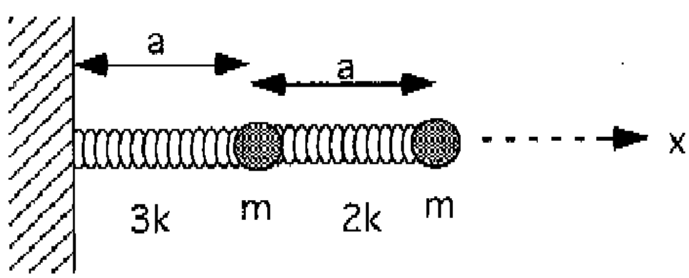
3. 2 objects, each of mass m, are attached to each other by a spring, and the left mass is also attached by a spring to a fixed wall. The springs are of equilibrium length a. The figure shows a top view. The masses are on a frictionless surface, and can only move along the x-axis . The left spring has spring constant 3k, and the right spring has spring constant 2k.



(a) Find the Lagrangian for this system.

(b) Find the normal modes and their frequencies.

Problem 4)

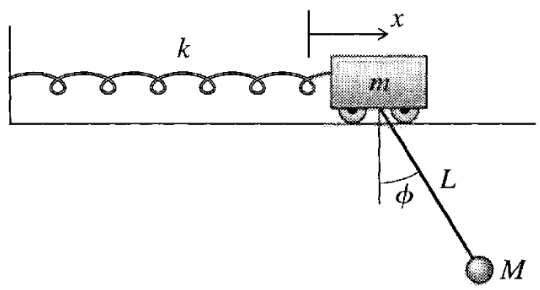


Figure 11.18 Problem 11.19

11.19 ★★★ A simple pendulum (mass M and length L) is suspended from a cart (mass m) that can oscillate on the end of a spring of force constant k, as shown in Figure 11.18. **(a)** Assuming that the angle ϕ remains small, write down the system's Lagrangian and the equations of motion for x and ϕ. **(b)** Assuming that $m = M = L = g = 1$ and $k = 2$ (all in appropriate units) find the normal frequencies, and for each normal frequency find and describe the motion of the corresponding normal mode.

Problem 5:

11.7 ★★ [Computer] The most general motion of the two carts of Section 11.2 is given by (11.21), with the constants A_1, A_2, δ_1, and δ_2 determined by the initial conditions. **(a)** Show that (11.21) can be rewritten as

$$\mathbf{x}(t) = (B_1 \cos \omega_1 t + C_1 \sin \omega_1 t)\begin{bmatrix} 1 \\ 1 \end{bmatrix} + (B_2 \cos \omega_2 t + C_2 \sin \omega_2 t)\begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

This form is usually a little more convenient for matching to given initial conditions. **(b)** If the carts are released from rest at positions $x_1(0) = x_2(0) = A$, find the coefficients B_1, B_2, C_1, and C_2 and plot $x_1(t)$ and $x_2(t)$. Take $A = \omega_1 = 1$ and $0 \le t \le 30$ for your plots. **(c)** Same as part (b), except that $x_1(0) = A$ but $x_2(0) = 0$.

NOTE: For both problem 5 and 6, assume w1 and w2 are related by Eq.11.15 as in the example in the text.

Problem 6:

a) Repeat problem 5, but take $x_1(0)=x_2(0)=0$ and $v_1(0)=v_2(0)=v_0$ and compute the coefficients and plot; you select v_0.

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Problem 7:

11.15 ★★ Write down the exact Lagrangian (good for all angles) for the double pendulum of Figure 11.9 and find the corresponding equations of motion. Show that they reduce to Equations (11.41) and (11.42) if both angles are small.