Problem #1)

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
$$T = \frac{1}{2} m\, b^2 (d\theta1^2 + d\theta2^2 + d\theta3^2)$$

Out[ ]=
$$\frac{1}{2} b^2 (d\theta1^2 + d\theta2^2 + d\theta3^2) m$$

In[ ]:=
$$V = \frac{1}{2} k\, b^2 ((\theta1 - \theta2)^2 + (\theta2 - \theta3)^2 + (\theta3 - \theta1)^2) \text{ // Expand}$$

Out[ ]=
$$b^2 k\, \theta1^2 - b^2 k\, \theta1\, \theta2 + b^2 k\, \theta2^2 - b^2 k\, \theta1\, \theta3 - b^2 k\, \theta2\, \theta3 + b^2 k\, \theta3^2$$

In[ ]:=
```
lag = T - V
```

Out[ ]=
$$\frac{1}{2} b^2 (d\theta1^2 + d\theta2^2 + d\theta3^2) m - b^2 k\, \theta1^2 + b^2 k\, \theta1\, \theta2 - b^2 k\, \theta2^2 + b^2 k\, \theta1\, \theta3 + b^2 k\, \theta2\, \theta3 - b^2 k\, \theta3^2$$

In[ ]:=
```
D[T, dθ1]
```

Out[ ]=
$$b^2\, d\theta1\, m$$

In[ ]:=
```
Tmat = m b^2 DiagonalMatrix[{1, 1, 1}];
Tmat // MatrixForm
```

Out[ ]//MatrixForm=
$$\begin{pmatrix} b^2 m & 0 & 0 \\ 0 & b^2 m & 0 \\ 0 & 0 & b^2 m \end{pmatrix}$$

In[ ]:=
```
D[V, θ1]
```

Out[ ]=
$$2 b^2 k\, \theta1 - b^2 k\, \theta2 - b^2 k\, \theta3$$

In[ ]:=
```
Vmat = k b^2 {{2, -1, -1}, {-1, 2, -1}, {-1, -1, 2}};
Vmat // MatrixForm
```

Out[ ]//MatrixForm=
$$\begin{pmatrix} 2 b^2 k & -b^2 k & -b^2 k \\ -b^2 k & 2 b^2 k & -b^2 k \\ -b^2 k & -b^2 k & 2 b^2 k \end{pmatrix}$$

In[ ]:=
```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```

Out[ ]//MatrixForm=
$$\begin{pmatrix} 2 b^2 k - b^2 m\, \omega2 & -b^2 k & -b^2 k \\ -b^2 k & 2 b^2 k - b^2 m\, \omega2 & -b^2 k \\ -b^2 k & -b^2 k & 2 b^2 k - b^2 m\, \omega2 \end{pmatrix}$$

In[ ]:= d[D[lag, dθ1], t] - D[lag, θ1]

Out[ ]= $2 b^2 k\, θ1 - b^2 k\, θ2 - b^2 k\, θ3 + d[b^2\, dθ1\, m, t]$

In[ ]:= d[D[lag, dθ2], t] - D[lag, θ2]

Out[ ]= $-b^2 k\, θ1 + 2 b^2 k\, θ2 - b^2 k\, θ3 + d[b^2\, dθ2\, m, t]$

In[ ]:= d[D[lag, dθ3], t] - D[lag, θ3]

Out[ ]= $-b^2 k\, θ1 - b^2 k\, θ2 + 2 b^2 k\, θ3 + d[b^2\, dθ3\, m, t]$

In[ ]:= sol = Solve[Det[mat] == 0, ω2]

Out[ ]= $\left\{\{ω2 \to 0\}, \left\{ω2 \to \frac{3 k}{m}\right\}, \left\{ω2 \to \frac{3 k}{m}\right\}\right\}$

In[ ]:= eq1 = mat .{a, b, c} == 0 // Thread;
eq1 // Column

Out[ ]=
$-b^3 k - b^2 c\, k + a\,(2 b^2 k - b^2 m\, ω2) == 0$
$-a\, b^2 k - b^2 c\, k + b\,(2 b^2 k - b^2 m\, ω2) == 0$
$-a\, b^2 k - b^3 k + c\,(2 b^2 k - b^2 m\, ω2) == 0$

In[ ]:= norm = a ^ 2 + b ^ 2 + c ^ 2 == 1

Out[ ]= $a^2 + b^2 + c^2 == 1$

In[ ]:= eq2 = Join[eq1, {norm}];
eq2 // Column

Out[ ]=
$-b^3 k - b^2 c\, k + a\,(2 b^2 k - b^2 m\, ω2) == 0$
$-a\, b^2 k - b^2 c\, k + b\,(2 b^2 k - b^2 m\, ω2) == 0$
$-a\, b^2 k - b^3 k + c\,(2 b^2 k - b^2 m\, ω2) == 0$
$a^2 + b^2 + c^2 == 1$

In[ ]:= sol1 = Solve[eq2 /. sol[[1]], {a, b, c}] // Last

••• Solve : Equations may not give solutions for all "solve" variables .

Out[ ]= $\left\{a \to \frac{1}{\sqrt{3}}, b \to \frac{1}{\sqrt{3}}, c \to \frac{1}{\sqrt{3}}\right\}$

In[ ]:= ev1 = {a, b, c} /. sol1

Out[ ]= $\left\{\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}$

In[ ]:=
```
sol2 = Solve[eq2 /. sol[[2]], {a, b, c}]
```

Solve : Equations may not give solutions for all "solve" variables .

Out[ ]= $\left\{\left\{b\to 0,\ c\to -\sqrt{1-a^2}\right\},\ \left\{b\to 0,\ c\to \sqrt{1-a^2}\right\},\ \left\{b\to \frac{1}{2}\left(-a-\sqrt{2-3\,a^2}\right),\ c\to \frac{1}{2}\left(-a+\sqrt{2-3\,a^2}\right)\right\},\ \left\{b\to \frac{1}{2}\left(-a+\sqrt{2-3\,a^2}\right),\ c\to \frac{1}{2}\left(-a-\sqrt{2-3\,a^2}\right)\right\}\right\}$

In[ ]:=
```
(* Let's look for a solution where one of them is at rest *)
ev2 = {a, b, c} /. Last[sol2] /. a -> 0
```

Out[ ]= $\left\{0,\ \frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right\}$

In[ ]:=
```
ev3 = Cross[ev1, ev2]
```

Out[ ]= $\left\{-\sqrt{\frac{2}{3}},\ \frac{1}{\sqrt{6}},\ \frac{1}{\sqrt{6}}\right\}$

In[ ]:=
```
eVecs = {ev1, ev2, ev3}
```

Out[ ]= $\left\{\left\{\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}}\right\},\ \left\{0,\ \frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right\},\ \left\{-\sqrt{\frac{2}{3}},\ \frac{1}{\sqrt{6}},\ \frac{1}{\sqrt{6}}\right\}\right\}$

Part B Look at motion

In[ ]:=
```
values = {k → 1, m → 1};
```

In[ ]:=
```
sol
```

Out[ ]= $\left\{\{\omega 2\to 0\},\ \left\{\omega 2\to \frac{3\,k}{m}\right\},\ \left\{\omega 2\to \frac{3\,k}{m}\right\}\right\}$

In[ ]:=
```
mode1 = ev1 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[1]]
```

Out[ ]= $\left\{\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}}\right\}$

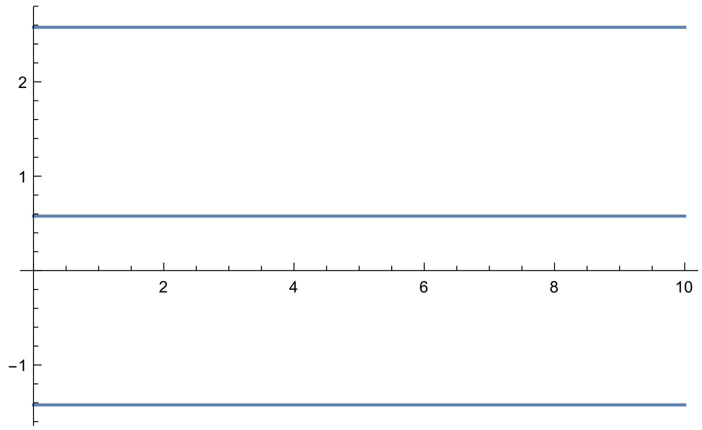
In[ ]:=
```
mode2 = ev2 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[2]]
```

Out[ ]= $\left\{0,\ \frac{e^{i\sqrt{3}\sqrt{\frac{k}{m}}\,t}}{\sqrt{2}},\ -\frac{e^{i\sqrt{3}\sqrt{\frac{k}{m}}\,t}}{\sqrt{2}}\right\}$

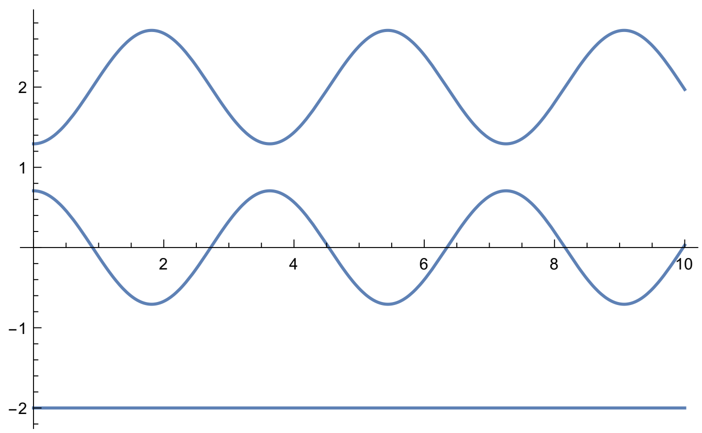
In[ ]:=
```
mode3 = ev3 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[3]]
```

Out[ ]= $\left\{-\sqrt{\frac{2}{3}}\,e^{i\sqrt{3}\sqrt{\frac{k}{m}}\,t},\ \frac{e^{i\sqrt{3}\sqrt{\frac{k}{m}}\,t}}{\sqrt{6}},\ \frac{e^{i\sqrt{3}\sqrt{\frac{k}{m}}\,t}}{\sqrt{6}}\right\}$

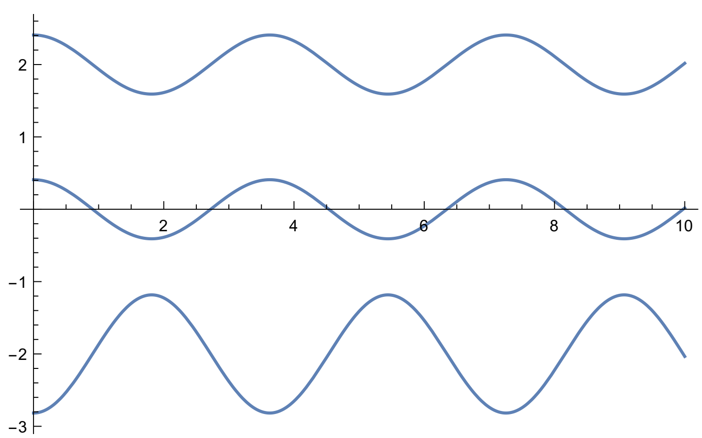
In[◦]:=
```
Plot[mode1 + {-2, 0, 2} /. values // Re, {t, 0, 10}]
```

Out[◦]=


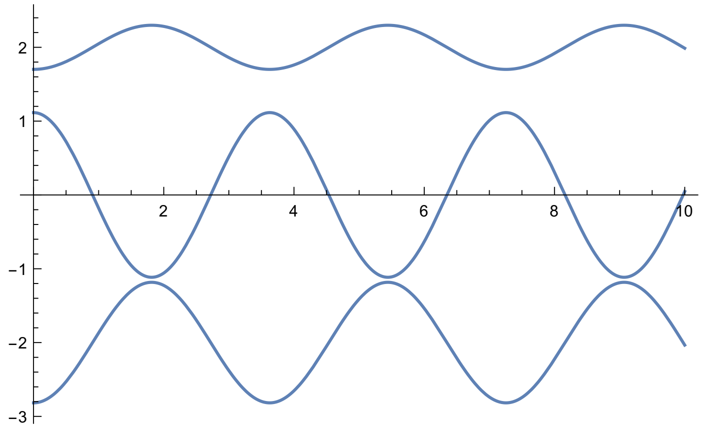
In[◦]:=
```
Plot[mode2 + {-2, 0, 2} /. values // Re, {t, 0, 10}]
```

Out[◦]=


In[◦]:=
```
Plot[mode3 + {-2, 0, 2} /. values // Re, {t, 0, 10}]
```

Out[◦]=


In[ ]:=
```
Plot[mode2 + mode3 + {-2, 0, 2} /. values // Re, {t, 0, 10}]
```

Out[ ]=


In[ ]:=
```
(* a cross check *)
Transpose[eVecs] == Inverse[eVecs]
eVecs.Transpose[eVecs]
```

Out[ ]= True

Out[ ]= {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

In[ ]:=
```
Vdiag = eVecs.Vmat.Transpose[eVecs] // Simplify;
Vdiag // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 b^2 k & 0 \\ 0 & 0 & 3 b^2 k \end{pmatrix}$$

In[ ]:=
```
Tdiag = eVecs.Tmat.Transpose[eVecs] // Simplify;
Tdiag // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} b^2 m & 0 & 0 \\ 0 & b^2 m & 0 \\ 0 & 0 & b^2 m \end{pmatrix}$$

Problem #2)

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:= $T = \frac{1}{2} m\, v2 + \frac{1}{2} Ibar\ \omega Bar\ ^\wedge 2;$

$\omega Bar = \frac{(x1\,'[t] - x2\,'[t])}{L};$

$Ibar = \frac{1}{12} m\, L\ ^\wedge 2;$

$vel = \frac{(x1\,'[t] + x2\,'[t])}{2};$

$v2 = vel^2$

T // Expand // Factor

Out[ ]= $\frac{1}{4} (x1'[t] + x2'[t])^2$

Out[ ]= $\frac{1}{6} m \left(x1'[t]^2 + x1'[t]\, x2'[t] + x2'[t]^2\right)$

In[ ]:= $V = -m\, g \frac{(x1[t] + x2[t])}{2} + \frac{1}{2} k\, (x1[t]\ ^\wedge 2 + x2[t]\ ^\wedge 2)$ // Simplify

Out[ ]= $\frac{1}{2} \left(-g\, m\, (x1[t] + x2[t]) + k \left(x1[t]^2 + x2[t]^2\right)\right)$

In[ ]:= lag = T – V;

In[ ]:= D[D[T, x1 '[t]], t] // Expand

Out[ ]= $\frac{1}{3} m\, x1''[t] + \frac{1}{6} m\, x2''[t]$

In[ ]:= D[D[T, x2 '[t]], t] // Expand

Out[ ]= $\frac{1}{6} m\, x1''[t] + \frac{1}{3} m\, x2''[t]$

In[ ]:= eq1 = D[D[lag, x1 '[t]], t] – D[lag, x1[t]] // Expand

Out[ ]= $-\frac{g\, m}{2} + k\, x1[t] + \frac{1}{3} m\, x1''[t] + \frac{1}{6} m\, x2''[t]$

In[ ]:= eq2 = D[D[lag, x2 '[t]], t] – D[lag, x2[t]] // Expand

Out[ ]= $-\frac{g\, m}{2} + k\, x2[t] + \frac{1}{6} m\, x1''[t] + \frac{1}{3} m\, x2''[t]$

In[ ]:= $Tmat = \frac{m}{6} \{\{2, 1\}, \{1, 2\}\};$

Tmat // MatrixForm

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{m}{3} & \frac{m}{6} \\ \frac{m}{6} & \frac{m}{3} \end{pmatrix}$$

In[ ]:=
```
D[V, x1[t]] // Expand
```
Out[ ]= $-\frac{g\,m}{2} + k\,x1[t]$

In[ ]:=
```
D[V, x2[t]] // Expand
```
Out[ ]= $-\frac{g\,m}{2} + k\,x2[t]$

In[ ]:=
```
Vmat = k {{1, 0}, {0, 1}};

Vmat // MatrixForm
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$$

In[ ]:=
```
Tmat // MatrixForm
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} \frac{m}{3} & \frac{m}{6} \\ \frac{m}{6} & \frac{m}{3} \end{pmatrix}$$

In[ ]:=
```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} k - \frac{m\,\omega2}{3} & -\frac{m\,\omega2}{6} \\ -\frac{m\,\omega2}{6} & k - \frac{m\,\omega2}{3} \end{pmatrix}$$

In[ ]:=
```
sol = Solve[Det[mat] == 0, ω2]
```
Out[ ]= $\left\{\left\{\omega2 \to \frac{2\,k}{m}\right\}, \left\{\omega2 \to \frac{6\,k}{m}\right\}\right\}$

In[ ]:=
```
eq0 = mat .{a, b} == 0 // Thread
```
Out[ ]= $\left\{-\frac{1}{6}\,b\,m\,\omega2 + a\left(k - \frac{m\,\omega2}{3}\right) == 0,\ -\frac{1}{6}\,a\,m\,\omega2 + b\left(k - \frac{m\,\omega2}{3}\right) == 0\right\}$

In[ ]:=
```
norm = a^2 + b^2 == 1
```
Out[ ]= $a^2 + b^2 == 1$

In[ ]:=
```
eqs = Join[eq0, {norm}]
```
Out[ ]= $\left\{-\frac{1}{6}\,b\,m\,\omega2 + a\left(k - \frac{m\,\omega2}{3}\right) == 0,\ -\frac{1}{6}\,a\,m\,\omega2 + b\left(k - \frac{m\,\omega2}{3}\right) == 0,\ a^2 + b^2 == 1\right\}$

In[ ]:=
```
sol1 = Solve[eqs /. sol[[1]], {a, b}]
```
Out[ ]= $\left\{\left\{a \to -\frac{1}{\sqrt{2}},\ b \to -\frac{1}{\sqrt{2}}\right\}, \left\{a \to \frac{1}{\sqrt{2}},\ b \to \frac{1}{\sqrt{2}}\right\}\right\}$

In[ ]:= `ev1 = {a, b} /. sol1[[1]]`

Out[ ]= $\left\{-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}$

In[ ]:= `sol2 = Solve[eqs /. sol[[2]], {a, b}]`

Out[ ]= $\left\{\left\{a \to -\frac{1}{\sqrt{2}}, b \to \frac{1}{\sqrt{2}}\right\}, \left\{a \to \frac{1}{\sqrt{2}}, b \to -\frac{1}{\sqrt{2}}\right\}\right\}$

In[ ]:= `ev2 = {a, b} /. sol2[[1]]`

Out[ ]= $\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$

In[ ]:= `ev1.ev2`

Out[ ]= 0

In[ ]:= `eVecs = {ev1, ev2}`

Out[ ]= $\left\{\left\{-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}, \left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

In[ ]:= `eVecs.Vmat.Transpose[eVecs] // Simplify // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$$

In[ ]:= `eVecs.Tmat.Transpose[eVecs] // Simplify // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{m}{2} & 0 \\ 0 & \frac{m}{6} \end{pmatrix}$$

Part B Look at motion

In[ ]:= `values = {k → 1, m → 1};`

In[ ]:= `sol`

Out[ ]= $\left\{\left\{\omega 2 \to \frac{2\,k}{m}\right\}, \left\{\omega 2 \to \frac{6\,k}{m}\right\}\right\}$

In[ ]:= `mode1 = ev1 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[1]]`

Out[ ]= $\left\{-\frac{e^{i \sqrt{2} \sqrt{\frac{k}{m}}\, t}}{\sqrt{2}}, -\frac{e^{i \sqrt{2} \sqrt{\frac{k}{m}}\, t}}{\sqrt{2}}\right\}$

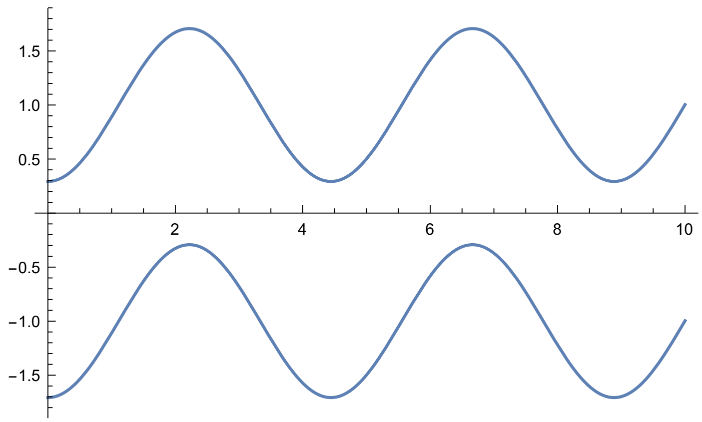
In[ ]:=
```
mode2 = ev2 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[1]]
```

Out[ ]= $\left\{-\frac{e^{i \sqrt{2} \sqrt{\frac{k}{m}} t}}{\sqrt{2}}, \frac{e^{i \sqrt{2} \sqrt{\frac{k}{m}} t}}{\sqrt{2}}\right\}$

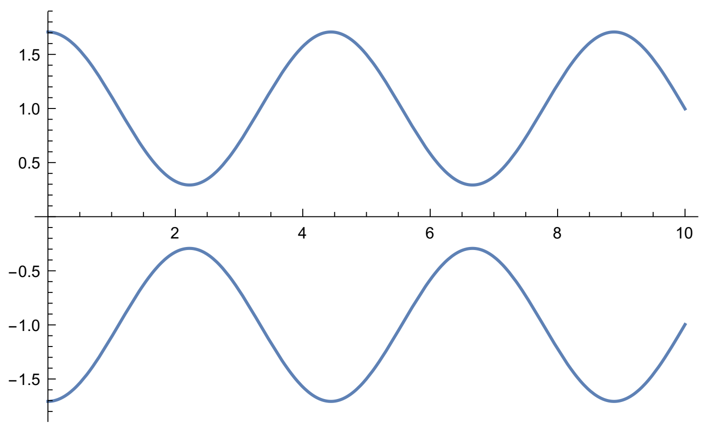
In[ ]:=
```
mode1 + {-1, 1} /. values
```

Out[ ]= $\left\{-1-\frac{e^{i \sqrt{2} t}}{\sqrt{2}}, 1-\frac{e^{i \sqrt{2} t}}{\sqrt{2}}\right\}$

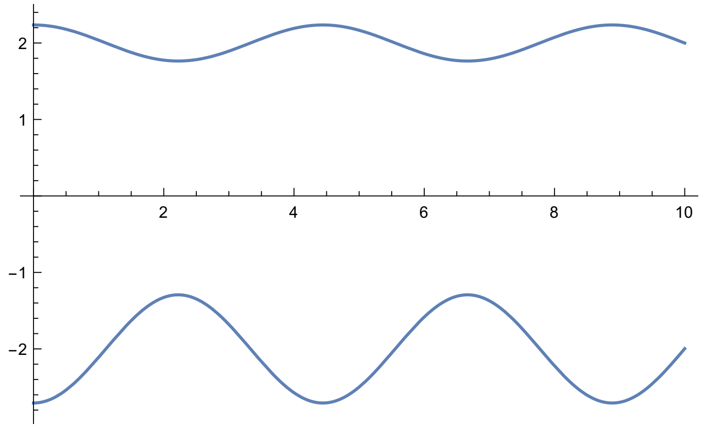
In[ ]:=
```
Plot[mode1 + {-1, 1} /. values // Re, {t, 0, 10}]
```

Out[ ]= 

In[ ]:=
```
Plot[mode2 + {-1, 1} /. values // Re, {t, 0, 10}]
```

Out[ ]= 

In[ ]:= Plot[$\frac{1}{3}$ mode1 + $\frac{2}{3}$ mode2 + {-2, 2} /. values // Re, {t, 0, 10}]

Out[ ]=


In[ ]:= {x1, x2} == eVecs.{n1, n2}

Out[ ]= $\{x1, x2\} == \left\{-\frac{n1}{\sqrt{2}} - \frac{n2}{\sqrt{2}}, -\frac{n1}{\sqrt{2}} + \frac{n2}{\sqrt{2}}\right\}$

In[ ]:= {x1, x2} == eVecs.{n1, n2} /. {n1 → 1/2, n2 → 1/2}

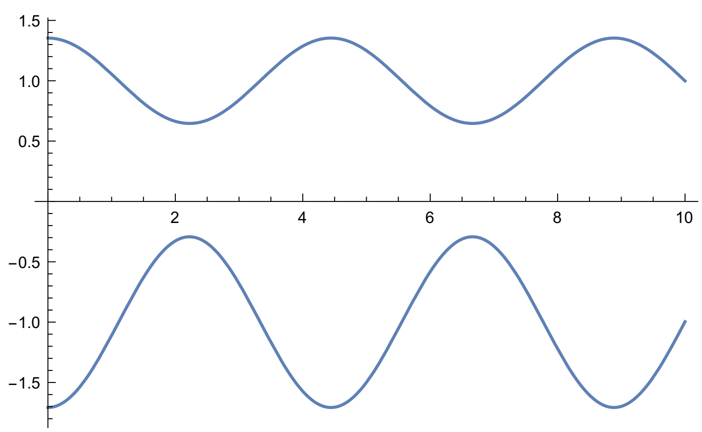
Out[ ]= $\{x1, x2\} == \left\{-\frac{1}{\sqrt{2}}, 0\right\}$

```
In[ ]:= mix[a_] := a mode1 + (1 - a) mode2 /. values // Re
        mix[1/2] /. {t → 0} // Simplify
```

Out[ ]= $\left\{-\frac{1}{\sqrt{2}}, 0\right\}$

```
In[ ]:= doPlot[a_ : 1/2] := Plot[a (mode1 + {-1, 1} /. values) + (1 - a) (mode2 + {-1, 1} /. values)
          // Re, {t, 0, 10}]
```

In[ ]:= doPlot[1/4]

Out[ ]=


Problem #3)

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
T = 1/2 m x1'[t]^2 + 1/2 m x2'[t]^2
```

Out[ ]= $\frac{1}{2} m\, x1'[t]^2 + \frac{1}{2} m\, x2'[t]^2$

In[ ]:=
```
V = 1/2 (3 k) x1[t]^2 + 1/2 (2 k) (x1[t] - x2[t])^2 // Expand
```

Out[ ]= $\frac{5}{2} k\, x1[t]^2 - 2 k\, x1[t] \times x2[t] + k\, x2[t]^2$

In[ ]:=
```
lag = T - V;
```

In[ ]:=
```
D[T, v1]
```

Out[ ]= 0

In[ ]:=
```
D[D[lag, x1'[t]], t] - D[lag, x1[t]]
```

Out[ ]= $5 k\, x1[t] - 2 k\, x2[t] + m\, x1''[t]$

In[ ]:=
```
D[D[lag, x2'[t]], t] - D[lag, x2[t]]
```

Out[ ]= $-2 k\, x1[t] + 2 k\, x2[t] + m\, x2''[t]$

In[ ]:=
```
Tmat = m DiagonalMatrix[{1, 1}];
Tmat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

In[ ]:=
```
D[V, x1]
```

Out[ ]= 0

In[ ]:=
```
D[V, x2]
```

Out[ ]= 0

In[ ]:=
```
Vmat = k {{5, -2}, {-2, 2}};
Vmat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 5 k & -2 k \\ -2 k & 2 k \end{pmatrix}$$

In[ ]:=
```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 5\,k - m\,\omega 2 & -2\,k \\ -2\,k & 2\,k - m\,\omega 2 \end{pmatrix}$$

In[ ]:=
```
sol = Solve[Det[mat] == 0, ω2]
```

Out[ ]= $\left\{\left\{\omega 2 \to \frac{k}{m}\right\}, \left\{\omega 2 \to \frac{6\,k}{m}\right\}\right\}$

In[ ]:=
```
eq1 = mat .{a, b} == 0 // Thread;
eq1 // Column
```

Out[ ]=

$-2\,b\,k + a\,(5\,k - m\,\omega 2) == 0$

$-2\,a\,k + b\,(2\,k - m\,\omega 2) == 0$

In[ ]:=
```
norm = a^2 + b^2 == 1
```

Out[ ]= $a^2 + b^2 == 1$

In[ ]:=
```
eq2 = Join[eq1, {norm}];
eq2 // Column
```

Out[ ]=

$-2\,b\,k + a\,(5\,k - m\,\omega 2) == 0$

$-2\,a\,k + b\,(2\,k - m\,\omega 2) == 0$

$a^2 + b^2 == 1$

In[ ]:=
```
sol1 = Solve[eq2 /. sol[[1]], {a, b}] // Last
```

Out[ ]= $\left\{a \to \frac{1}{\sqrt{5}}, b \to \frac{2}{\sqrt{5}}\right\}$

In[ ]:=
```
ev1 = {a, b} /. sol1
```

Out[ ]= $\left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\}$

In[ ]:=
```
sol2 = Solve[eq2 /. sol[[2]], {a, b}]
```

Out[ ]= $\left\{\left\{a \to -\frac{2}{\sqrt{5}}, b \to \frac{1}{\sqrt{5}}\right\}, \left\{a \to \frac{2}{\sqrt{5}}, b \to -\frac{1}{\sqrt{5}}\right\}\right\}$

In[ ]:=
```
ev2 = {a, b} /. Last[sol2]
```

Out[ ]= $\left\{\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right\}$

In[ ]:=
```
eVecs = {ev1, ev2} // Simplify
```

Out[ ]= $\left\{\left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\}, \left\{\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right\}\right\}$

In[ ]:= eVecs.eVecs

Out[ ]= {{1, 0}, {0, 1}}

In[ ]:= Vdiag = eVecs.Vmat.Transpose[eVecs] // Simplify ;
Vdiag // MatrixForm

Out[ ]//MatrixForm=
$$\begin{pmatrix} k & 0 \\ 0 & 6\,k \end{pmatrix}$$

In[ ]:= Tdiag = eVecs.Tmat.Transpose[eVecs] // Simplify;
Tdiag // MatrixForm

Out[ ]//MatrixForm=
$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

Part B Look at motion

In[ ]:= values = {k → 1, m → 1};

In[ ]:= sol

Out[ ]= $\left\{\left\{\omega 2 \to \frac{k}{m}\right\}, \left\{\omega 2 \to \frac{6\,k}{m}\right\}\right\}$

In[ ]:= mode1 = ev1 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[1]]

Out[ ]= $\left\{\frac{e^{i \sqrt{\frac{k}{m}}\, t}}{\sqrt{5}}, \frac{2\, e^{i \sqrt{\frac{k}{m}}\, t}}{\sqrt{5}}\right\}$

In[ ]:= mode2 = ev2 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[2]]

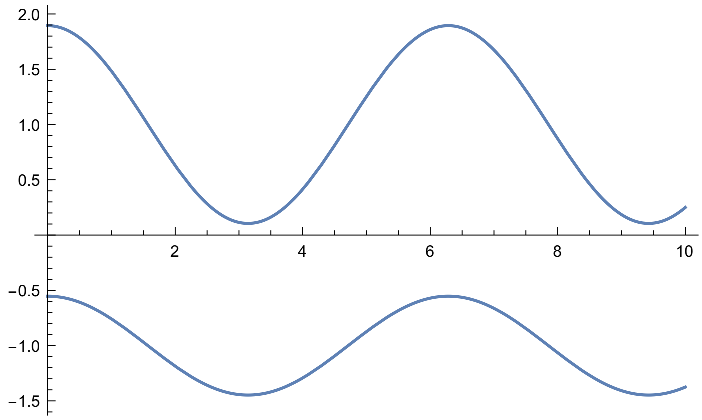
Out[ ]= $\left\{\frac{2\, e^{i \sqrt{6} \sqrt{\frac{k}{m}}\, t}}{\sqrt{5}}, -\frac{e^{i \sqrt{6} \sqrt{\frac{k}{m}}\, t}}{\sqrt{5}}\right\}$

In[ ]:= mode1 + {-1, 1} /. values

Out[ ]= $\left\{-1 + \frac{e^{i\, t}}{\sqrt{5}}, 1 + \frac{2\, e^{i\, t}}{\sqrt{5}}\right\}$

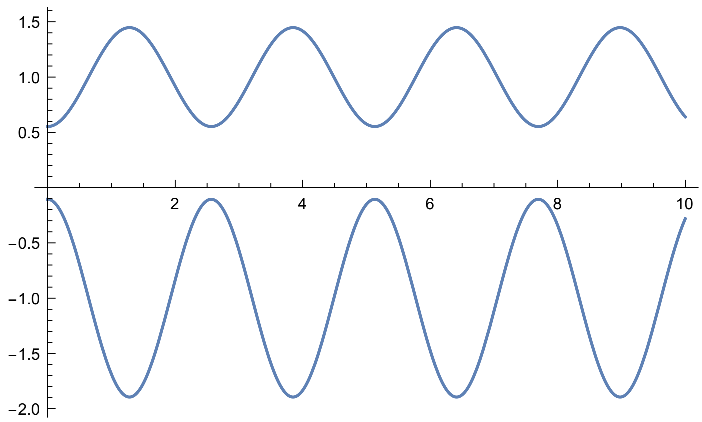
In[ ]:= Plot[mode1 + {-1, 1} /. values // Re, {t, 0, 10}]

Out[ ]=



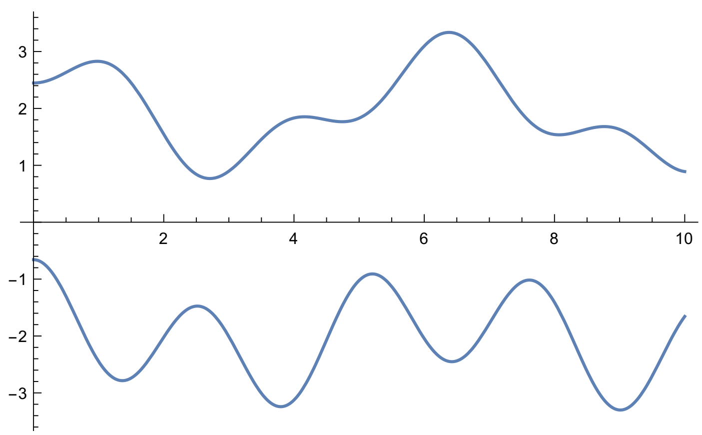
In[ ]:= Plot[mode2 + {-1, 1} /. values // Re, {t, 0, 10}]

Out[ ]=



In[ ]:= Plot[mode1 + mode2 + {-2, 2} /. values // Re, {t, 0, 10}]

Out[ ]=



In[ ]:= {x1, x2} == eVecs.{n1, n2}

Out[ ]= $\{x1, x2\} == \left\{\frac{n1}{\sqrt{5}} + \frac{2\,n2}{\sqrt{5}}, \frac{2\,n1}{\sqrt{5}} - \frac{n2}{\sqrt{5}}\right\}$

In[ ]:= `{x1, x2} == eVecs.{n1, n2} /. {n1 → 1/2, n2 → 1/2}`

Out[ ]= $\{x1, x2\} == \left\{\frac{3}{2\sqrt{5}}, \frac{1}{2\sqrt{5}}\right\}$

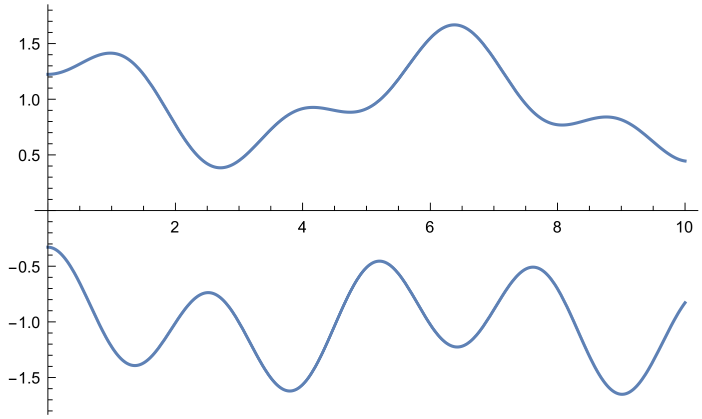
In[ ]:=
```
mix[a_] := a mode1 + (1 - a) mode2 /. values // Re
mix[1/2] /. {t → 0} // Simplify
```

Out[ ]= $\left\{\frac{3}{2\sqrt{5}}, \frac{1}{2\sqrt{5}}\right\}$

In[ ]:=
```
doPlot[a_ : 1/2] := Plot[a (mode1 + {-1, 1} /. values) + (1 - a) (mode2 + {-1, 1} /. values)
    // Re, {t, 0, 10}]
```

In[ ]:= `doPlot[1/2]`

Out[ ]=



Problem #4)

In[ ]:= `Clear["Global`*"]`

In[ ]:= $T = \frac{1}{2} m1\, x'[t]^2 + \frac{1}{2} m2\, (x'[t] + L\, \theta'[t])^2$ // Expand

Out[ ]= $\frac{1}{2} m1\, x'[t]^2 + \frac{1}{2} m2\, x'[t]^2 + L\, m2\, x'[t]\, \theta'[t] + \frac{1}{2} L^2\, m2\, \theta'[t]^2$

In[ ]:= $V = m2\, g\, L\, (1 - \mathrm{Cos}[\theta[t]]) + \frac{1}{2} k\, x[t]^2$ // Expand

Out[ ]= $g\, L\, m2 - g\, L\, m2\, \mathrm{Cos}[\theta[t]] + \frac{1}{2} k\, x[t]^2$

In[ ]:= `lag = T - V`

Out[ ]= $-g\, L\, m2 + g\, L\, m2\, \mathrm{Cos}[\theta[t]] - \frac{1}{2} k\, x[t]^2 + \frac{1}{2} m1\, x'[t]^2 + \frac{1}{2} m2\, x'[t]^2 + L\, m2\, x'[t]\, \theta'[t] + \frac{1}{2} L^2\, m2\, \theta'[t]^2$

In[ ]:=
```
rule = Cos[θ[t]] -> Series[Cos[θ[t]], {θ[t], 0, 2}] // Normal
```
Out[ ]= $\mathrm{Cos}[\theta[t]] \to 1 - \frac{\theta[t]^2}{2}$

In[ ]:=
```
V = V /. rule // Simplify
```
Out[ ]= $\frac{1}{2}\left(k\, x[t]^2 + g\, L\, m2\, \theta[t]^2\right)$

In[ ]:=
```
D[T, x'[t]]
```
Out[ ]= $m1\, x'[t] + m2\, x'[t] + L\, m2\, \theta'[t]$

In[ ]:=
```
D[T, θ'[t]]
```
Out[ ]= $L\, m2\, x'[t] + L^2\, m2\, \theta'[t]$

In[ ]:=
```
t11 = Coefficient[D[T, x'[t]], x'[t]]
```
Out[ ]= $m1 + m2$

In[ ]:=
```
t12 = t21 = Coefficient[D[T, x'[t]], θ'[t]]
```
Out[ ]= $L\, m2$

In[ ]:=
```
t22 = Coefficient[D[T, θ'[t]], θ'[t]]
```
Out[ ]= $L^2\, m2$

In[ ]:=
```
Tmat = {{t11, t12}, {t21, t22}};
Tmat // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} m1 + m2 & L\, m2 \\ L\, m2 & L^2\, m2 \end{pmatrix}$$

In[ ]:=
```
D[V, x[t]]
```
Out[ ]= $k\, x[t]$

In[ ]:=
```
D[V, θ[t]]
```
Out[ ]= $g\, L\, m2\, \theta[t]$

In[ ]:=
```
v11 = Coefficient[D[V, x[t]], x[t]]
```
Out[ ]= k

In[ ]:=
```
v12 = v21 = Coefficient[D[V, x[t]], θ[t]]
```
Out[ ]= 0

In[ ]:=
```
v22 = Coefficient[D[V, θ[t]], θ[t]]
```
Out[ ]= $g\, L\, m2$

In[ ]:=
```
Vmat = {{v11, v12}, {v21, v22}};
Vmat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} k & 0 \\ 0 & g\,L\,m2 \end{pmatrix}$$

In[ ]:=
```
Tmat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} m1 + m2 & L\,m2 \\ L\,m2 & L^2\,m2 \end{pmatrix}$$

In[ ]:=
```
(* x Equation *)
D[D[lag, x'[t]], t] - D[lag, x[t]] // Simplify
```

Out[ ]= $k\,x[t] + (m1 + m2)\,x''[t] + L\,m2\,\theta''[t]$

In[ ]:=
```
(* θ Equation *)
D[D[lag, θ'[t]], t] - D[lag, θ[t]]
```

Out[ ]= $g\,L\,m2\,\mathrm{Sin}[\theta[t]] + L\,m2\,x''[t] + L^2\,m2\,\theta''[t]$

In[ ]:=
```
values = {m1 → 1, m2 → 1, L → 1, g → 1, k → 2};
```

In[ ]:=
```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} k - (m1 + m2)\,\omega 2 & -L\,m2\,\omega 2 \\ -L\,m2\,\omega 2 & g\,L\,m2 - L^2\,m2\,\omega 2 \end{pmatrix}$$

In[ ]:=
```
mat = mat //. values // Simplify;
mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 2 - 2\,\omega 2 & -\omega 2 \\ -\omega 2 & 1 - \omega 2 \end{pmatrix}$$

In[ ]:=
```
sol = Solve[Det[mat] == 0, ω2] // Simplify
```

Out[ ]= $\{\{\omega 2 \to 2 - \sqrt{2}\}, \{\omega 2 \to 2 + \sqrt{2}\}\}$

In[ ]:=
```
eq1 = mat .{a, b} == 0 // Thread;
eq1 // Column
```

Out[ ]=

$a\,(2 - 2\,\omega 2) - b\,\omega 2 == 0$

$b\,(1 - \omega 2) - a\,\omega 2 == 0$

In[ ]:=
```
sol1 = Solve[eq1 /. sol[[1]], {a, b}][[1]] /. values // Simplify
```

••• Solve : Equations may not give solutions for all "solve" variables .

Out[ ]= $\{b \to \sqrt{2}\,a\}$

In[ ]:= `ev1 = {a, b} /. sol1 /. {a → 1} // Simplify`

Out[ ]= $\{1, \sqrt{2}\}$

In[ ]:= `sol2 = Solve[eq1 /. sol[[2]], {a, b}][[1]] /. values // Simplify // PowerExpand`

••• Solve : Equations may not give solutions for all "solve " variables .

Out[ ]= $\{b \to -\sqrt{2}\, a\}$

In[ ]:= `ev2 = {a, b} /. sol2 /. {a → 1} // Simplify`

Out[ ]= $\{1, -\sqrt{2}\}$

In[ ]:= `eVecs = Normalize /@ {ev1, ev2} // Simplify`

Out[ ]= $\left\{\left\{\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right\}, \left\{\frac{1}{\sqrt{3}}, -\sqrt{\frac{2}{3}}\right\}\right\}$

In[ ]:= `Vdiag = eVecs.Vmat.Transpose[eVecs] /. values // FullSimplify ;`
`Vdiag // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{4}{3} & 0 \\ 0 & \frac{4}{3} \end{pmatrix}$$

In[ ]:= `Tdiag = eVecs.Tmat.Transpose[eVecs] /. values // FullSimplify ;`
`Tdiag // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{2}{3}\left(2+\sqrt{2}\right) & 0 \\ 0 & -\frac{2}{3}\left(-2+\sqrt{2}\right) \end{pmatrix}$$

Part B Look at motion

In[ ]:= `sol /. values`

Out[ ]= $\{\{\omega 2 \to 2-\sqrt{2}\}, \{\omega 2 \to 2+\sqrt{2}\}\}$

In[ ]:= `mode1 = ev1 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[1]] /. values`

Out[ ]= $\left\{e^{i\sqrt{2-\sqrt{2}}\, t}, \sqrt{2}\, e^{i\sqrt{2-\sqrt{2}}\, t}\right\}$

In[ ]:= `mode2 = ev2 Exp[I ω t] /. {ω → Sqrt[ω2]} /. sol[[2]] /. values`

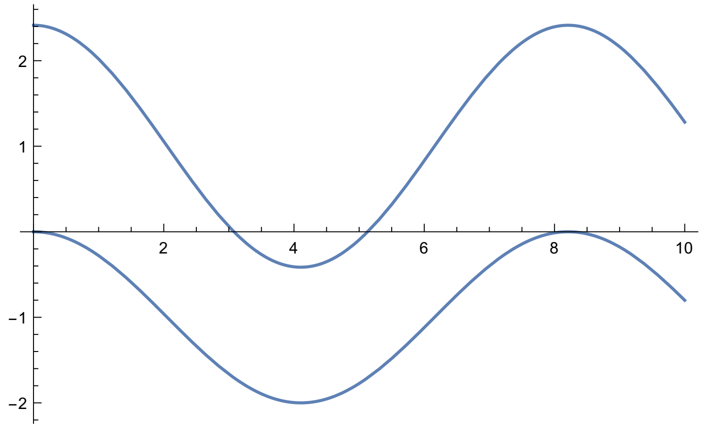
Out[ ]= $\left\{e^{i\sqrt{2+\sqrt{2}}\, t}, -\sqrt{2}\, e^{i\sqrt{2+\sqrt{2}}\, t}\right\}$

In[ ]:= `mode1 + {-1, 1} /. values`

Out[ ]= $\left\{-1+e^{i\sqrt{2-\sqrt{2}}\, t}, 1+\sqrt{2}\, e^{i\sqrt{2-\sqrt{2}}\, t}\right\}$

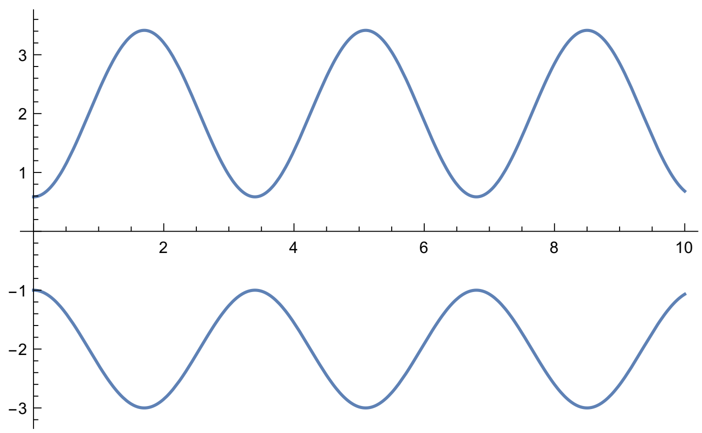
In[ ]:=
```
Plot[mode1 + {-1, 1} /. values // Re, {t, 0, 10}]
```

Out[ ]=



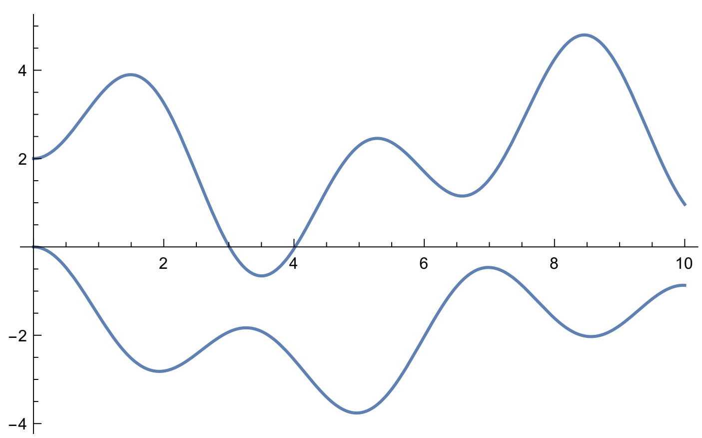
In[ ]:=
```
Plot[mode2 + {-2, 2} /. values // Re, {t, 0, 10}]
```

Out[ ]=



In[ ]:=
```
Plot[mode1 + mode2 + {-2, 2} /. values // Re, {t, 0, 10}]
```

Out[ ]=



In[ ]:=
```
{x1, x2} == eVecs.{n1, n2}
```

Out[ ]= $\{x1, x2\} == \left\{\frac{n1}{\sqrt{3}} + \sqrt{\frac{2}{3}}\, n2,\ \frac{n1}{\sqrt{3}} - \sqrt{\frac{2}{3}}\, n2\right\}$

In[∘]:=
```
{x1, x2} == eVecs.{n1, n2} /. {n1 → 1/2, n2 → 1/2}
```

Out[∘]= $\{x1, x2\} == \left\{\frac{1}{2\sqrt{3}} + \frac{1}{\sqrt{6}}, \frac{1}{2\sqrt{3}} - \frac{1}{\sqrt{6}}\right\}$

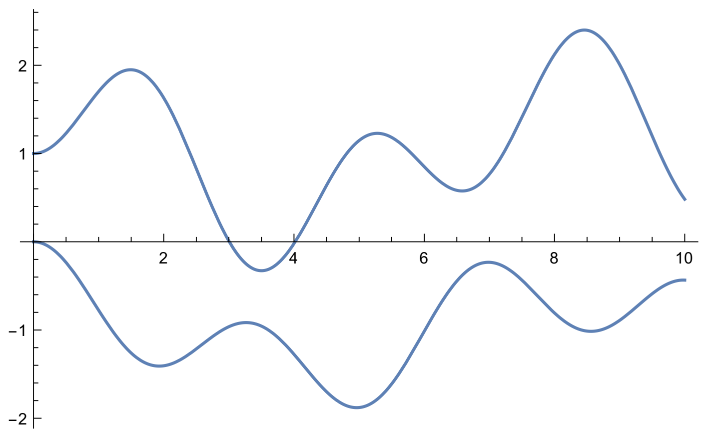
In[∘]:=
```
mix[a_] := a mode1 + (1 - a) mode2 /. values // Re
mix[1/2] /. {t → 0} // Simplify
```

Out[∘]= {1, 0}

In[∘]:=
```
doPlot[a_ : 1/2] := Plot[a (mode1 + {-1, 1} /. values) + (1 - a) (mode2 + {-1, 1} /. values)
    // Re, {t, 0, 10}]
```

In[∘]:=
```
doPlot[1/2]
```

Out[∘]=



Problem #5 & 6)

In[1]:=
```
Clear["Global`*"]
```

In[2]:=
```
mass = m DiagonalMatrix[{1, 1}];
kmat = k {{2, -1}, {-1, 2}};
mass // MatrixForm
kmat // MatrixForm
```

Out[4]//MatrixForm=

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

Out[5]//MatrixForm=

$$\begin{pmatrix} 2\,k & -k \\ -k & 2\,k \end{pmatrix}$$

In[6]:=
```
mat = kmat - w2 mass;
mat // MatrixForm
```

Out[7]//MatrixForm=

$$\begin{pmatrix} 2\,k - m\,w2 & -k \\ -k & 2\,k - m\,w2 \end{pmatrix}$$

In[8]:= **Det[mat]**

Out[8]= $3 k^2 - 4 k m w2 + m^2 w2^2$

In[9]:= **sol = Solve[Det[mat] == 0, w2]**

Out[9]= $\left\{\left\{w2 \to \frac{k}{m}\right\}, \left\{w2 \to \frac{3 k}{m}\right\}\right\}$

In[10]:=
```
vec = {a, b};
eqs = mat.vec == 0 /. sol[[1]] // Thread
```

Out[11]= {a k - b k == 0, - a k + b k == 0}

In[12]:= **Solve[eqs, {a, b}]**

Solve : Equations may not give solutions for all "solve" variables .

Out[12]= {{b → a}}

In[13]:= **ev1 = {1, 1}**

Out[13]= {1, 1}

In[14]:=
```
vec = {a, b};
eqs = mat.vec == 0 /. sol[[2]] // Thread
```

Out[15]= {- a k - b k == 0, - a k - b k == 0}

In[16]:= **Solve[eqs, {a, b}]**

Solve : Equations may not give solutions for all "solve" variables .

Out[16]= {{b → - a}}

In[17]:= **ev2 = {1, -1}**

Out[17]= {1, - 1}

Part B Look at motion

In[18]:= **tmp1 = a1 Cos[w1 t - δ1]**

Out[18]= a1 Cos[t w1 - δ1]

In[19]:= **tmp1 // TrigExpand**

Out[19]= a1 Cos[t w1] Cos[δ1] + a1 Sin[t w1] Sin[δ1]

In[20]:= **tmp2 = b1 Cos[w1 t] + c1 Sin[w1 t]**

Out[20]= b1 Cos[t w1] + c1 Sin[t w1]

In[21]:= **list1 = CoefficientList [tmp2, {Cos[t w1], Sin[t w1]}]**

Out[21]= {{0, c1}, {b1, 0}}

In[22]:=
```
list2 = CoefficientList [tmp1 // TrigExpand , {Cos[t w1], Sin[t w1]}]
```
Out[22]= {{0, a1 Sin[δ1]}, {a1 Cos[δ1], 0}}

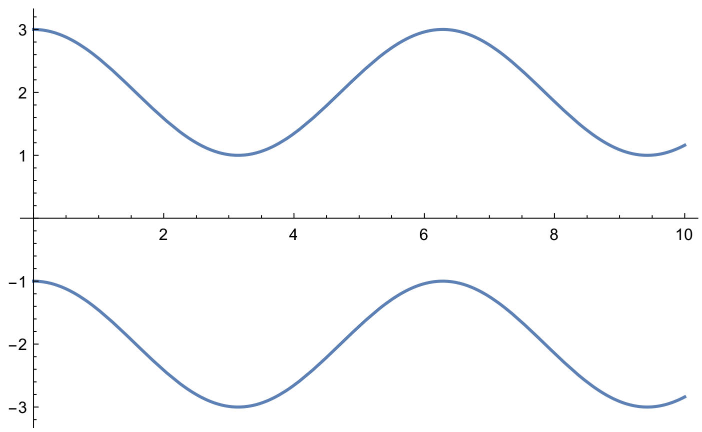
Part B Look at motion

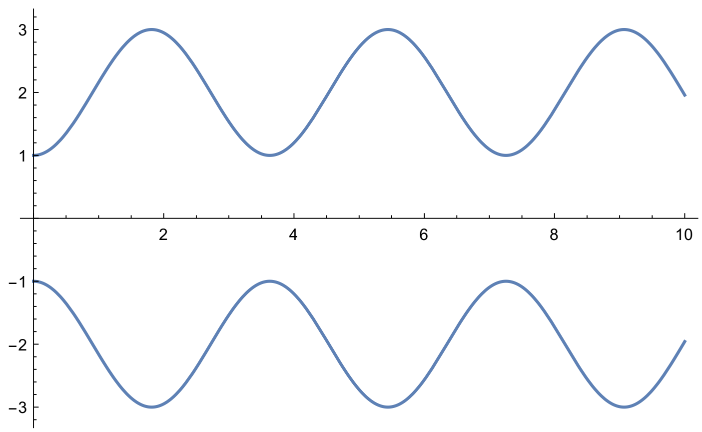
In[23]:=
```
values = {k → 1, m → 1};
```
In[24]:=
```
sol /. values
```
Out[24]= {{w2 → 1}, {w2 → 3}}

In[25]:=
```
mode1 = ev1 Exp[I w t] /. {w → Sqrt[w2]} /. sol[[1]] /. values
```
Out[25]= $\{e^{i t}, e^{i t}\}$

In[26]:=
```
mode2 = ev2 Exp[I w t] /. {w → Sqrt[w2]} /. sol[[2]] /. values
```
Out[26]= $\{e^{i \sqrt{3} t}, -e^{i \sqrt{3} t}\}$

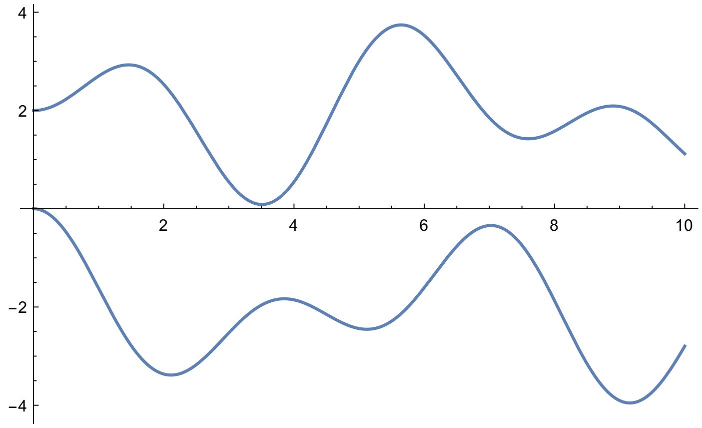
In[27]:=
```
mode1 + {-1, 1} /. values
```
Out[27]= $\{-1 + e^{i t}, 1 + e^{i t}\}$

In[28]:=
```
Plot[mode1 + {-2, 2} /. values // Re, {t, 0, 10}]
```
Out[28]=


In[29]:=
```
Plot[mode2 + {-2, 2} /. values // Re, {t, 0, 10}]
```
Out[29]=


In[30]:=
```
Plot[mode1 + mode2 + {-2, 2} /. values // Re, {t, 0, 10}]
```

Out[30]=



Part B Look at motion

In[31]:=
```
mode1 = ev1 (b1 Cos[w t] + c1 Sin[w t]) /. {w → Sqrt[w2]} /. sol[[1]] /. values;
mode1 // MatrixForm
```

Out[32]//MatrixForm=

$$\begin{pmatrix} b1\ Cos[t] + c1\ Sin[t] \\ b1\ Cos[t] + c1\ Sin[t] \end{pmatrix}$$

In[33]:=
```
mode2 = ev2 (b2 Cos[w t] + c2 Sin[w t]) /. {w → Sqrt[w2]} /. sol[[2]] /. values;
mode2 // MatrixForm
```

Out[34]//MatrixForm=

$$\begin{pmatrix} b2\ Cos\left[\sqrt{3}\ t\right] + c2\ Sin\left[\sqrt{3}\ t\right] \\ -b2\ Cos\left[\sqrt{3}\ t\right] - c2\ Sin\left[\sqrt{3}\ t\right] \end{pmatrix}$$

In[35]:=
```
x[t_] = mode1 + mode2
```

Out[35]= $\left\{b1\ Cos[t] + b2\ Cos\left[\sqrt{3}\ t\right] + c1\ Sin[t] + c2\ Sin\left[\sqrt{3}\ t\right],\ b1\ Cos[t] - b2\ Cos\left[\sqrt{3}\ t\right] + c1\ Sin[t] - c2\ Sin\left[\sqrt{3}\ t\right]\right\}$

In[36]:=
```
v[t_] = D[x[t], t]
```

Out[36]= $\left\{c1\ Cos[t] + \sqrt{3}\ c2\ Cos\left[\sqrt{3}\ t\right] - b1\ Sin[t] - \sqrt{3}\ b2\ Sin\left[\sqrt{3}\ t\right],\ c1\ Cos[t] - \sqrt{3}\ c2\ Cos\left[\sqrt{3}\ t\right] - b1\ Sin[t] + \sqrt{3}\ b2\ Sin\left[\sqrt{3}\ t\right]\right\}$

In[37]:=
```
x[0]
```

Out[37]= $\{b1 + b2,\ b1 - b2\}$

In[38]:=
```
v[0]
```

Out[38]= $\left\{c1 + \sqrt{3}\ c2,\ c1 - \sqrt{3}\ c2\right\}$

Find x={a,a} v={0,0}

In[ ]:=
```
eq1 = x[0] == {a, a} // Thread
```
Out[ ]= {b1 + b2 == a, b1 - b2 == a}

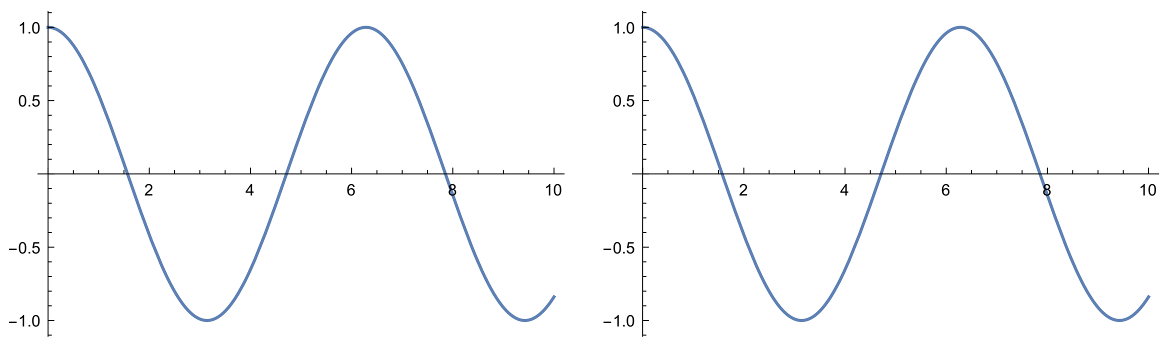
In[ ]:=
```
solb = Solve[eq1, {b1, b2}][[1]]
```
Out[ ]= {b1 → a, b2 → 0}

In[ ]:=
```
eq1v = v[0] == {0, 0} // Thread
```
Out[ ]= $\{c1 + \sqrt{3}\ c2 == 0,\ c1 - \sqrt{3}\ c2 == 0\}$

In[ ]:=
```
solc = Solve[eq1v, {c1, c2}][[1]]
```
Out[ ]= {c1 → 0, c2 → 0}

In[ ]:=
```
x[t] /. solb /. solc /. {a → 1}
```
Out[ ]= {Cos[t], Cos[t]}

In[ ]:=
```
p1 = Plot[x[t][[1]] /. solb /. solc /. {a → 1}, {t, 0, 10}];
p2 = Plot[x[t][[2]] /. solb /. solc /. {a → 1}, {t, 0, 10}];
GraphicsGrid[{{p1, p2}}]
```
Out[ ]=



Find x={a,0} v={0,0}

In[74]:=
```
eq1 = x[0] == {a, 0} // Thread
```
Out[74]= {b1 + b2 == a, b1 - b2 == 0}

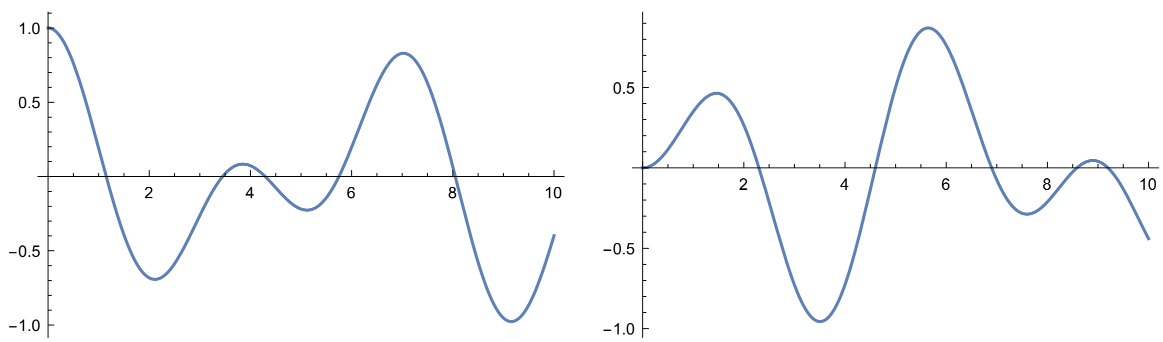
In[75]:=
```
solb = Solve[eq1, {b1, b2}][[1]]
```
Out[75]= $\{b1 \to \frac{a}{2},\ b2 \to \frac{a}{2}\}$

In[76]:=
```
eq1v = v[0] == {0, 0} // Thread
```
Out[76]= $\{c1 + \sqrt{3}\ c2 == 0,\ c1 - \sqrt{3}\ c2 == 0\}$

In[77]:=
```
solc = Solve[eq1v, {c1, c2}][[1]]
```
Out[77]= {c1 → 0, c2 → 0}

In[78]:=
```
x[t] /. solb /. solc /. {a → 1}
```
Out[78]= $\left\{\frac{\text{Cos}[t]}{2} + \frac{1}{2}\text{Cos}\left[\sqrt{3}\ t\right], \frac{\text{Cos}[t]}{2} - \frac{1}{2}\text{Cos}\left[\sqrt{3}\ t\right]\right\}$

In[79]:=
```
p1 = Plot[x[t][[1]] /. solb /. solc /. {a → 1}, {t, 0, 10}];
p2 = Plot[x[t][[2]] /. solb /. solc /. {a → 1}, {t, 0, 10}];
GraphicsGrid[{{p1, p2}}]
```

Out[81]=


Problem 6)

Find x={0,0} v={a,a}

In[ ]:=
```
eq1 = x[0] == {0, 0} // Thread
```
Out[ ]= {b1 + b2 == 0, b1 − b2 == 0}

In[ ]:=
```
solb = Solve[eq1, {b1, b2}][[1]]
```
Out[ ]= {b1 → 0, b2 → 0}

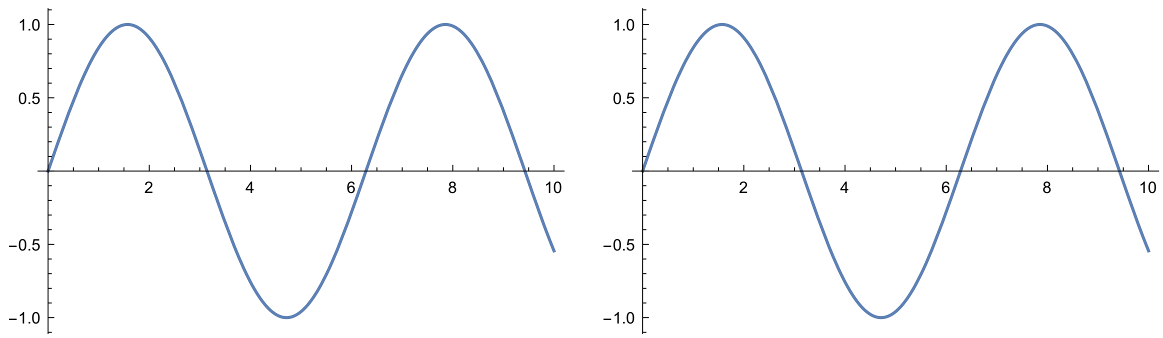
In[ ]:=
```
eq1v = v[0] == {a, a} // Thread
```
Out[ ]= $\left\{c1 + \sqrt{3}\ c2 == a, c1 - \sqrt{3}\ c2 == a\right\}$

In[ ]:=
```
solc = Solve[eq1v, {c1, c2}][[1]]
```
Out[ ]= {c1 → a, c2 → 0}

In[ ]:=
```
x[t] /. solb /. solc /. {a → 1}
```
Out[ ]= {Sin[t], Sin[t]}

In[ ]:=
```
p1 = Plot[ x[t][[1]] /. solb /. solc /. {a → 1}, {t, 0, 10}];
p2 = Plot[ x[t][[2]] /. solb /. solc /. {a → 1}, {t, 0, 10}];
GraphicsGrid [{{p1, p2}}]
```

Out[ ]=



Find x={0,0} v={0,a}

In[90]:=
```
eq1 = x[0] == {0, 0} // Thread
```

Out[90]= {b1 + b2 == 0, b1 − b2 == 0}

In[91]:=
```
solb = Solve[eq1, {b1, b2}][[1]]
```

Out[91]= {b1 → 0, b2 → 0}

In[92]:=
```
eq1v = v[0] == {0, a} // Thread
```

Out[92]= $\left\{c1 + \sqrt{3}\ c2 == 0,\ c1 - \sqrt{3}\ c2 == a\right\}$

In[93]:=
```
solc = Solve[eq1v, {c1, c2}][[1]]
```

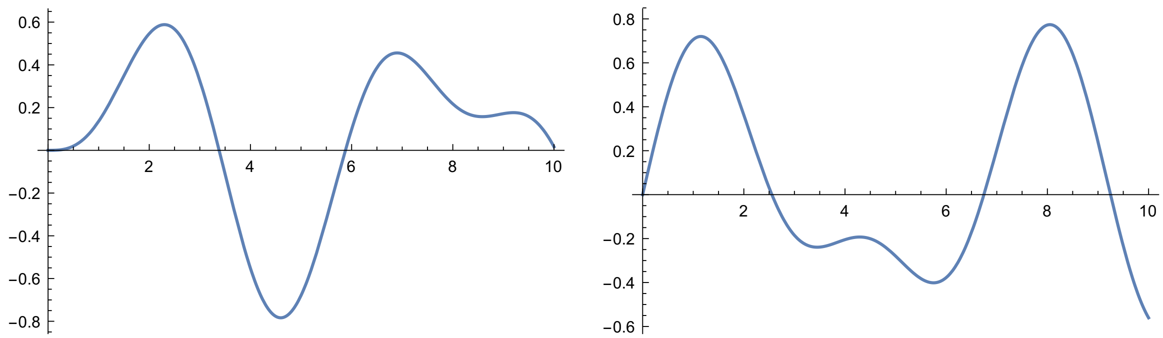
Out[93]= $\left\{c1 \to \frac{a}{2},\ c2 \to -\frac{a}{2\sqrt{3}}\right\}$

In[94]:=
```
x[t] /. solb /. solc /. {a → 1}
```

Out[94]= $\left\{\frac{\text{Sin}[t]}{2} - \frac{\text{Sin}\left[\sqrt{3}\ t\right]}{2\sqrt{3}},\ \frac{\text{Sin}[t]}{2} + \frac{\text{Sin}\left[\sqrt{3}\ t\right]}{2\sqrt{3}}\right\}$

In[95]:=
```
p1 = Plot[x[t][[1]] /. solb /. solc /. {a → 1}, {t, 0, 10}];
p2 = Plot[x[t][[2]] /. solb /. solc /. {a → 1}, {t, 0, 10}];
GraphicsGrid[{{p1, p2}}]
```

Out[97]=



Problem #7)

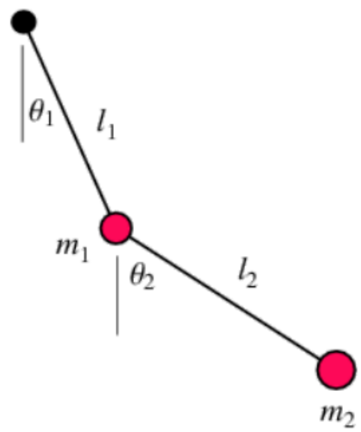
In[99]:=
```
Clear["Global`*"]
```

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}_1} = \frac{\partial \mathcal{L}}{\partial \phi_1} \quad \text{or} \quad (m_1+m_2)L_1^2\ddot{\phi}_1 + m_2 L_1 L_2 \ddot{\phi}_2 = -(m_1+m_2)gL_1\phi_1 \qquad (11.41)$$

and

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}_2} = \frac{\partial \mathcal{L}}{\partial \phi_2} \quad \text{or} \quad m_2 L_1 L_2 \ddot{\phi}_1 + m_2 L_2^2 \ddot{\phi}_2 = -m_2 g L_2 \phi_2. \qquad (11.42)$$

Double Pendulum



A double pendulum consists of one pendulum attached to another. Double pendula are an example of a simple physical system which can exhibit chaotic behavior. Consider a double bob pendulum with masses m_1 and m_2 attached by rigid massless wires of lengths l_1 and l_2. Further, let the angles the two wires make with the vertical be denoted θ_1 and θ_2, as illustrated above. Finally, let gravity be given by *g*. Then the positions of the bobs are given by

$$x_1 = l_1 \sin\theta_1 \quad (1)$$

$$y_1 = -l_1 \cos\theta_1 \quad (2)$$

$$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2 \quad (3)$$

$$y_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2. \quad (4)$$

The potential energy of the system is then given by

$$V = m_1 g y_1 + m_2 g y_2 \quad (5)$$

$$= -(m_1 + m_2) g l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2, \quad (6)$$

and the kinetic energy by

$$T = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 \quad (7)$$

$$= \tfrac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \tfrac{1}{2} m_2 [l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)]. \quad (8)$$

In[100]:=
```
x1[t_] = l1 Sin[θ1[t]];
y1[t_] = -l1 Cos[θ1[t]];

x2[t_] = l1 Sin[θ1[t]] + l2 Sin[θ2[t]];
y2[t_] = -l1 Cos[θ1[t]] - l2 Cos[θ2[t]];

x3[t_] = l1 Sin[θ1[t]] + l2 Sin[θ2[t]] + l3 Sin[θ3[t]];
y3[t_] = -l1 Cos[θ1[t]] - l2 Cos[θ2[t]] - l3 Cos[θ3[t]];
```

In[106]:=
```
v1[t_] = Sqrt[x1'[t]^2 + y1'[t]^2] // Simplify // PowerExpand
```
Out[106]= $l1\ \theta1'[t]$

In[107]:=
```
v2[t_] = Sqrt[x2'[t]^2 + y2'[t]^2] // Simplify // PowerExpand
```
Out[107]= $\sqrt{l1^2\ \theta1'[t]^2 + 2\ l1\ l2\ \text{Cos}[\theta1[t] - \theta2[t]]\ \theta1'[t]\ \theta2'[t] + l2^2\ \theta2'[t]^2}$

In[108]:=
```
v3[t_] = Sqrt[x3'[t]^2 + y3'[t]^2] // Simplify // PowerExpand
```
Out[108]= $\sqrt{((l1\ \text{Cos}[\theta1[t]]\ \theta1'[t] + l2\ \text{Cos}[\theta2[t]]\ \theta2'[t] + l3\ \text{Cos}[\theta3[t]]\ \theta3'[t])^2 + (l1\ \text{Sin}[\theta1[t]]\ \theta1'[t] + l2\ \text{Sin}[\theta2[t]]\ \theta2'[t] + l3\ \text{Sin}[\theta3[t]]\ \theta3'[t])^2)}$

Look at single pendulum case

In[109]:=
```
tKinetic1 = 1/2 m1 v1[t]^2 // Simplify
```
Out[109]= $\frac{1}{2}\ l1^2\ m1\ \theta1'[t]^2$

In[110]:=
```
vPot1 = m1 g y1[t]
```
Out[110]= $-g\ l1\ m1\ \text{Cos}[\theta1[t]]$

In[111]:=
```
lag1 = tKinetic1 - vPot1;
```

In[112]:=
```
eq1 = D[D[lag1, θ1'[t]], t] - D[lag1, θ1[t]]
```
Out[112]= $g\ l1\ m1\ \text{Sin}[\theta1[t]] + l1^2\ m1\ \theta1''[t]$

In[113]:=
```
Series[eq1, {θ1[t], 0, 1}] // Normal // Simplify
```
Out[113]= $l1\ m1\ (g\ \theta1[t] + l1\ \theta1''[t])$

Look at double pendulum case

In[114]:=
```
list = {θ1[t], θ1'[t], θ2[t], θ2'[t]};
rule2 = (Outer[Times, list, list] // Flatten) → 0 // Thread
```
Out[115]= $\{\theta1[t]^2 \to 0,\ \theta1[t]\ \theta1'[t] \to 0,\ \theta1[t] \times \theta2[t] \to 0,\ \theta1[t]\ \theta2'[t] \to 0,\ \theta1[t]\ \theta1'[t] \to 0,\ \theta1'[t]^2 \to 0,\ \theta2[t]\ \theta1'[t] \to 0,\ \theta1'[t]\ \theta2'[t] \to 0,\ \theta1[t] \times \theta2[t] \to 0,\ \theta2[t]\ \theta1'[t] \to 0,\ \theta2[t]^2 \to 0,\ \theta2[t]\ \theta2'[t] \to 0,\ \theta1[t]\ \theta2'[t] \to 0,\ \theta1'[t]\ \theta2'[t] \to 0,\ \theta2[t]\ \theta2'[t] \to 0,\ \theta2'[t]^2 \to 0\}$

In[116]:=
```
tKinetic2 = 1/2 m1 v1[t]^2 + 1/2 m2 v2[t]^2 // Simplify
```
Out[116]= $\frac{1}{2}\ (l1^2\ (m1 + m2)\ \theta1'[t]^2 + 2\ l1\ l2\ m2\ \text{Cos}[\theta1[t] - \theta2[t]]\ \theta1'[t]\ \theta2'[t] + l2^2\ m2\ \theta2'[t]^2)$

```
In[117]:= vPot2 = m1 g y1[t] + m2 g y2[t]
Out[117]= - g l1 m1 Cos[θ1[t]] + g m2 (- l1 Cos[θ1[t]] - l2 Cos[θ2[t]])

In[118]:= lag2 = tKinetic2 - vPot2;

In[119]:= D[lag2, θ1[t]] // Simplify
Out[119]= - l1 (g (m1 + m2) Sin[θ1[t]] + l2 m2 Sin[θ1[t] - θ2[t]] θ1′[t] θ2′[t])

In[120]:= D[lag2, θ1'[t]] // Simplify
Out[120]= l1 (l1 (m1 + m2) θ1′[t] + l2 m2 Cos[θ1[t] - θ2[t]] θ2′[t])

In[121]:= eq21 = D[D[lag2, θ1'[t]], t] - D[lag2, θ1[t]]  // Simplify
Out[121]= l1 (g m1 Sin[θ1[t]] + g m2 Sin[θ1[t]] +
            l2 m2 Sin[θ1[t] - θ2[t]] θ2′[t]^2 + l1 (m1 + m2) θ1″[t] + l2 m2 Cos[θ1[t] - θ2[t]] θ2″[t])

In[122]:= eq21a = Series[eq21, {θ1[t], 0, 1}, {θ2[t], 0, 1}] // Normal // ExpandAll
Out[122]= g l1 m1 θ1[t] + g l1 m2 θ1[t] + l1 l2 m2 θ1[t] θ2′[t]^2 - l1 l2 m2 θ2[t] θ2′[t]^2 +
           l1^2 m1 θ1″[t] + l1^2 m2 θ1″[t] + l1 l2 m2 θ2″[t] + l1 l2 m2 θ1[t] × θ2[t] θ2″[t]

In[123]:= eq21a = eq21a  //. rule2 // Simplify
Out[123]= l1 (g (m1 + m2) θ1[t] + l1 (m1 + m2) θ1″[t] + l2 m2 θ2″[t])

In[124]:= eq22 = D[D[lag2, θ2'[t]], t] - D[lag2, θ2[t]]  // Simplify
Out[124]= l2 m2 (g Sin[θ2[t]] - l1 Sin[θ1[t] - θ2[t]] θ1′[t]^2 + l1 Cos[θ1[t] - θ2[t]] θ1″[t] + l2 θ2″[t])

In[125]:= eq22a = Series[eq22, {θ1[t], 0, 1}, {θ2[t], 0, 1}] // Normal // ExpandAll
Out[125]= g l2 m2 θ2[t] - l1 l2 m2 θ1[t] θ1′[t]^2 + l1 l2 m2 θ2[t] θ1′[t]^2 +
           l1 l2 m2 θ1″[t] + l1 l2 m2 θ1[t] × θ2[t] θ1″[t] + l2^2 m2 θ2″[t]

In[126]:= eq22a = eq22a  //. rule2 // Simplify
Out[126]= l2 m2 (g θ2[t] + l1 θ1″[t] + l2 θ2″[t])
```

Double Pendulum: find eigensystem

```
In[169]:= {eq21a, eq22a}
          ------------- //. {m1 → m, m2 → m, l1 → L, l2 → L} // Simplify // TableForm
               L m
Out[169]//TableForm=
          2 g θ1[t] + L (2 θ1″[t] + θ2″[t])
          g θ2[t] + L (θ1″[t] + θ2″[t])

          mMat = L {{2, 1}, {1, 1}};
          mMat // MatrixForm
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 2\,L & L \\ L & L \end{pmatrix}$$

In[172]:=
```
vMat = g {{2, 0}, {0, 1}};
vMat // MatrixForm
```

Out[173]//MatrixForm=

$$\begin{pmatrix} 2\,g & 0 \\ 0 & g \end{pmatrix}$$

In[175]:=
```
vMat - w2 mMat // MatrixForm
```

Out[175]//MatrixForm=

$$\begin{pmatrix} 2\,g - 2\,L\,w2 & -L\,w2 \\ -L\,w2 & g - L\,w2 \end{pmatrix}$$

In[179]:=
```
sol = Solve[Det[vMat - w2 mMat ] == 0, w2] // Simplify
```

Out[179]=

$$\left\{\left\{w2 \to -\frac{\left(-2+\sqrt{2}\right)g}{L}\right\}, \left\{w2 \to \frac{\left(2+\sqrt{2}\right)g}{L}\right\}\right\}$$

In[∘]:=
```
sol1 = Solve[(vMat - w2 mMat).{a, b} == 0 /. sol[[1]] // Thread , {a, b}][[1]]
```

Out[∘]=

$$\left\{b \to -\frac{2\left(-1+\sqrt{2}\right)a}{-2+\sqrt{2}}\right\}$$

In[∘]:=
```
ev1 = {a, b} /. sol1 /. {a → 1} // Simplify
```

Out[∘]=

$$\left\{1, \sqrt{2}\right\}$$

In[189]:=
```
sol2 = Solve[(vMat - w2 mMat).{a, b} == 0 /. sol[[2]] // Thread , {a, b}][[1]]
```

Out[189]=

$$\left\{b \to -\frac{2\left(1+\sqrt{2}\right)a}{2+\sqrt{2}}\right\}$$

In[190]:=
```
ev2 = {a, b} /. sol2 /. {a → 1} // Simplify
```

Out[190]=

$$\left\{1, -\sqrt{2}\right\}$$

In[∘]:=
```
evMat = {ev1, ev2};
evMat.mMat.Transpose[evMat] // Simplify // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 2\left(2+\sqrt{2}\right)L & 0 \\ 0 & -2\left(-2+\sqrt{2}\right)L \end{pmatrix}$$

In[197]:=
```
evMat.vMat.Transpose[evMat] // Simplify // MatrixForm
```

Out[197]//MatrixForm=

$$\begin{pmatrix} 4\,g & 0 \\ 0 & 4\,g \end{pmatrix}$$

Look at triple pendulum case

```
In[128]:= list = {θ1[t], θ1'[t], θ2[t], θ2'[t], θ3[t], θ3'[t]};
          rule3 = (Outer[Times, list, list] // Flatten) → 0 // Thread

Out[129]= {θ1[t]^2 → 0, θ1[t] θ1'[t] → 0, θ1[t] × θ2[t] → 0, θ1[t] θ2'[t] → 0, θ1[t] × θ3[t] → 0, θ1[t] θ3'[t] → 0,
           θ1[t] θ1'[t] → 0, θ1'[t]^2 → 0, θ2[t] θ1'[t] → 0, θ1'[t] θ2'[t] → 0, θ3[t] θ1'[t] → 0, θ1'[t] θ3'[t] → 0,
           θ1[t] × θ2[t] → 0, θ2[t] θ1'[t] → 0, θ2[t]^2 → 0, θ2[t] θ2'[t] → 0, θ2[t] × θ3[t] → 0, θ2[t] θ3'[t] → 0,
           θ1[t] θ2'[t] → 0, θ1'[t] θ2'[t] → 0, θ2[t] θ2'[t] → 0, θ2'[t]^2 → 0, θ3[t] θ2'[t] → 0, θ2'[t] θ3'[t] → 0,
           θ1[t] × θ3[t] → 0, θ3[t] θ1'[t] → 0, θ2[t] × θ3[t] → 0, θ3[t] θ2'[t] → 0, θ3[t]^2 → 0, θ3[t] θ3'[t] → 0,
           θ1[t] θ3'[t] → 0, θ1'[t] θ3'[t] → 0, θ2[t] θ3'[t] → 0, θ2'[t] θ3'[t] → 0, θ3[t] θ3'[t] → 0, θ3'[t]^2 → 0}

In[130]:= tKinetic3 = 1/2 m1 v1[t]^2 + 1/2 m2 v2[t]^2 + 1/2 m3 v3[t]^2 // Simplify

Out[130]= 1/2 (l1^2 m1 θ1'[t]^2 + m2 (l1^2 θ1'[t]^2 + 2 l1 l2 Cos[θ1[t] - θ2[t]] θ1'[t] θ2'[t] + l2^2 θ2'[t]^2) +
             m3 ((l1 Cos[θ1[t]] θ1'[t] + l2 Cos[θ2[t]] θ2'[t] + l3 Cos[θ3[t]] θ3'[t])^2 +
                 (l1 Sin[θ1[t]] θ1'[t] + l2 Sin[θ2[t]] θ2'[t] + l3 Sin[θ3[t]] θ3'[t])^2))

In[131]:= vPot3 = m1 g y1[t] + m2 g y2[t] + m3 g y3[t]

Out[131]= -g l1 m1 Cos[θ1[t]] + g m2 (-l1 Cos[θ1[t]] - l2 Cos[θ2[t]]) +
           g m3 (-l1 Cos[θ1[t]] - l2 Cos[θ2[t]] - l3 Cos[θ3[t]])

In[132]:= lag3 = tKinetic3 - vPot3;

In[133]:= D[lag3, θ1[t]] // Simplify

Out[133]= -l1 (g (m1 + m2 + m3) Sin[θ1[t]] + θ1'[t] (l2 (m2 + m3) Sin[θ1[t] - θ2[t]] θ2'[t] + l3 m3 Sin[θ1[t] - θ3[t]] θ3'[t]))

In[134]:= D[lag3, θ1'[t]] // Simplify

Out[134]= l1 (l1 (m1 + m2 + m3) θ1'[t] + l2 (m2 + m3) Cos[θ1[t] - θ2[t]] θ2'[t] + l3 m3 Cos[θ1[t] - θ3[t]] θ3'[t])

In[135]:= eq31 = D[D[lag3, θ1'[t]], t] - D[lag3, θ1[t]] // Simplify

Out[135]= l1 (g m1 Sin[θ1[t]] + g m2 Sin[θ1[t]] + g m3 Sin[θ1[t]] + l2 (m2 + m3) Sin[θ1[t] - θ2[t]] θ2'[t]^2 +
             l3 m3 Sin[θ1[t] - θ3[t]] θ3'[t]^2 + l1 m1 θ1''[t] + l1 m2 θ1''[t] + l1 m3 θ1''[t] +
             l2 m2 Cos[θ1[t] - θ2[t]] θ2''[t] + l2 m3 Cos[θ1[t] - θ2[t]] θ2''[t] + l3 m3 Cos[θ1[t] - θ3[t]] θ3''[t])

In[136]:= eq31a = Series[eq31, {θ1[t], 0, 1}, {θ2[t], 0, 1}, {θ3[t], 0, 1}] // Normal // ExpandAll

Out[136]= g l1 m1 θ1[t] + g l1 m2 θ1[t] + g l1 m3 θ1[t] + l1 l2 m2 θ1[t] θ2'[t]^2 + l1 l2 m3 θ1[t] θ2'[t]^2 -
           l1 l2 m2 θ2[t] θ2'[t]^2 - l1 l2 m3 θ2[t] θ2'[t]^2 + l1 l3 m3 θ1[t] θ3'[t]^2 - l1 l3 m3 θ3[t] θ3'[t]^2 +
           l1^2 m1 θ1''[t] + l1^2 m2 θ1''[t] + l1^2 m3 θ1''[t] + l1 l2 m2 θ2''[t] + l1 l2 m3 θ2''[t] +
           l1 l2 m2 θ1[t] × θ2[t] θ2''[t] + l1 l2 m3 θ1[t] × θ2[t] θ2''[t] + l1 l3 m3 θ3''[t] + l1 l3 m3 θ1[t] × θ3[t] θ3''[t]

In[137]:= eq31a = eq31a //. rule3 // Simplify

Out[137]= l1 (g (m1 + m2 + m3) θ1[t] + l1 (m1 + m2 + m3) θ1''[t] + l2 m2 θ2''[t] + l2 m3 θ2''[t] + l3 m3 θ3''[t])
```

In[138]:= `eq32 = D[D[lag3, θ2'[t]], t] - D[lag3, θ2[t]] // Simplify`

Out[138]= l2 (g m2 Sin[θ2[t]] + g m3 Sin[θ2[t]] - l1 (m2 + m3) Sin[θ1[t] - θ2[t]] θ1′[t]^2 + l3 m3 Sin[θ2[t] - θ3[t]] θ3′[t]^2 + l1 m2 Cos[θ1[t] - θ2[t]] θ1″[t] + l1 m3 Cos[θ1[t] - θ2[t]] θ1″[t] + l2 m2 θ2″[t] + l2 m3 θ2″[t] + l3 m3 Cos[θ2[t] - θ3[t]] θ3″[t])

In[139]:= `eq32a = Series[eq32, {θ1[t], 0, 1}, {θ2[t], 0, 1}, {θ3[t], 0, 1}] // Normal // ExpandAll`

Out[139]= g l2 m2 θ2[t] + g l2 m3 θ2[t] - l1 l2 m2 θ1[t] θ1′[t]^2 - l1 l2 m3 θ1[t] θ1′[t]^2 + l1 l2 m2 θ2[t] θ1′[t]^2 + l1 l2 m3 θ2[t] θ1′[t]^2 + l2 l3 m3 θ2[t] θ3′[t]^2 - l2 l3 m3 θ3[t] θ3′[t]^2 + l1 l2 m2 θ1″[t] + l1 l2 m3 θ1″[t] + l1 l2 m2 θ1[t] × θ2[t] θ1″[t] + l1 l2 m3 θ1[t] × θ2[t] θ1″[t] + l2^2 m2 θ2″[t] + l2^2 m3 θ2″[t] + l2 l3 m3 θ3″[t] + l2 l3 m3 θ2[t] × θ3[t] θ3″[t]

In[140]:= `eq32a = eq32a //. rule3 // Simplify`

Out[140]= l2 (g (m2 + m3) θ2[t] + l1 (m2 + m3) θ1″[t] + l2 m2 θ2″[t] + l2 m3 θ2″[t] + l3 m3 θ3″[t])

In[141]:= `eq33 = D[D[lag3, θ3'[t]], t] - D[lag3, θ3[t]] // Simplify`

Out[141]= l3 m3 (g Sin[θ3[t]] - l1 Sin[θ1[t] - θ3[t]] θ1′[t]^2 - l2 Sin[θ2[t] - θ3[t]] θ2′[t]^2 + l1 Cos[θ1[t] - θ3[t]] θ1″[t] + l2 Cos[θ2[t] - θ3[t]] θ2″[t] + l3 θ3″[t])

In[142]:= `eq33a = Series[eq33, {θ1[t], 0, 1}, {θ2[t], 0, 1}, {θ3[t], 0, 1}] // Normal // ExpandAll`

Out[142]= g l3 m3 θ3[t] - l1 l3 m3 θ1[t] θ1′[t]^2 + l1 l3 m3 θ3[t] θ1′[t]^2 - l2 l3 m3 θ2[t] θ2′[t]^2 + l2 l3 m3 θ3[t] θ2′[t]^2 + l1 l3 m3 θ1″[t] + l1 l3 m3 θ1[t] × θ3[t] θ1″[t] + l2 l3 m3 θ2″[t] + l2 l3 m3 θ2[t] × θ3[t] θ2″[t] + l3^2 m3 θ3″[t]

In[143]:= `eq33a = eq33a //. rule3 // Simplify`

Out[143]= l3 m3 (g θ3[t] + l1 θ1″[t] + l2 θ2″[t] + l3 θ3″[t])

In[144]:= `{eq31a, eq32a, eq33a} // TableForm`

Out[144]//TableForm=

l1 (g (m1 + m2 + m3) θ1[t] + l1 (m1 + m2 + m3) θ1″[t] + l2 m2 θ2″[t] + l2 m3 θ2″[t] + l3 m3 θ3″[t])
l2 (g (m2 + m3) θ2[t] + l1 (m2 + m3) θ1″[t] + l2 m2 θ2″[t] + l2 m3 θ2″[t] + l3 m3 θ3″[t])
l3 m3 (g θ3[t] + l1 θ1″[t] + l2 θ2″[t] + l3 θ3″[t])

Look at double pendulum case in matrix form

In[145]:= `eq21a`
`eq22a`

Out[145]= l1 (g (m1 + m2) θ1[t] + l1 (m1 + m2) θ1″[t] + l2 m2 θ2″[t])

Out[146]= l2 m2 (g θ2[t] + l1 θ1″[t] + l2 θ2″[t])

In[147]:=
```
mMat = {{D[eq21a, θ1''[t]], D[eq21a, θ2''[t]]},
    {D[eq22a, θ1''[t]], D[eq22a, θ2''[t]]}};
mMat // MatrixForm
```

Out[148]//MatrixForm=

$$\begin{pmatrix} l1^2 (m1 + m2) & l1\, l2\, m2 \\ l1\, l2\, m2 & l2^2\, m2 \end{pmatrix}$$

In[149]:=
```
kMat = {{D[eq21a, θ1[t]], D[eq21a, θ2[t]]},
    {D[eq22a, θ1[t]], D[eq22a, θ2[t]]}};
kMat // MatrixForm
```

Out[150]//MatrixForm=

$$\begin{pmatrix} g\, l1 (m1 + m2) & 0 \\ 0 & g\, l2\, m2 \end{pmatrix}$$

In[151]:=
```
values = {m1 → m, m2 → m, g → 1, l1 → len, l2 → len, len → 1};
```

In[152]:=
```
mMat //. values // MatrixForm
kMat //. values // MatrixForm
```

Out[152]//MatrixForm=

$$\begin{pmatrix} 2\, m & m \\ m & m \end{pmatrix}$$

Out[153]//MatrixForm=

$$\begin{pmatrix} 2\, m & 0 \\ 0 & m \end{pmatrix}$$

In[154]:=
```
eq = Det[kMat - w2 mMat] == 0 //. values
```

Out[154]= $2\, m^2 - 4\, m^2\, w2 + m^2\, w2^2 == 0$

In[155]:=
```
sol = Solve[eq, w2]
```

Out[155]= $\{\{w2 \to 2 - \sqrt{2}\}, \{w2 \to 2 + \sqrt{2}\}\}$

In[156]:=
```
vec = {a, b};
eq1 = (kMat - w2 mMat).vec == 0 /. sol[[1]] //. values
```

Out[157]= $\{-(2 - \sqrt{2})\, b\, m + a\, (2\, m - 2\, (2 - \sqrt{2})\, m), -(2 - \sqrt{2})\, a\, m + b\, (m - (2 - \sqrt{2})\, m)\} == 0$

In[158]:=
```
sol1 = Solve[eq1, {a, b}] // Simplify
```

••• Solve : Equations may not give solutions for all "solve" variables.

Out[158]= $\left\{\left\{b \to -\frac{2\,(-1 + \sqrt{2})\, a}{-2 + \sqrt{2}}\right\}\right\}$

In[159]:=
```
ev1 = {a, b} /. sol1[[1]] /. {a → 1}
```

Out[159]= $\left\{1, -\frac{2\left(-1+\sqrt{2}\right)}{-2+\sqrt{2}}\right\}$

In[160]:=
```
vec = {a, b};
eq2 = (kMat - w2 mMat).vec == 0 /. sol[[2]] //. values
```

Out[161]= $\left\{-\left(2+\sqrt{2}\right) b\, m + a\left(2 m - 2\left(2+\sqrt{2}\right) m\right), -\left(2+\sqrt{2}\right) a\, m + b\left(m - \left(2+\sqrt{2}\right) m\right)\right\} == 0$

In[162]:=
```
sol2 = Solve[eq2, {a, b}] // Simplify
```

••• Solve : Equations may not give solutions for all "solve" variables .

Out[162]= $\left\{\left\{b \to -\frac{2\left(1+\sqrt{2}\right) a}{2+\sqrt{2}}\right\}\right\}$

In[163]:=
```
ev2 = {a, b} /. sol2[[1]] /. {a → 1}
```

Out[163]= $\left\{1, -\frac{2\left(1+\sqrt{2}\right)}{2+\sqrt{2}}\right\}$

In[164]:=
```
ev1 // N
ev2 // N
```

Out[164]= {1., 1.41421}

Out[165]= {1., -1.41421}