1 & 2) PROBLEM

13.3 ⋆ Consider the Atwood machine of Figure 13.2, but suppose that the pulley is a uniform disc of mass M and radius R. Using x as your generalized coordinate, write down the Lagrangian, the generalized momentum p, and the Hamiltonian $\mathcal{H} = p\dot{x} - \mathcal{L}$. Find Hamilton's equations and use them to find the acceleration $\ddot{x}$.

13.4 ⋆ The Hamiltonian $\mathcal{H}$ is always given by $\mathcal{H} = pq - \mathcal{L}$ (in one dimension), and this is the form you should use if in doubt. However, if your generalized coordinate q is "natural" (relation between q and the underlying Cartesian coordinates is independent of time) then $\mathcal{H} = T + U$, and this form is almost always easier to write down. Therefore, in solving any problem you should quickly check to see if the generalized coordinate is "natural," and if it is you can use the simpler form $\mathcal{H} = T + U$. For the Atwood machine of Example 13.2 (page 527), check that the generalized coordinate was "natural." [*Hint:* There are one generalized coordinate x and two underlying Cartesian coordinates x and y. You have only to write equations for the two Cartesians in terms of the one generalized coordinate and check that they don't involve the time, so it's safe to use $\mathcal{H} = T + U$. This is ridiculously easy!]

3) PROBLEM

14.7 ⋆ Calculate the solid angles subtended by the moon and by the sun, both as seen from the earth. Comment on your answers. (The radii of the moon and sun are $R_m = 1.74 \times 10^6$ m and $R_s = 6.96 \times 10^8$ m. Their distances from earth are $d_m = 3.84 \times 10^8$ m and $d_s = 1.50 \times 10^{11}$ m.)

4) PROBLEM

14.11 ⋆⋆ The differential cross section for scattering 6.5-MeV alpha particles at 120° off a silver nucleus is about 0.5 barns/sr. If a total of 10^{10} alphas impinge on a silver foil of thickness 1 μm and if we detect the scattered particles using a counter of area 0.1 mm^2 at 120° and 1 cm from the target, about how many scattered alphas should we expect to count? (Silver has a specific gravity of 10.5, and atomic mass of 108.)

5) PROBLEM

For the exam problem 5b) we ASSUMED the scattering was isotropic as in example 14.5. For Rutherford scattering this is NOT correct as the correct relation is the differential cross section is below. Let's re-do this a bit more correctly.

a) Show that if we compute the total cross section using the Rutherford formula, we get infinity.

b) Plot the cross section as a function of angle.

c) We will now neglect scattering ± 10 Degrees. Calculate the total cross section for scattering between 10 and 180 degrees. We'll define this as our EFFECTIVE total cross section. Now compute the percentage of particles scattered in bins of 10 degrees; i.e. [10,20], [20,30]...[170,180]
Check that the total adds to 100%

The Rutherford Formula

The differential cross section for scattering a charge q off a fixed charge Q is given by the **Rutherford formula**

$$\frac{d\sigma}{d\Omega} = \left(\frac{kqQ}{4E\sin^2(\theta/2)}\right)^2. \qquad \text{[Eq. (14.32)]}$$

6) PROBLEM

a) The luminosity at the LHC is approximately L=10^{34} cm^{-2} sec^{-1}. Assume the Higgs cross section is 50 pb, where pb is pico-barns. compute the rate at which Higgs bosons are produced.
b) Assume the machine runs for a year, and it is operational 1/3'rd of the time. Compute the total number of Higgs bosons produced.
c) For comparison, the event rate (for any ordinary collision) at the LHC is 40MHz. Compute the number of such collisions at the LHC in a year, again assuming the machine is operational 1/3'rd of the time.
d) Take the ratio of "ordinary" events vs "Higgs" events. That means that out of every XXX events, there will be one Higgs event.
e) The event rate at the LHC is 40MHz. Compute the time between events, and the distance light will travel during this time.

Notes: Pico = 10^{-12}. 1 barn = 10^{-28}m^2

For reference, an atom is about 1 Angstrom = 10^{-10}m, a proton is about 1 Fermi = 10^{-15}m, and a nuclei is about 10 Fermi = 10^{-14}m; thus 1 barn is the cross sectional area of a nuclei 10^{-28}m^2.