Homework 11

Problem 3

In[1]:= **14.7** ⋆ Calculate the solid angles subtended by the moon and by the sun, both as seen from the earth. Comment on your answers. (The radii of the moon and sun are $R_m = 1.74 \times 10^6$ m and $R_s = 6.96 \times 10^8$ m. Their distances from earth are $d_m = 3.84 \times 10^8$ m and $d_s = 1.50 \times 10^{11}$ m.)

Out[1]= **14.7** ⋆ Calculate the solid angles subtended by the moon and by the sun, both as seen from the earth. Comment on your answers. (The radii of the moon and sun are $R_m = 1.74 \times 10^6$ m and $R_s = 6.96 \times 10^8$ m. Their distances from earth are $d_m = 3.84 \times 10^8$ m and $d_s = 1.50 \times 10^{11}$ m.)

In[2]:=
```
(* solid angle *)
Ω = area/d^2 /. {area → π r^2}
```

Out[2]= $\frac{\pi r^2}{d^2}$

In[3]:=
```
moon = Ω /. {d → 3.84 × 10^8, r → 1.74 × 10^6};
moon // ScientificForm
```

Out[4]//ScientificForm= 6.45039×10^{-5}

In[5]:=
```
sun = Ω /. {d → 1.5 × 10^11, r → 6.96 × 10^8};
sun // ScientificForm
```

Out[6]//ScientificForm= 6.76372×10^{-5}

In[7]:= $\frac{\text{moon}}{\text{sun}}$

Out[7]= 0.953674

Problem 4

In[8]:=
```
Clear["Global`*"]
```

14.11 ⋆⋆ The differential cross section for scattering 6.5-MeV alpha particles at 120° off a silver nucleus is about 0.5 barns/sr. If a total of 10^{10} alphas impinge on a silver foil of thickness 1 μm and if we detect the scattered particles using a counter of area 0.1 mm^2 at 120° and 1 cm from the target, about how many scattered alphas should we expect to count? (Silver has a specific gravity of 10.5, and atomic mass of 108.)

In[9]:=
```
ntar = ρ t / m /. {ρ → (10.5 × 10^3), t → (10^-6), m → (108 *(1.66 × 10^-27))}
```
Out[9]= 5.85676×10^{22}

In[10]:=
```
(* solid angle *)
Ω = area / d^2
```
Out[10]= $\frac{\text{area}}{d^2}$

In[11]:=
```
Ω = Ω /. {area → 0.1 × 10^-6, d → 1 × 10^-2}
```
Out[11]= 0.001

In[12]:=
```
Nsc = Nin ntar sig Ω /. {Nin → 10^10, sig → 0.5 × 10^-28}
```
Out[12]= 29.2838

Problem 5

5) PROBLEM

For the exam problem 5b) we ASSUMED the scattering was isotropic as in example 14.5. For Rutherford scattering this is NOT correct as the correct relation is the differential cross section is below. Let's re-do this a bit more correctly.

a) Show that if we compute the total cross section using the Rutherford formula, we get infinity.

b) Plot the cross section as a function of angle.

c) We will now neglect scattering ±10 Degrees. Calculate the total cross section for scattering between 10 and 180 degrees. We'll define this as our EFFECTIVE total cross section. Now compute the percentage of particles scattered in bins of 10 degrees; i.e. [10,20], [20,30]...[170,180]
Check that the total adds to 100%

The Rutherford Formula

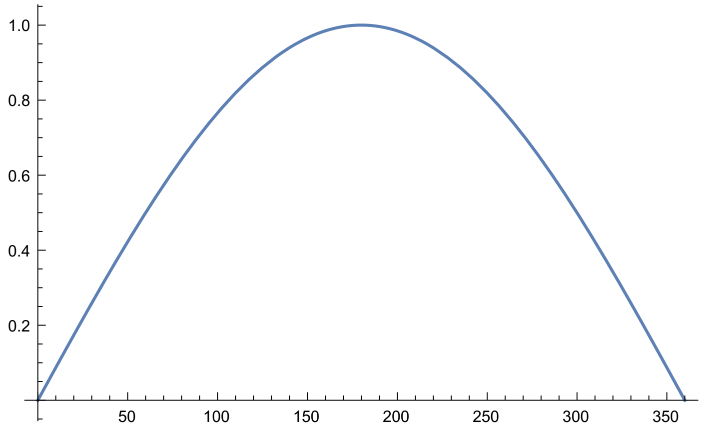
The differential cross section for scattering a charge q off a fixed charge Q is given by the **Rutherford formula**

$$\frac{d\sigma}{d\Omega} = \left(\frac{kqQ}{4E\sin^2(\theta/2)}\right)^2. \qquad \text{[Eq. (14.32)]}$$

In[58]:=
```
π / Degree * 1.
```
Out[58]= 180.

In[59]:= `Plot[Sin[θ / 2 Degree], {θ, 0, 360}]`

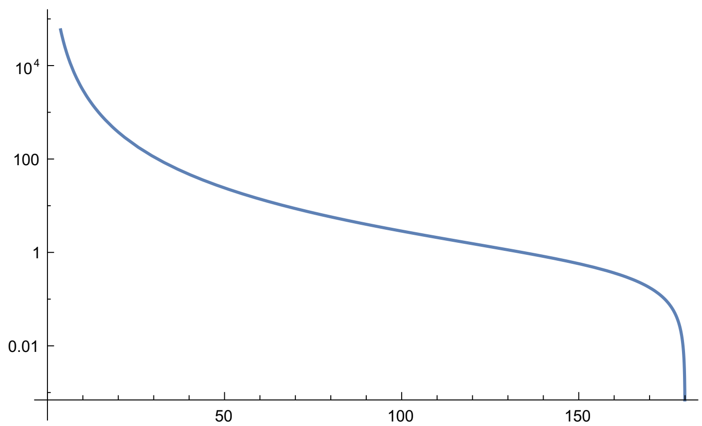
Out[59]=



In[60]:= $f[\theta\_] = \frac{1}{\text{Sin}[\theta / 2\ \text{Degree}]^4};$

In[61]:= `LogPlot[f[θ] Sin[θ Degree], {θ, 0, 180}]`

Out[61]=



In[62]:= $\text{tmp} = \text{Integrate}\left[\frac{\text{Sin}[\theta]}{\text{Sin}[\theta / 2]^4}, \theta\right]$

Out[62]= $-2\ \text{Csc}\left[\frac{\theta}{2}\right]^2$

In[63]:= `Limit[tmp , θ → π]`

Out[63]= -2

In[64]:= `Series[tmp , {θ, 0, 1}]`

Out[64]= $-\frac{8}{\theta^2} - \frac{2}{3} + O[\theta]^2$

In[65]:= `int[a_, b_] := NIntegrate[f[θ] Sin[θ Degree], {θ, a, b}, {ϕ, 0, 2 π}]`

In[66]:= `total = int[10, 180]`

Out[66]= 94 065.2

In[67]:=
```
Table[xint[a, a + 10], {a, 10, 170, 10}]
```

Out[67]=
```
{xint[10, 20], xint[20, 30], xint[30, 40], xint[40, 50], xint[50, 60], xint[60, 70],
 xint[70, 80], xint[80, 90], xint[90, 100], xint[100, 110], xint[110, 120], xint[120, 130],
 xint[130, 140], xint[140, 150], xint[150, 160], xint[160, 170], xint[170, 180]}
```

In[68]:=

$$tab = Table\left[\frac{int[a, a+10]}{total}, \{a, 10, 170, 10\}\right]$$

Out[68]=
```
{0.753812 , 0.139577 , 0.0488309 , 0.022578 , 0.0122385 , 0.00735111 ,
 0.00474052 , 0.00321691 , 0.00226499 , 0.00163646 , 0.00120139 , 0.000887057 ,
 0.000650372 , 0.000464442 , 0.000311571 , 0.000179393 , 0.0000585878 }
```

In[69]:=
```
labels = Table[a, {a, 10, 170, 10}]
```

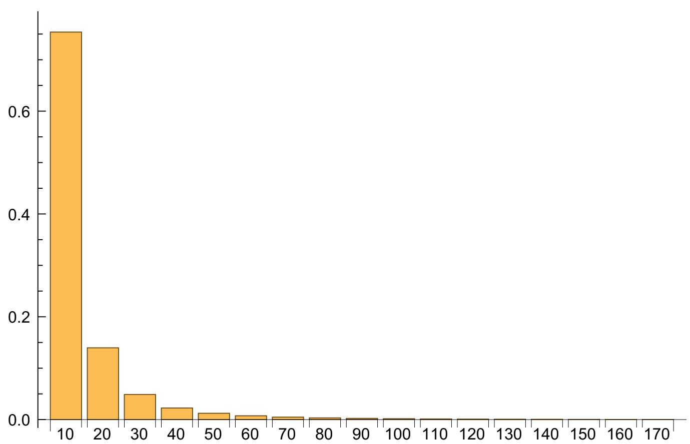
Out[69]=
```
{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170}
```

In[70]:=
```
Length /@ {labels, tab}
```

Out[70]=
```
{17, 17}
```

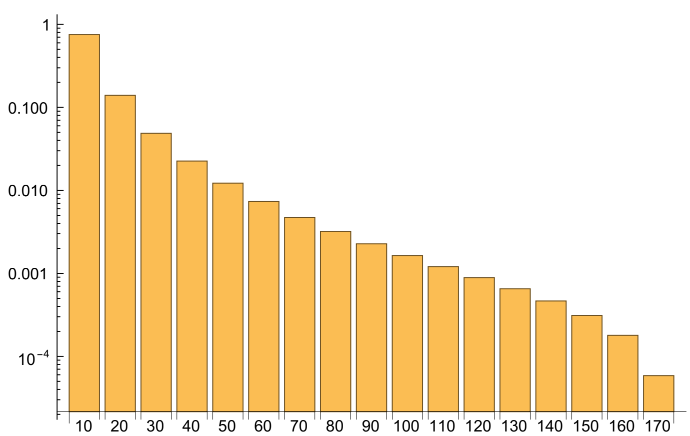
In[71]:=
```
BarChart[tab, ScalingFunctions → "Linear", ChartLabels → labels]
```

Out[71]=



In[72]:=
```
BarChart[tab, ScalingFunctions → "Log", ChartLabels → labels]
```

Out[72]=



Problem 5
Repeat for Hard Sphere

In[73]:=
```
Clear["Global`*"]
```

5) PROBLEM
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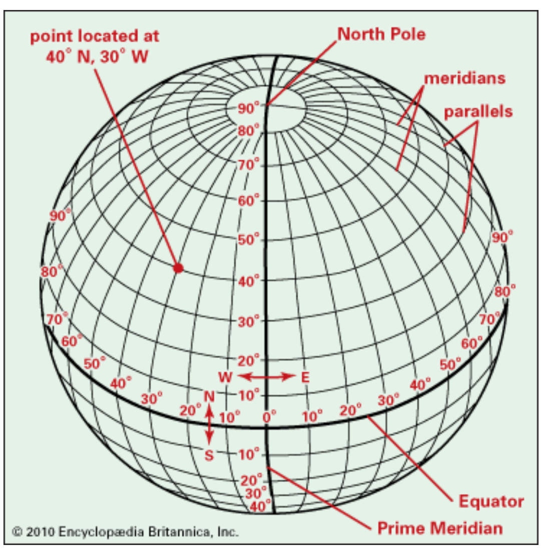
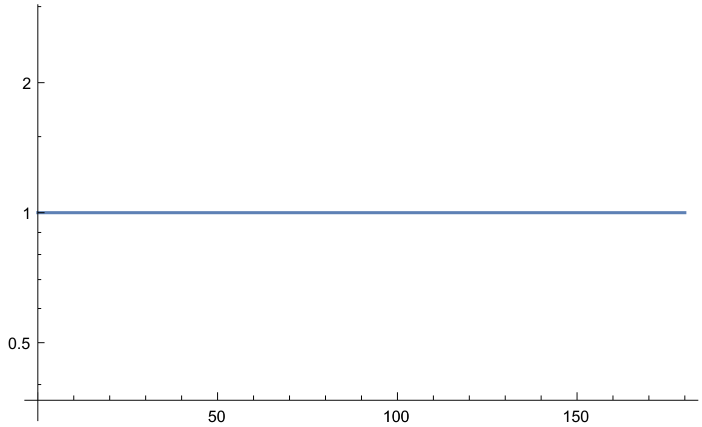
In[74]:=
```
π / Degree * 1.
```

Out[74]=
```
180.
```

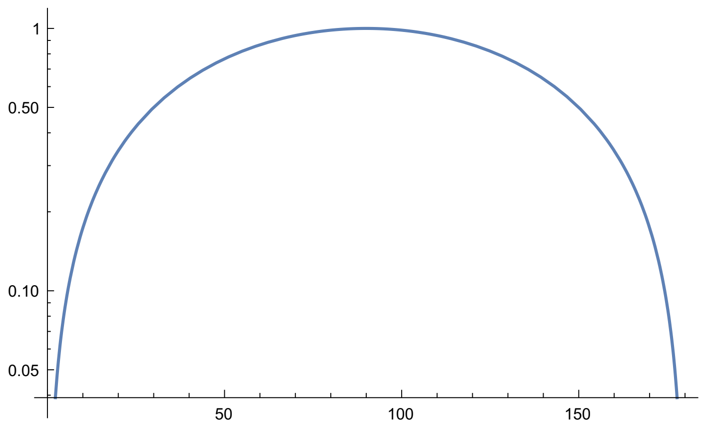
In[75]:=
```
f[θ_] = 1;
```



In[76]:=
```
LogPlot[f[θ], {θ, 0, 180}]
```

Out[76]= 

In[77]:=
```
LogPlot[f[θ] Sin[θ Degree], {θ, 0, 180}]
```

Out[77]= 

In[78]:=
```
tmp = Integrate[Sin[θ], {θ, 0, π}, {ϕ, 0, 2 π}]
```

Out[78]=
```
4 π
```

In[79]:=
```
int[a_, b_] := NIntegrate[f[θ] Sin[θ Degree], {θ, a, b}, {ϕ, 0, 2 π}]
```

In[80]:=
```
total = int[0, 180]
```

Out[80]=
```
720.
```

In[81]:=
```
Table[xint[a, a + 10], {a, 00, 170, 10}]
```

Out[81]=
```
{xint[0, 10], xint[10, 20], xint[20, 30], xint[30, 40], xint[40, 50],
 xint[50, 60], xint[60, 70], xint[70, 80], xint[80, 90], xint[90, 100],
 xint[100, 110], xint[110, 120], xint[120, 130], xint[130, 140],
 xint[140, 150], xint[150, 160], xint[160, 170], xint[170, 180]}
```

In[82]:=
```
tab = Table[int[a, a + 10] / total, {a, 00, 170, 10}]
```

Out[82]=
```
{0.00759612, 0.0225576, 0.0368336, 0.0499905, 0.0616284, 0.0713938,
 0.0789899, 0.084186, 0.0868241, 0.0868241, 0.084186, 0.0789899,
 0.0713938, 0.0616284, 0.0499905, 0.0368336, 0.0225576, 0.00759612}
```

In[83]:=
```
labels = Table[a, {a, 00, 170, 10}]
```

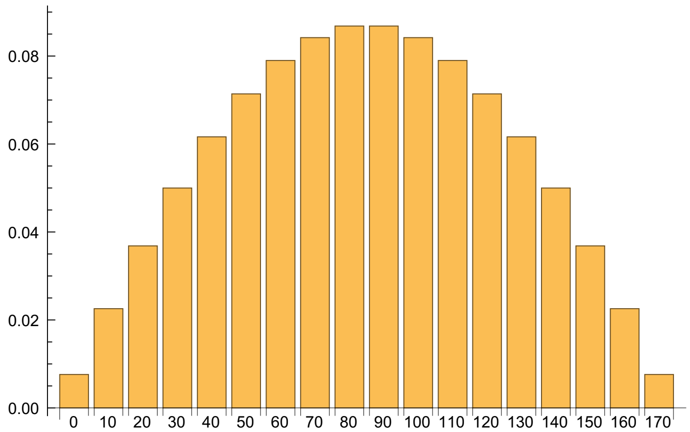
Out[83]=
```
{0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170}
```

In[84]:=
```
Length /@ {labels, tab}
```

Out[84]=
```
{18, 18}
```

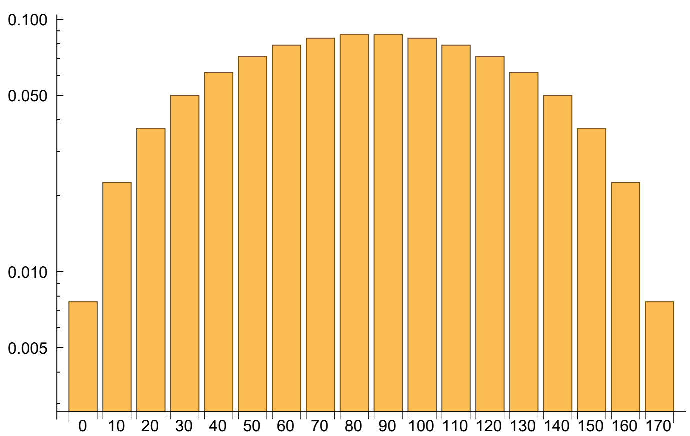
In[85]:=
```
BarChart[tab, ScalingFunctions → "Linear", ChartLabels → labels]
```

Out[85]=



In[86]:=
```
BarChart[tab, ScalingFunctions → "Log", ChartLabels → labels]
```

Out[86]=



Problem 6

In[126]:=
```
Clear["Glboal`*"]
```

6) PROBLEM

a) The luminosity at the LHC is approximately L=10^{34} cm^{-2} sec^{-1}. Assume the Higgs cross section is 50 pb, where pb is pico-barns. compute the rate at which Higgs bosons are produced.
b) Assume the machine runs for a year, and it is operational 1/3'rd of the time. Compute the total number of Higgs bosons produced.
c) For comparison, the event rate (for any ordinary collision) at the LHC is 40MHz. Compute the number of such collisions at the LHC in a year, again assuming the machine is operational 1/3'rd of the time.
d) Take the ratio of "ordinary" events vs "Higgs" events. That means that out of every XXX events, there will be one Higgs event.
e) The event rate at the LHC is 40MHz. Compute the time between events, and the distance light will travel during this time.

Notes: Pico = 10^{-12}. 1 barn = 10^{-28}m^2

For reference, an atom is about 1 Angstrom = 10^{-10}m, a proton is about 1 Fermi = 10^{-15}m, and a nuclei is about 10 Fermi = 10^{-14}m; thus 1 barn is the cross sectional area of a nuclei 10^{-28}m^2.

Part a

In[127]:= $\text{lum} = 10.^{34}\,\frac{1}{\text{cm}^2}\,\frac{1}{\text{sec}}$

Out[127]= $\frac{1.\times 10^{34}}{\text{cm}^2\,\text{sec}}$

In[128]:= `higgsCS = 50. pb //. {pb → 10^-12 barn, barn → 10^-28 m^2, m → 10^2 cm}`

Out[128]= $5.\times 10^{-35}\,\text{cm}^2$

In[129]:= `hrate = lum higgsCS`

Out[129]= $\frac{0.5}{\text{sec}}$

In[130]:= `1 / hrate`

Out[130]= 2. sec

Part b

In[131]:= `UnitConvert[Quantity[1, "Years"], "Seconds"]`

Out[131]= 31 536 000 s

In[132]:= $\text{oneYear} = 1\,\text{year}\left(\frac{365.25\,\text{days}}{1\,\text{year}}\right)\left(\frac{24\,\text{hour}}{1\,\text{days}}\right)\left(\frac{60\,\text{min}}{1\,\text{hour}}\right)\left(\frac{60\,\text{sec}}{1\,\text{min}}\right)$

Out[132]= $3.15576\times 10^{7}\,\text{sec}$

In[133]:=
```
hPerYear = 1/3 hrate oneYear;
hPerYear // EngineeringForm
```

Out[134]//EngineeringForm=

5.2596×10^{6}

Part c

In[135]:=
```
rate = 40. × 10^6 1/sec
```

Out[135]= $\frac{4. \times 10^{7}}{\text{sec}}$

In[136]:=
```
normalPerYear = 1/3 rate oneYear
```

Out[136]= 4.20768×10^{14}

Part d

In[137]:=
```
normalPerYear/hPerYear // EngineeringForm
```

Out[137]//EngineeringForm=

$80. \times 10^{6}$

Part e

In[138]:=
```
rate
```

Out[138]= $\frac{4. \times 10^{7}}{\text{sec}}$

In[139]:=
```
rate // EngineeringForm
```

Out[139]//EngineeringForm=

$\frac{40. \times 10^{6}}{\text{sec}}$

```
1 / rate
```

Out[140]= 2.5×10^{-8} sec

In[141]:=
```
1 / rate // EngineeringForm
```

Out[141]//EngineeringForm=

$(25. \times 10^{-9})$ sec

In[142]:=
```
c = 3.0 × 10^8 meter/sec;
```

In[143]:= $\frac{c}{rate}$

Out[143]= 7.5 meter

In[144]:= $\frac{c}{rate}\left(\frac{3.3\ feet}{1\ meter}\right)$

Out[144]= 24.75 feet