Problem 1

In[51]:=
```
Clear["Global`*"]
```

Vertical falling:

In[52]:=
```
eq1 = m v'[t] == m g - b v[t] - c v[t]^2
```

Out[52]= $m\,v'[t] == g\,m - b\,v[t] - c\,v[t]^2$

In[53]:=
```
bc = v[0] == v0
```

Out[53]= v[0] == v0

In[54]:=
```
eq2 = eq1 /. {v'[t] → 0}
```

Out[54]= $0 == g\,m - b\,v[t] - c\,v[t]^2$

In[55]:=
```
eq2b = eq2 /. {c → 0}
```

Out[55]= 0 == g m - b v[t]

In[56]:=
```
eq2c = eq2 /. {b → 0}
```

Out[56]= $0 == g\,m - c\,v[t]^2$

In[57]:=
```
bSol = Solve[eq2b, v[t]][[1]]
```

Out[57]= $\left\{v[t] \to \frac{g\,m}{b}\right\}$

In[58]:=
```
cSol = Solve[eq2c, v[t]][[2]]
```

Out[58]= $\left\{v[t] \to \frac{\sqrt{g}\,\sqrt{m}}{\sqrt{c}}\right\}$

In[59]:=
```
values = {b → β D, c → γ D^2, β → 1.6×10^-4, γ → 0.25};
```

In[60]:=
```
vb = v[t] /. bSol //. values /. {g → 10, m → 100 , D → 1}
```

Out[60]= 6.25×10^6

In[61]:=
```
vc = v[t] /. cSol //. values /. {g → 10, m → 100 , D → 1}
```

Out[61]= 63.2456

In[62]:=
```
Needs["Units`"]
```

In[63]:=
```
Convert[vb Meter / Second, Mile / Hour]
```

Out[63]= $\frac{1.39809 \times 10^7\ \text{Mile}}{\text{Hour}}$

In[64]:=
```
Convert[vc Meter / Second , Mile / Hour]
```
Out[64]=
$$\frac{141.476\ \text{Mile}}{\text{Hour}}$$

Problem 2

Horizontal

In[122]:=
```
Clear["Global`*"]
```
In[123]:=
```
(* To start, we'll zero gravity  *)
eq1 = m v '[t] == m g - b v[t] - c v[t]^2 /. {g → 0}
```
Out[123]= $m\, v'[t] == -b\, v[t] - c\, v[t]^2$

In[124]:=
```
bc = v[0] == v0 (* Boundary Conditions *)
```
Out[124]= v[0] == v0

In[125]:=
```
dsol1 = DSolve[{eq1 , bc} , v[t], t][[1]] // FullSimplify
```
••• Solve : Inverse functions are being used by Solve , so some solutions may not be found ; use Reduce for complete solution information .

Out[125]=
$$\left\{v[t] \to \frac{b\, v0}{-c\, v0 + e^{\frac{b\,t}{m}}\,(b + c\, v0)}\right\}$$

In[126]:=
```
(* Look at ONLY LINEAR term *)
dsol2 = DSolve[{eq1 , bc} /. {c → 0}, v[t], t][[1]]
```
Out[126]=
$$\left\{v[t] \to e^{-\frac{b\,t}{m}}\, v0\right\}$$

In[127]:=
```
(* Look at ONLY QUADRATIC term *)
dsol3 = DSolve[{eq1 , bc} /. {b → 0}, v[t], t][[1]]
```
Out[127]=
$$\left\{v[t] \to \frac{m\, v0}{m + c\, t\, v0}\right\}$$

In[128]:=
```
v[t] /. dsol1
```
Out[128]=
$$\frac{b\, v0}{-c\, v0 + e^{\frac{b\,t}{m}}\,(b + c\, v0)}$$

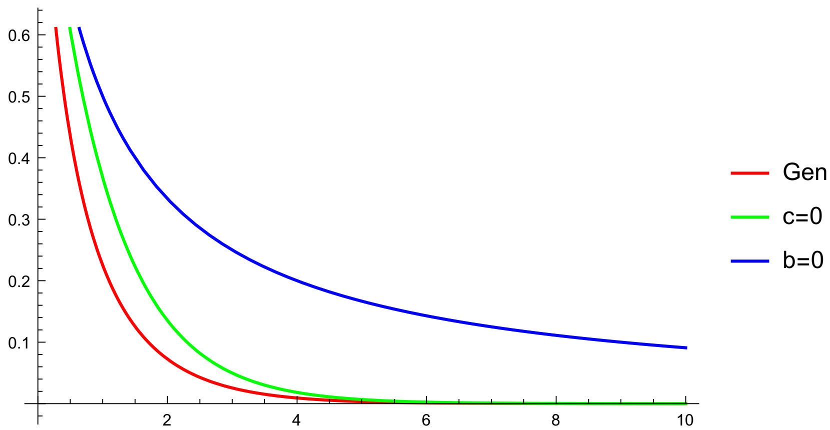
In[129]:=
```
v[t] /. dsol2
```
Out[129]=
$$e^{-\frac{b\,t}{m}}\, v0$$

In[130]:=
```
v[t] /. dsol3
```
Out[130]= $\frac{m\, v0}{m + c\, t\, v0}$

In[131]:=
```
(* Pick some values *)
values = {v0 → 1, m → 1};
```

In[132]:=
```
v[t] /. dsol1 /. values
```
Out[132]= $\frac{b}{-c + (b + c)\, e^{b\, t}}$

In[133]:=
```
tmp1 = v[t] /. {dsol1, dsol2, dsol3} /. values
```
Out[133]= $\left\{\frac{b}{-c + (b + c)\, e^{b\, t}},\ e^{-b\, t},\ \frac{1}{1 + c\, t}\right\}$

In[134]:=
```
Plot[tmp1 /. {b → 1, c → 1} // Evaluate, {t, 0, 10},
 PlotStyle → {Red, Green, Blue},
 PlotLegends → LineLegend[{Red, Green, Blue}, {"Gen", "c=0", "b=0"}]]
```
Out[134]=


In[135]:=
```
fun[b_, c_] = v[t] /. {dsol1, dsol2, dsol3} /. values
```
Out[135]= $\left\{\frac{b}{-c + (b + c)\, e^{b\, t}},\ e^{-b\, t},\ \frac{1}{1 + c\, t}\right\}$

In[136]:=
```
? Manipulate
```

Symbol

Out[136]=

Manipulate [*expr*, {u, u_{min}, u_{max}}] generates a version of *expr* with controls added to allow interactive manipulation of the value of u.

Manipulate [*expr*, {u, u_{min}, u_{max}, du}] allows the value of u to vary between u_{min} and u_{max} in steps du.

Manipulate [*expr*, {{u, u_{init}}, u_{min}, u_{max}, …}] takes the initial value of u to be u_{init}.

Manipulate [*expr*, {{u, u_{init}, u_{lbl}}, …}] labels the controls for u with u_{lbl}.

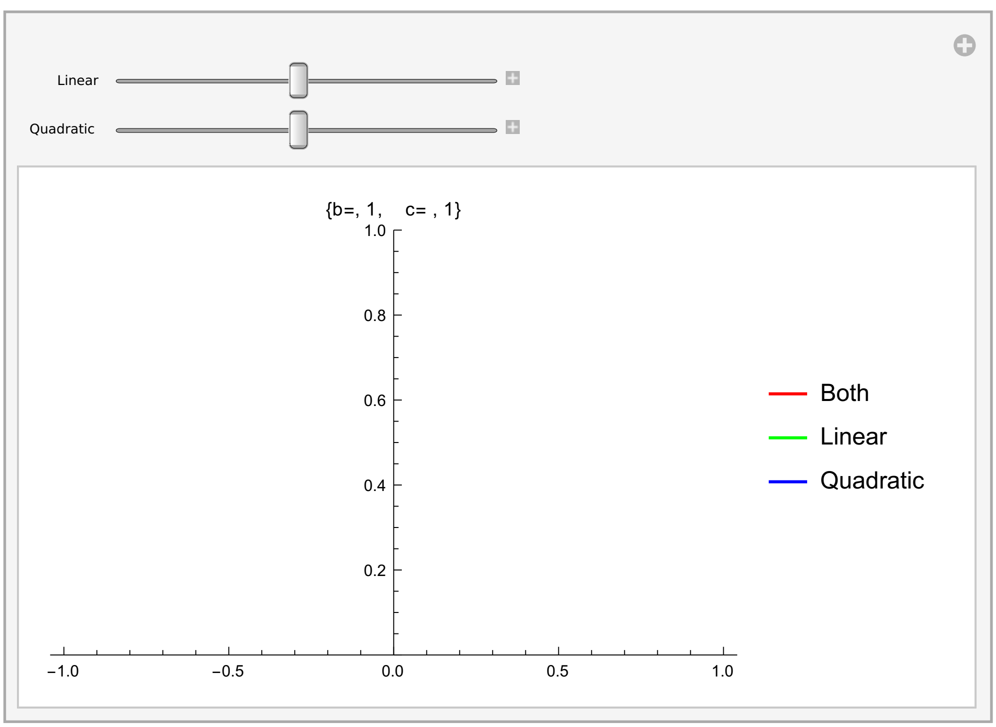
Manipulate [*expr*, {u, {u_1, u_2, …}}] allows u to take on discrete values u_1, u_2, ….

Manipulate [*expr*, {u, …}, {v, …}, …] provides controls to manipulate each of the u, v, ….

Manipulate [*expr*, c_u → {u, …}, c_v → {v, …}, …] links the controls to the specified controllers on an external device.

In[137]:=
```
Manipulate[
 Plot[fun[b, c] // Evaluate, {t, 0, 10},
  PlotStyle → {{Thickness[0.015], Red}, Green, Blue},
  PlotLabel → {"b=", b, "    c= ", c},
  PlotLegends → LineLegend[{Red, Green, Blue}, {"Both", "Linear", "Quadratic"}],
  PlotRange → {Automatic, {0, 1}}
 ]
 , {{b, 1, "Linear"}, 0.1, 2, 0.1}, {{c, 1, "Quadratic"}, 0.1, 2, 0.1}]
```

Out[137]=



Vertical
Now add gravity

In[138]:=
```
Clear["Global`*"]
```

In[139]:=
```
(* To start, we'll zero gravity  *)
eq1 = m v'[t] == m g - b v[t] - c v[t]^2
```

Out[139]= $m\, v'[t] == g\, m - b\, v[t] - c\, v[t]^2$

In[140]:=
```
bc = v[0] == v0 (* Boundary Conditions *)
```

Out[140]= v[0] == v0

In[141]:=
```
dsol1 = DSolve[{eq1, bc}, v[t], t][[1]] // FullSimplify
```

Solve : Inverse functions are being used by Solve , so some solutions may not be found ; use Reduce for complete solution information .

Out[141]= $$\left\{v[t] \to -\frac{b + \sqrt{-b^2 - 4\, c\, g\, m}\ \mathrm{Tan}\left[\frac{\sqrt{-b^2-4\, c\, g\, m}\ t}{2\, m} - \mathrm{ArcTan}\left[\frac{b+2\, c\, v0}{\sqrt{-b^2-4\, c\, g\, m}}\right]\right]}{2\, c}\right\}$$

In[142]:=
```
(* Look at ONLY LINEAR term *)
dsol2 = DSolve[{eq1, bc} /. {c → 0}, v[t], t][[1]] // FullSimplify
```

Out[142]= $$\left\{v[t] \to \frac{g\, m + e^{-\frac{b\, t}{m}}\, (-g\, m + b\, v0)}{b}\right\}$$

In[143]:=
```
(* Look at ONLY QUADRATIC term *)
dsol3 = DSolve[{eq1, bc} /. {b → 0}, v[t], t][[1]] // FullSimplify
```

Solve : Inverse functions are being used by Solve , so some solutions may not be found ; use Reduce for complete solution information .

Out[143]= $$\left\{v[t] \to \frac{\sqrt{g}\ \sqrt{m}\ \mathrm{Tanh}\left[\frac{\sqrt{c}\ \sqrt{g}\ t}{\sqrt{m}} + \mathrm{ArcTanh}\left[\frac{\sqrt{c}\ v0}{\sqrt{g}\ \sqrt{m}}\right]\right]}{\sqrt{c}}\right\}$$

In[144]:=
```
v[t] /. dsol1
```

Out[144]= $$-\frac{b + \sqrt{-b^2 - 4\, c\, g\, m}\ \mathrm{Tan}\left[\frac{\sqrt{-b^2-4\, c\, g\, m}\ t}{2\, m} - \mathrm{ArcTan}\left[\frac{b+2\, c\, v0}{\sqrt{-b^2-4\, c\, g\, m}}\right]\right]}{2\, c}$$

In[145]:=
```
v[t] /. dsol2
```

Out[145]= $$\frac{g\, m + e^{-\frac{b\, t}{m}}\, (-g\, m + b\, v0)}{b}$$

In[146]:=
```
v[t] /. dsol3
```

Out[146]=
$$\frac{\sqrt{g}\ \sqrt{m}\ \text{Tanh}\left[\frac{\sqrt{c}\ \sqrt{g}\ t}{\sqrt{m}} + \text{ArcTanh}\left[\frac{\sqrt{c}\ v0}{\sqrt{g}\ \sqrt{m}}\right]\right]}{\sqrt{c}}$$

In[147]:=
```
(* Pick some values *)
values = {m → 1, g → 1};
```

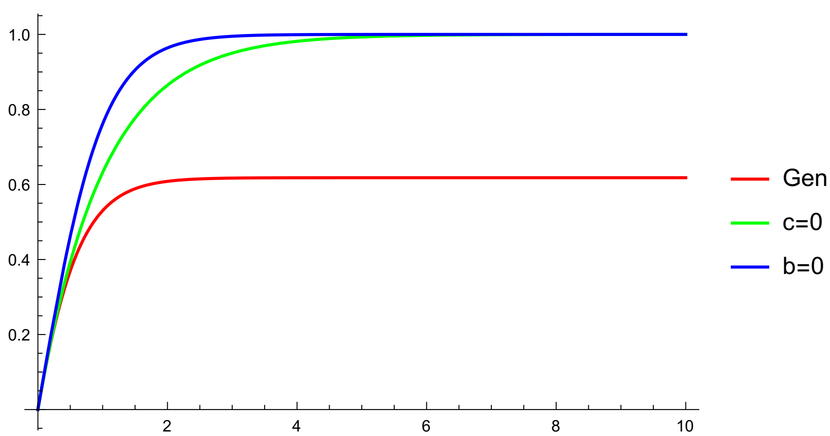
In[148]:=
```
v[t] /. dsol1 /. values
```

Out[148]=
$$-\frac{b + \sqrt{-b^2 - 4\,c}\ \text{Tan}\left[\frac{1}{2}\ \sqrt{-b^2 - 4\,c}\ t - \text{ArcTan}\left[\frac{b+2\,c\,v0}{\sqrt{-b^2-4\,c}}\right]\right]}{2\,c}$$

In[149]:=
```
tmp1 = v[t] /. {dsol1, dsol2, dsol3} /. values
```

Out[149]=
$$\left\{-\frac{b + \sqrt{-b^2 - 4\,c}\ \text{Tan}\left[\frac{1}{2}\ \sqrt{-b^2 - 4\,c}\ t - \text{ArcTan}\left[\frac{b+2\,c\,v0}{\sqrt{-b^2-4\,c}}\right]\right]}{2\,c},\ \frac{1 + e^{-b\,t}\,(-1 + b\,v0)}{b},\ \frac{\text{Tanh}\left[\sqrt{c}\ t + \text{ArcTanh}\left[\sqrt{c}\ v0\right]\right]}{\sqrt{c}}\right\}$$

In[150]:=
```
Plot[tmp1 /. {v0 → 0} /. {b → 1, c → 1} // Evaluate, {t, 0, 10},
 PlotStyle → {Red, Green, Blue},
 PlotLegends → LineLegend[{Red, Green, Blue}, {"Gen", "c=0", "b=0"}]]
```

Out[150]=


In[151]:=
```
fun[b_, c_, v0_] = v[t] /. {dsol1, dsol2, dsol3} /. values
```

Out[151]=
$$\left\{-\frac{b + \sqrt{-b^2 - 4\,c}\ \text{Tan}\left[\frac{1}{2}\ \sqrt{-b^2 - 4\,c}\ t - \text{ArcTan}\left[\frac{b+2\,c\,v0}{\sqrt{-b^2-4\,c}}\right]\right]}{2\,c},\ \frac{1 + e^{-b\,t}\,(-1 + b\,v0)}{b},\ \frac{\text{Tanh}\left[\sqrt{c}\ t + \text{ArcTanh}\left[\sqrt{c}\ v0\right]\right]}{\sqrt{c}}\right\}$$

In[152]:=
```
? Manipulate
```

Symbol

Out[152]=

Manipulate [$expr$, {u, u_{min}, u_{max}}] generates a version of $expr$

with controls added to allow interactive manipulation of the value of u.

Manipulate [$expr$, {u, u_{min}, u_{max}, du}] allows the value of u

to vary between u_{min} and u_{max} in steps du.

Manipulate [$expr$, {{u, u_{init}}, u_{min}, u_{max}, …}] takes the initial value of u to be u_{init}.

Manipulate [$expr$, {{u, u_{init}, u_{lbl}}, …}] labels the controls for u with u_{lbl}.

Manipulate [$expr$, {u, {u_1, u_2, …}}] allows u to take on discrete values u_1, u_2, ….

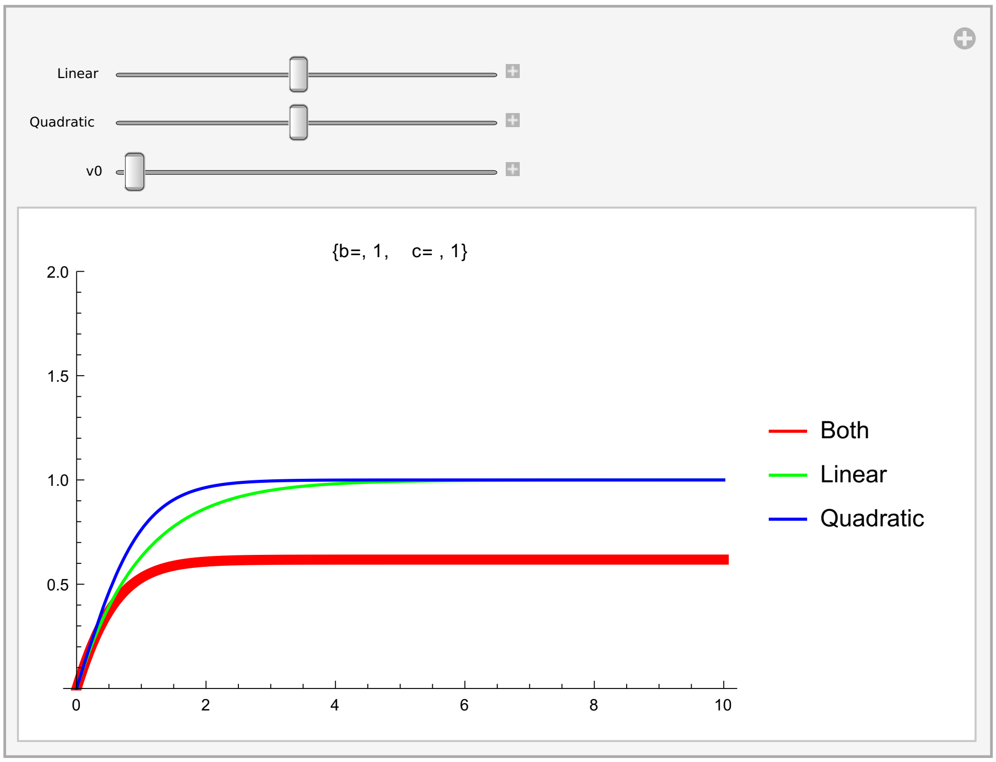
Manipulate [$expr$, {u, …}, {v, …}, …] provides controls to manipulate each of the u, v, ….

Manipulate [$expr$, $c_u \to$ {u, …}, $c_v \to$ {v, …}, …] links

the controls to the specified controllers on an external device .

In[153]:=
```
Manipulate[
 Plot[fun[b, c, v0] // Evaluate, {t, 0, 10},
  PlotStyle → {{Thickness[0.015], Red}, Green, Blue},
  PlotLabel → {"b=", b, "    c= ", c},
  PlotLegends → LineLegend[{Red, Green, Blue}, {"Both", "Linear", "Quadratic"}],
  PlotRange → {Automatic, {0, 2}}
 ], {{b, 1, "Linear"}, 0.1, 2, 0.1},
 {{c, 1, "Quadratic"}, 0.1, 2, 0.1},
 {{v0, 0}, 0, 2, 0.2}
]
```

Out[153]=



Problem 3

In[97]:=
```
Clear["Global`*"]
```

Part 1 :

In[98]:=
```
eq1 = {m a == m g Sin[θ] - μ normal, normal == m g Cos[θ]};
```

In[99]:=
```
eq1
```

Out[99]=
```
{a m == -normal μ + g m Sin[θ], normal == g m Cos[θ]}
```

In[100]:=
```
sol = Solve[eq1, {a, normal}][[1]]
```
Out[100]= $\{a \to -g\,\mu\,\text{Cos}[\theta] + g\,\text{Sin}[\theta],\ \text{normal} \to g\,m\,\text{Cos}[\theta]\}$

In[101]:=
```
sol /. {θ → π / 2}
```
Out[101]= $\{a \to g,\ \text{normal} \to 0\}$

In[102]:=
```
work = μ normal d /. sol
```
Out[102]= $d\,g\,m\,\mu\,\text{Cos}[\theta]$

In[103]:=
```
vSol = Solve[2 a d == v^2, v][[2]]
```
Out[103]= $\{v \to \sqrt{2}\,\sqrt{a}\,\sqrt{d}\}$

In[104]:=
```
kinetic = (1 / 2) m v^2 /. vSol /. sol // Expand
```
Out[104]= $-d\,g\,m\,\mu\,\text{Cos}[\theta] + d\,g\,m\,\text{Sin}[\theta]$

Part 2 :

In[105]:=
```
eq = energy == m g h - μ normal d /. {h → d Sin[θ]} /. sol
```
Out[105]= $\text{energy} == -d\,g\,m\,\mu\,\text{Cos}[\theta] + d\,g\,m\,\text{Sin}[\theta]$

In[106]:=
```
eq = eq /. {energy → 1/2 m v^2}
```
Out[106]= $\frac{m\,v^2}{2} == -d\,g\,m\,\mu\,\text{Cos}[\theta] + d\,g\,m\,\text{Sin}[\theta]$

In[107]:=
```
Solve[eq, v]
```
Out[107]= $\{\{v \to -\sqrt{2}\,\sqrt{-d\,g\,\mu\,\text{Cos}[\theta] + d\,g\,\text{Sin}[\theta]}\},\ \{v \to \sqrt{2}\,\sqrt{-d\,g\,\mu\,\text{Cos}[\theta] + d\,g\,\text{Sin}[\theta]}\}\}$

Problem 5

In[108]:=
```
Clear["Global`*"]
```

In[109]:=
```
PlanetData[]
```
Out[109]= {Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune}

In[110]:=
```
mass = PlanetData["Earth", "Mass"]
```
Out[110]= 5.97×10^{24} kg

In[111]:=
```
rTmp = PlanetData["Earth", "Radius"]
```
Out[111]= 3958.761 mi

In[112]:= radius = UnitConvert [rTmp , "SI"]

Out[112]= 6371.009 km

In[113]:= radius = UnitConvert [radius , "Meter "]

Out[113]= 6.371009×10^{6} m

In[114]:= momInertia = (2 / 5) mass radius^{2}

Out[114]= 9.70×10^{37} kg m^{2}

In[115]:= day = UnitConvert [Quantity [1, "Days "], "Seconds "] // N

Out[115]= 86 400. s

In[116]:= omega = 2 π / day

Out[116]= 0.0000727221 per second

In[117]:= angMom = momInertia omega

Out[117]= 7.05142×10^{33} kg m^{2}/s

In[118]:= angAcc = omega / day

Out[118]= 8.4169×10^{-10} per second^{2}

In[119]:= torque = momInertia angAcc

Out[119]= 8.16137×10^{28} kg m^{2}/s^{2}

In[120]:= force = torque / radius

Out[120]= 1.28102×10^{22} kg m/s^{2}

In[121]:= UnitConvert [force , "Newton "]

Out[121]= 1.28102×10^{22} N

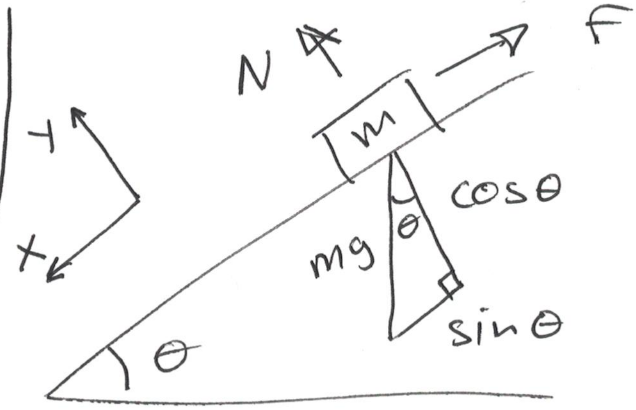
③ $F = ma$

Vertical: $N - mg\cos\theta = 0$

Horiz: $mg\sin\theta - F = ma$

Friction: $F = \mu N$

Unknowns: F, N, a



Solve: $a = g(\sin\theta - \mu\cos\theta)$

$N = gm\cos\theta \qquad F = \mu mg\cos\theta$

Work $=$ $W = Fd = \mu N d = \mu mg\cos\theta\, d$

Velocity: $2ad = v^2 - v_0^2$

$v = \sqrt{2ad}$

Energy: $E_{before} = E_{after}$

$U = K + W$

potential Kinetic Friction

$\frac{1}{2}mv^2 = K = U - W = mgh - \mu mg\cos\theta\, d$